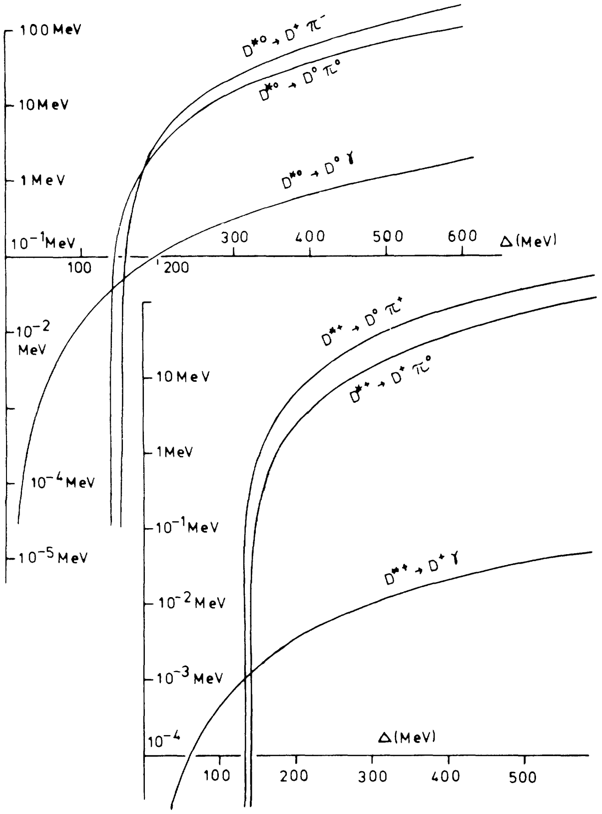

FIG. 1. Decay rates $\Gamma_{D^*\to D\pi}$ and $\Gamma_{D^*\to D\gamma}$ in the charmed quark model: $\Delta \equiv D^{*0}-D^0$.

For $\Delta > 200$ MeV one gets

$$\Gamma_{D^{*0}\to D^+\pi^-}:\Gamma_{D^{*0}\to D^0\pi^0}:\Gamma_{D^{*0}\to D^0\gamma} \simeq 1:0.5:10^{-2},$$
$$\Gamma_{D^{*+}\to D^0\pi^+}:\Gamma_{D^{*+}\to D^+\pi^0}:\Gamma_{D^{*+}\to D^+\gamma} \simeq 1:0.5:3\times 10^{-3}. \quad (17)$$

In view of the fact that, experimentally, Δ appears to be about 140 MeV, very nearly a pion mass, and is not measured with high accuracy, one may not be able to calculate the decay rates of $D^*\to D\pi$ very accurately.

The author would like to thank the Alexander von Humboldt Foundation for financial support.

---

*Alexander von Humboldt Foundation Fellow.

[1] V. Lüth, in Proceedings of the International Neutrino Conference, Aachen, West Germany, 1976 (to be published).

[2] A. De Rújula, in Proceedings of the International Neutrino Conference, Aachen, West Germany, 1976 (to be published).

[3] S. Ono, Nucl. Phys. B107, 522 (1976).

[4] S. Ono, to be published.

[5] D. B. Lichtenberg, Phys. Rev. D 12, 3760 (1975).

[6] A. Le Yaounac, L. Oliver, O. Pène, and J. C. Raynal, Nucl. Phys. B37, 552 (1972).

[7] S. Ono, Phys. Rev. D 9, 2005, 2670 (1974).

Gauge Theory of *CP* Nonconservation*

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 2 July 1976)

It is proposed that *CP* nonconservation arises purely from the exchange of Higgs bosons.

Ever since the discovery[1] that *CP* conservation is not exact, the mystery has been why it is so feebly violated.[2] In many proposed theories,[3] one must arrange to have *CP* to be approximately conserved, by making the appropriate constants in the Lagrangian to have sufficiently small values. However, one would prefer a more natural explanation.

Renormalizable gauge theories[4] of the weak and electromagnetic interactions provide a mechanism which could violate *CP* conservation with about the right strength: the Higgs boson. The coupling strength of a Higgs boson to a quark or lepton of mass m is of order $mG_F^{1/2}$ (where G_F is the Fermi coupling constant), so that the exchange of a Higgs boson of mass m_H produces an effective Fermi interaction with coupling of order $G_F m^2/m_H^2$. For reasonable mass values,[5] this is "milliweak." However, in order for the Higgs exchange to appear as a natural explanation for a feeble *CP* nonconservation, one must understand why *CP* conservation is strongly violated in the Higgs exchange, *and nowhere else*. In this paper, I wish to present a realistic gauge theory, in which *CP* nonconservation automatically arises in just this way.[6]

We assume an SU(2)⊗U(1) gauge theory,[4] with the usual four quarks[7]: $\mathcal{P}_1$ and $\mathcal{P}_2$ have charge $+\frac{2}{3}$

and $\mathcal{N}_1$ and $\mathcal{N}_2$ have charge $-\frac{1}{3}$. The left-handed quarks form two doublets $(\mathcal{P}_1, \mathcal{N}_2)$, while all right-handed quarks are SU(2) singlets. The only scalar bosons which can couple to the quarks then are the doublets $(\varphi_r^+, \varphi_r^0)$, and their charge conjugates. In order to insure that strangeness and charm conservation are not violated by φ^0 exchange, we must assume that there are *at most* two scalar doublets coupled to the quarks, with a Yukawa interaction of the form[8]

$$\mathcal{L}_Y = \sum_{i,j=1}^{2} \Gamma_{ij}^{(1)} \bar{\mathcal{N}}_{iR}(\varphi_1^{+*}\mathcal{P}_{jL} + \varphi_1^{0*}\mathcal{N}_{jL}) + \sum_{i,j=1}^{2} \Gamma_{ij}^{(2)} \bar{\mathcal{P}}_{iR}(\varphi_2^{0}\mathcal{P}_{jL} - \varphi_2^{+}\mathcal{N}_{jL}) + \text{H.c.} \tag{1}$$

[A label L or R denotes multiplication with $(1+\gamma_5)/2$ or $(1-\gamma_5)/2$.] In general, φ_2 and φ_1 could be charge conjugates, but I assume here that they are not.[9] The interaction will automatically take this general form, provided we assume that there is some sort of discrete symmetry which only allows φ_1 (φ_2) to couple to $\mathcal{N}_R$ ($\mathcal{P}_R$) quarks.

I am not yet imposing either exact or approximate CP conservation on the Lagrangian. In particular, the matrices $\Gamma_{ij}^{(r)}$ are still perfectly arbitrary. The φ_r^0 will have vacuum expectation values λ_r, yielding mass matrices which in general are neither diagonal nor even Hermitian. However, we can always define new quark fields

$$\begin{pmatrix} u \\ c \end{pmatrix}_{L,R} = U_{L,R}\mathcal{P}_{L,R}, \quad \begin{pmatrix} d \\ s \end{pmatrix}_{L,R} = V_{L,R}\mathcal{N}_{L,R} \tag{2}$$

(with unitary U and V matrices), so that the corresponding mass matrices are real and diagonal:

$$V_R\Gamma^{(1)}\lambda_1 V_L^{-1} = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}, \quad U_R\Gamma^{(2)}\lambda_2^* U_L^{-1} = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}. \tag{3}$$

Further, we can define the phases of the new quark fields so that the 2×2 matrix $U_L V_L^{-1}$ is real and orthogonal,

$$U_L V_L^{-1} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}. \tag{4}$$

The gauge interactions can therefore be rewritten in terms of the usual Cabibbo-mixed doublets, so that they automatically conserve CP whether or not we assume that CP is a good symmetry of the Lagrangian or the vacuum.[10]

The Yukawa interaction (1) can now be rewritten in terms of the new quarks. Using Eqs. (2)–(4), I find

$$\begin{aligned}\mathcal{L}_Y = {} & (\lambda_1^*)^{-1}\varphi_1^{0*}(m_d\bar{d}_R d_L + m_s\bar{s}_R s_L) + \lambda_2^{-1}\varphi_2^{0}(m_u\bar{u}_R u_L + m_c\bar{c}_R c_L) \\ & + (\lambda_1^*)^{-1}\varphi_1^{+*}[m_d\cos\theta_c\,\bar{d}_R u_L - m_d\sin\theta_c\,\bar{d}_R c_L + m_s\sin\theta_c\,\bar{s}_R u_L + m_s\cos\theta_c\,\bar{s}_R c_L) \\ & - \lambda_2^{-1}\varphi_2^{+}(m_u\cos\theta_c\,\bar{u}_R d_L + m_u\sin\theta_c\,\bar{u}_R s_L - m_c\sin\theta_c\,\bar{c}_R d_L + m_c\cos\theta_c\,\bar{c}_R s_L) + \text{H.c.}\end{aligned} \tag{5}$$

This interaction will also conserve CP, provided that one assigns to the scalar fields a suitable CP-transformation phase. [The CP properties of the quark fields are fixed by Eqs. (2)–(4).] However, for a wide class of theories, *CP will not be conserved by the scalar propagators.* In particular, the exchange of a single Higgs boson leads to an effective milliweak Fermi interaction

$$\begin{aligned} & -A(m_d\cos\theta_c\,\bar{d}_R u_L - m_d\sin\theta_c\,\bar{d}_R c_L + m_s\sin\theta_c\,\bar{s}_R u_L + m_s\cos\theta_c\,\bar{s}_R c_L) \\ & \quad \times (m_u\cos\theta_c\,\bar{u}_R d_L + m_u\sin\theta_c\,\bar{u}_R s_L - m_c\sin\theta_c\,\bar{c}_R d_L + m_c\cos\theta_c\,\bar{c}_R s_L) + \text{H.c.}\end{aligned} \tag{6}$$

with a single unknown parameter

$$A \equiv \langle T\{\varphi_1^{+*}\varphi_2^{+}\}\rangle_{0,q=0}/\lambda_1^*\lambda_2. \tag{7}$$

This interaction will violate CP conservation, because A is in general complex.

The nonconservation of CP can be easily checked for a fairly broad class of models. In order to preserve the rather successful SU(2)⊗U(1) predictions[4] for the strengths of neutral currents, I now assume that *all* the scalar fields form SU(2) doublets $(\varphi_r^+, \varphi_r^0)$. For simplicity, I also suppose that the Lagrangian is invariant under separate reflections under which any one of the φ_r (and perhaps some fermions) changes sign. Discrete symmetries of this kind are needed anyway to insure that it is only φ_1 and φ_2 that couple to $\mathfrak{N}_R$ and $\mathcal{P}_R$. The most general quartic polynomial that is invariant under these reflections and SU(2)⊗U(1) is of the form

$$P(\varphi)=\sum_r M_r^2(\varphi_r^\dagger\varphi_r)+\sum_{rs}a_{rs}(\varphi_r^\dagger\varphi_r)(\varphi_s^\dagger\varphi_s)+\sum_{rs}b_{rs}(\varphi_r^\dagger\vec{\tau}\varphi_r)(\varphi_s^\dagger\vec{\tau}\varphi_s)+\sum_{rs}c_{rs}(\varphi_r^\dagger\varphi_s)(\varphi_r^\dagger\varphi_s). \qquad (8)$$

Hermiticity requires that M_r^2 be real and a_{rs} and b_{rs} be real and symmetric, while c_{rs} need only be Hermitian.

For three or more φ doublets, this polynomial need not conserve CP. It is invariant under a CP transformation $\varphi_r \to \exp(i\theta_r)\varphi_r^*$ if and only if c_{rs} has phase $\pm\exp(i\theta_r - i\theta_s)$. For this to be possible for some set of θ_r, all the quantities $c_{rs}c_{st}c_{tr}$ would have to be real. A direct calculation for the case of three doublets shows that if $c_{12}c_{23}c_{31}$ is not real, then CP is actually not conserved, and the quantity (7) is complex. Further, even if $P(\varphi)$ *is* required to conserve CP, it can be shown that there will always (for three or more doublets) be at least a finite range of the parameters a, b, c, and M for which CP conservation is spontaneously broken.

Let us now consider the experimental implications of this sort of theory:

(1) The Higgs exchange produces a CP-nonconserving milliweak interaction with $\Delta S=1$, $\Delta C=0$. Inspection of (6) shows that $\Delta I=\frac{1}{2}$ terms dominate by large factors $m_c/m_{u,d}$, as required[11] if a milliweak interaction is to imitate the successful predictions of the superweak theory. The effect of terms involving c quarks may be dynamically suppressed, but if we ignore such effects, the dominant interaction here is $Am_s m_c \sin\theta_c \cos\theta_c \bar{s}_R c_L \bar{c}_R \times d_L$. In addition, there is a superweak interaction produced by exchange of a Higgs plus a W boson; this contribution to $\bar{s}d \to \bar{d}s$ (at energies and momentum transfers of order m_s) is less than the usual $2W$ contribution by a factor of order $m_s^2 A/G_\mathrm{F}$. It is not clear how to use these remarks to calculate observable parameters, but if we take (6) at face value, we would expect $m_s m_c \times \mathrm{Im}A/G_\mathrm{F}$ to be of the order of the phase of $K^0 \to 2\pi$ (relative to $\langle \bar{K}^0|M|K^0\rangle^{1/2}$), or 3.25×10^{-3}. If we also assume a maximal CP violation then $\mathrm{Im}A \approx |A| \approx G_\mathrm{F}/m_\mathrm{H}^2$; with $m_c=3m_s=1500$ MeV, this gives $m_\mathrm{H} \approx 15$ GeV, not an unreasonable value.[5]

(2) There is also a CP- and P-nonconserving milliweak interaction with $\Delta S=\Delta C=0$, so that one expects the neutron to have a detectable electric dipole moment. To get a rough idea of its magnitude, we can neglect the strong interactions, and calculate the electric dipole moments of the d and u quarks produced by the virtual Higgs exchange. The results[12] for $m_\mathrm{H}^2 \gg m_\mathrm{quark}$ are

$$D_d=\frac{em_d\mathrm{Im}A}{24\pi^2}\left(m_u^2\cos^2\theta_c \ln\frac{m_\mathrm{H}^2}{m_u^2}+m_c^2\sin^2\theta_c \ln\frac{m_\mathrm{H}^2}{m_c^2}\right),$$

$$D_u=-\frac{em_u\mathrm{Im}A}{48\pi^2}\left(m_d^2\cos^2\theta_c \ln\frac{m_\mathrm{H}^2}{m_d^2}+m_s^2\sin^2\theta_c \ln\frac{m_\mathrm{H}^2}{m_s^2}\right).$$

For illustration I take $m_u \approx m_d \approx 300$ MeV, $m_s \approx 500$ MeV, and $m_c \approx 1500$ MeV. Using previous estimates $\mathrm{Im}A \approx 3.2\times10^{-3}G_\mathrm{F}/m_s m_c$ and $m_\mathrm{H} \approx 15$ GeV, I find $D_d \simeq 1.6\times10^{-24}$ e·cm and $D_u \simeq -0.5\times10^{-24}$ e·cm. A simple quark-model calculation then gives the neutron electric dipole moment as $(4D_d - D_u)/3$, or about 2.3×10^{-24} e·cm. The most recent experimental result[13] is $(0.4\pm1.1)\times10^{-24}$ e·cm. In view of the great uncertainties surrounding the calculation, it would be premature to call this a real contradiction, but any appreciable further improvement in the precision of these measurements will be likely to confirm or refute the existence of a milliweak interaction of the type predicted here.

(3) By applying to the leptons the same considerations as to the quarks, we can infer that the leptons will have CP-conserving gauge interactions, and a CP-nonconserving Higgs interaction of the form

$$\mathcal{L}_Y' = (\lambda_3^*)^{-1}m_\mu(\varphi_3^{0*}\bar{\mu}_R\mu_L + \varphi_3^{+*}\bar{\mu}_R\nu_{\mu L}) + \mathrm{H.c.}$$

(Higgs couplings are negligible for electron-type leptons.) Here φ_3 may or may not be independent of φ_1 and φ_2. It is tempting to suppose that φ_3 is the third independent doublet needed to allow CP violation, but this is not necessary. In general the quantities $\langle T\{\varphi_{1,2}^{+*}, \varphi_3^{+}\}\rangle/\lambda_{1,2}^*\lambda_3$ will be complex, so that there will be a semileptonic CP-nonconserving milliweak interaction. This will produce small triple-scalar-product correlations in muonic decay modes. In particular, the Higgs exchange would produce an effective f_- form factor in $K_{\mu3}$ decay, with $\mathrm{Im}(f_-/f_+)$ being of order 10^{-3}. The observation of such a CP violation in $K_{\mu3}$ decay, together with the nonobservation of similar CP-violating effects in electronic processes such as the β decay, would be a clear indication that it is the Higgs bosons that are responsible for CP violation.

(4) The acceptability of this theory of CP violation could be affected by the experimental evidence on the number of quark flavors. At present, the situation is unclear. Even if it were found that new quarks must be introduced in order to account for bumps in e^+e^- or $\mu^+\mu^-$ channels or to cancel Bell-Jackiw anomalies associated with new heavy leptons, the essential features of the present theory could still easily be preserved, provided that there is some unbroken conservation law which prevents mixing of the additional quarks with u, d, s, and c. The present theory would, however, be ruled out if the reported anomalies in high-energy antineutrino-nucleon scattering were found definitely to be associated with new quarks or right-handed currents.

Suppose that Higgs exchange really does turn out to be the source of CP violation. Gauge theories would then provide a very attractive interpretation of all the inversions. It may be that none of them, C, P, T, or CP, is an *a priori* symmetry of the Lagrangian. The fact that the strong and electromagnetic interactions conserve C, P, and T would be explained by the circumstance that the right- and left-handed fermions happen to furnish isomorphic representations of the unbroken gauge subgroups of color and charge.[14] The fact that the weak interactions conserve CP approximately would be explained by the circumstances that there are four quarks, and that the quarks are lighter than the Higgs bosons. The polynomial part of the Lagrangian would violate CP conservation strongly, because there is no reason for it to behave otherwise.

I am grateful for valuable conversations with many colleagues, especially Alvaro De Rújula, Howard Georgi, Sheldon Glashow, Roman Jackiw, Ben Lee, T. D. Lee, and Gerard 't Hooft.

---

*Work supported in part by the National Science Foundation under Grant No. MPS75-20427.

[1] J. H. Christensen, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. 13, 138 (1964).

[2] The various possible interpretations of the small $(K_L\to 2\pi)/(K_S\to 2\pi)$ ratio are discussed by T. D. Lee and L. Wolfenstein, Phys. Rev. 138, B1490 (1965).

[3] Gauge theories of CP nonconservation are reviewed by R. N. Mohapatra, in *Particles and Fields—1974*, edited by C. E. Carlson, AIP Conference Proceedings No. 23 (American Institute of Physics, New York, 1974), p. 127.

[4] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Physics*, edited by N. Svartholm (Almquist and Wiksells, Stockholm, 1968).

[5] The masses of the Higgs bosons are unknown, but they are expected to be comparable to those of the intermediate vector bosons if the scalar self-coupling is comparable with the gauge couplings. In one class of theories, there is one "light" Higgs boson with mass 7–10 GeV, but this boson's couplings necessarily conserve CP; see E. Gildener and S. Weinberg, Phys. Rev. D 13, 3333 (1976). There is in any case a lower bound of order 5 GeV on the Higgs boson masses; see S. Weinberg, Phys. Rev. Lett. 36, 294 (1976); A. D. Linde, to be published.

[6] The idea that the Higgs bosons are responsible for a milliweak CP violation was suggested a few years ago by T. D. Lee, Phys. Rev. D 8, 1226 (1973), and Phys. Rep. 9C, 143 (1974). There are several significant differences between Lee's work and the present paper: (i) Lee developed his ideas most fully in the context of a "modified Georgi-Glashow" model, which was subsequently ruled out by the discovery of neutral currents. He also considered a general class of $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ models, but not specifically with four quarks, so that the problem of strangeness-changing neutral currents was left unresolved. In particular, Lee attributed CP violation to the exchange of the *neutral* Higgs bosons, which as shown here is only possible if the neutral Higgs exchange is allowed to violate strangeness and/or charm conservation. (ii) In some of the models considered by Lee, the CP violation does *not* occur only in the Higgs exchange; it also occurs in the gauge couplings of light to heavy quarks. Such couplings can produce large CP-violation effects involving only the light quarks when the strong interactions are taken into account; see, e.g., H. Fritzsch and P. Minkowski, California Institute of Technology Report No. CALT-68-537 (to be published). (iii) Lee assumed that CP is a good symmetry of the Lagrangian, while the present paper leaves open the possibility that CP is not an *a priori* symmetry at all.

[7] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970). The incorporation of a fourth quark in renormalizable gauge theories was suggested by S. Weinberg, Phys. Rev. D 5, 1412 (1972).

[8] S. L. Glashow and S. Weinberg, to be published. This paper points out that we do not yet know (but soon will!) whether neutral-current processes conserve charm, so it is possible that there is more than one scalar doublet coupled to the $\mathcal{P}_R$ quarks. This would complicate the theory developed here, but would not change its general features.

[9] If φ_2 is the charge conjugate of φ_1, then CP nonconservation can still occur in Higgs self-couplings, but not in the charged Higgs propagators. It is difficult to see in this case how the $K_L\to 2\pi$ decay could have its observed rate.

[10] This observation is originally due to M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973); see also L. Maiani, unpublished; and S. Pakvasa and

H. Sugawara, to be published. It has led some authors to suppose that there must be more than four quarks (or right-handed currents), to allow for *CP*-nonconserving gauge couplings. From the viewpoint of the present paper, it is hoped that there are *not* more than four quarks, to insure that *CP* violation arises only from Higgs exchange.

[11] T. T. Wu and C. N. Yang, Phys. Rev. Lett. 13, 180 (1964).

[12] These formulas are obtained by making suitable adjustments in the results of T. D. Lee, Ref. 6. Terms of order unity are neglected in comparison with the logarithms; in particular, m_H is an average Higgs mass, and I neglect logarithms of Higgs boson mass ratios.

[13] N. F. Ramsey, in *Proceedings of the 1975 Neutrino Conference* (Hungarian Physical Society, Budapest, 1975), Vol. I, p. 307.

[14] See, e.g., S. Weinberg, Phys. Rev. Lett. 31, 494 (1973). One possible objection to this viewpoint arises from the fact that it is possible to add a term to the strong interaction Lagrangian (the curl of the gluon field times its dual) that violates *CP* and *P* conservation. This term is formally a total divergence, but triangle anomalies may give it a finite effect; see G. 't Hooft, to be published; R. Jackiw and C. Rebbi, to be published; C. Callen, R. Dashen, and D. Gross, to be published.

Use of a Relativistic Oscillator Model to Explain the Sea-Gull Effect and Anomalous Charged-Lepton Production*

Louis S. Osborne
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 14 June 1976)

A relativistic oscillator model gives an expression for meson emission as a function of the fractional de-excitation α, $k_\perp^2$, and the meson mass. From this, a form for $\langle k_\perp^2\rangle$ in terms of z is derived; it agrees with electroproduction experiments. The same expression predicts enhanced single-photon and, thereby, anomalous charged-lepton pair production.

A relativistic oscillator model, ROM I, had been found to describe quantitatively many features of hadron-hadron[1] and lepton-hadron collisions.[2] I proceed to exploit its predictive powers to explain some old phenomena (the sea-gull effect)[3] and to predict new ones (anomalous photon and charged-lepton emission).

The model envisages the de-excitation of a hadron as proceeding through successive energy states, q_i, of a one-dimensional oscillator. The energy of the *i*th meson, k_i, is $q_{i-1}-q_i$.

In Ref. 1, I derived the differential probability, d^2P, describing each emission in terms of the square of the transverse momentum, $k_\perp^2$, given to the meson and the fractional energy, α, given to the meson.[4]

$$d^2P(1-\alpha)/d\alpha dk_\perp^2 = C(1-\alpha)^2\alpha^{-1}(1+\omega^2)^{-2}, \quad (1)$$

where

$$\omega = (k_\perp^2+m^2)/4\alpha m_0^2, \quad \alpha = k_0/q,$$

m is the mass of the emitted meson, and C and m_0^2 are constants of the model.[5]

In Ref. 1, I showed that this expression gives rise to an average $k_\perp^2$, which increases with the number of emissions which, in turn, increases logarithmically with the square of the center-of-mass energy s. Using a best fit to the data in this reference, I obtain

$$m_0 = 0.30 \pm 0.05 \text{ GeV}/c^2. \quad (2)$$

In the next section, I will simplify by taking $m=0$; this approximation is justified for π mesons.

From Eq. (1), one can obtain the average $\langle k_\perp^2(\alpha)\rangle$ per emission:

$$\langle k_\perp^2(\alpha)\rangle = \tfrac{16}{3}\pi\alpha m_0 = \alpha p_0^2. \quad (3)$$

I also calculate the emission probability, P', obtained by integrating Eq. (1) over $k_\perp^2$,

$$P' = dP/d\alpha = \pi m_0^2 C(1-\alpha)^2. \quad (4)$$

This is not directly comparable with the data. We must average (3) over all α's occurring over many events. Starting at an initial excitation energy, q_0, the hadron de-excites through successive states, q_i.

$$1 \rightarrow y_1 \rightarrow y_2 \cdots \rightarrow y_{n-1} \rightarrow y_n \equiv y,$$

$$y_i = q_i/q_0, \quad \alpha_i = k_i/q_{i-1} = 1 - y_i/y_{i-1}.$$

The average contribution to the mean-square