(ii) polarized and unpolarized proton elastic scattering via isobaric analog resonances. The former experiment is capable of yielding some measure of Z and $N+Z$ with the presently available targets, provided that certain experimental difficulties can be overcome. The latter experiment requires an appropriately enriched target. Calculations to predict the form of the proton resonances from our single particle spectra are in progress. An important consequence of the existence of naturally occurring superheavy nuclei is the possibility of investigating superheavy electromagnetic effects[14] by bombardment with medium mass projectiles such as Ag.

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[1]R. V. Gentry, T. A. Cahill, N. R. Fletcher, H. C. Kaufmann, L. R. Medsker, J. W. Nelson, and R. C. Flocchini, Phys. Rev. Lett. 37, 11 (1976).

[2]R. V. Gentry, Annu. Rev. Nucl. Sci. 23, 347 (1973).

[3]E. Rost, Phys. Lett. 26B, 184 (1968).

[4]J. Blomqvist and S. Wahlborn, Ark. Fys. 16, 545 (1959).

[5]S. G. Nilsson *et al.*, Nucl. Phys. A131, 1 (1969); E. O. Fiset and J. R. Nix, Nucl. Phys. A193, 647 (1972); J. Gruman *et al.*, Z. Phys. 228, 371 (1969).

[6]A. Sobiczewski *et al.*, Nucl. Phys. A168, 519 (1971); A. Lukasiak and A. Sobiczewski, Acta Phys. Pol. B 6, 147 (1975).

[7]D. Vautherin, M. Veneroni, and D. M. Brink, Phys. Lett. 33B, 381 (1970).

[8]W. D. Myers and W. J. Swiatecki, Nucl. Phys. 81, 1 (1966).

[9]V. M. Strutinsky, Nucl. Phys. A95, 420 (1967), and A122, 1 (1968).

[10]A. Bohr and B. R. Mottelson, Annu. Rev. Nucl. Sci. 23, 363 (1973).

[11]A. E. S. Green, *Nuclear Physics* (McGraw-Hill, New York, 1955).

[12]B. R. Doe, *Lead Isotopes* (Springer, New York, 1970).

[13]G. T. Seaborg, Annu. Rev. Nucl. Sci. 18, 53 (1968); B. Fricke, W. Greiner, and J. T. Waber, Theor. Chim. Acta 21, 235 (1971).

[14]B. Müller, J. Rafelski, and W. Greiner, Z. Phys. 257, 62, 183 (1972).

Reflectivity of Liquid ^{4}He Surfaces to ^{4}He Atoms

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We explain recent results on the reflectivity of liquid ^{4}He surfaces to externally incident ^{4}He atoms in terms of a Van der Waals interaction and strong coupling to quantized surface-tension waves which suppresses all other influences on the reflectivity. The model gives numerical agreement with experiment and it is found that a proper treatment of density variation at the liquid ^{4}He surface is essential.

In a recent experiment Edwards *et al.*[1] measured the reflection coefficient for ^{4}He atoms incident on the surface of liquid ^{4}He, the liquid being at 30 mK and the range of incident energies lying between 0.1 and 3.0 K relative to vacuum (or 7.26 and 10.16 K relative to the binding energy of ^{4}He in the liquid). It might be expected that a finite elastic, specular, reflection coefficient would be seen, less than unity because many of the incident atoms will lose energy to elementary excitations of the liquid, but rising to unity at low energies as the wavelength becomes long compared with the onset of the surface potential. At higher energies, in particular at the roton threshold,[2] a drop in the elastic reflectivity should be seen associated with the new channel for inelastic decay.

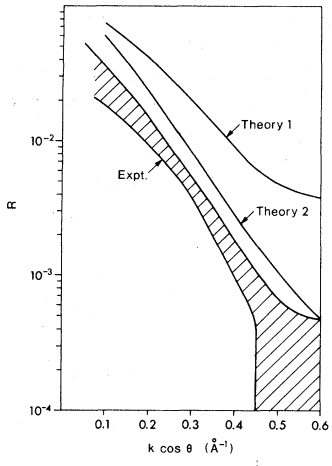

FIG. 1. The probability of specular reflection. The experimental results fall inside the shaded area shown. "Theory 1" assumes a step surface and "theory 2" a surface diffuse on a scale of 5 Å.

These expectations failed to materialize and a summary of the remarkable experimental results follows. (i) The probability of elastic reflection remained small even at small momenta: see Fig. 1. (ii) The inelastic and diffuse scattering probability was very small, less than the noise level in the experiment (2×10^{-3}) and much less than the elastic scattering probability. (iii) No structure was observed in the reflectivity near the roton threshold corresponding to an incident momentum of $k=0.5$ Å^{-1}. (iv) The elastic reflection coefficient, $R(k,\theta)$, within small deviations was a function only of the momentum perpendicular to the surface, $k\cos\theta$.

Edwards *et al.* noted in their paper that several quantities can in principle affect the reflectivity but we wish to suggest that just three suffice to explain the data: the Van der Waals interaction of the incident atom with the liquid,

$$-\lambda/Z^3, \tag{1}$$

the excitation of quantized surface-tension waves (ripplons),[3] and the thickness of the surface region.

In our model the incident ^{4}He atom first interacts with the weak tail of the Van der Waals potential. Elastic reflection takes place but is small because of the weakness of the tail. Next, through the Van der Waals interaction, the atom couples to the ripplons, and our calculations show that the coupling increases very rapidly from a negligible to a large value. Any atoms that penetrate beyond this region are almost certain to lose at least one quantum of energy to a ripplon and in fact will probably lose so many that they are almost inevitably captured in the attractive well of the liquid. Not only will elastic reflectivities be small, but also very few inelastically scattered atoms will lose sufficiently small amounts of energy to be seen outside the liquid again. All this takes place well removed from the physical surface of the helium. To excite a roton the atom must be able to tunnel into the liquid to find the required energy. Therefore rotons with their large quanta of energy will only be excited when the incident atom is very close to the surface and has already lost many quanta to the ripplons. Hence rotons have no effect on the elastically scattered flux.

In our calculation the ripplons are given a classical dispersion relationship

$$\omega^2 = q^3T/\rho, \tag{2}$$

where T is the surface tension and ρ the density of the liquid. More sophisticated descriptions have been given[4] which change things at large q, but because of the very rapid rise in the excitation probability on approaching the surface, our calculations are insensitive to such subtle considerations which merely serve to move the onset of ripplon excitations by a fractional amount. If the surface waves given rise to a displacement of the surface

$$\zeta = \Omega^{-1/2}A_{\vec{q}}\cos(\vec{q}\cdot\vec{r}), \tag{3}$$

where Ω is the surface area, we can write down a Lagrangian for the system:

$$\mathcal{L} = \sum_{\vec{q}}[(\rho/4q)(\dot{A}_{\vec{q}}^2 - \omega^2 A_{\vec{q}}^2)] - \sum_{\vec{q}}[\varphi_{\vec{q}}(\vec{R})A_{\vec{q}}] + [\tfrac{1}{2}m|\dot{\vec{R}}|^2 + \lambda Z^{-3}], \tag{4}$$

where m is the mass of ^{4}He and $\vec{R}$ is the position of the incident atom, the Z coordinate being perpendicular to the surface. The first term is the Lagrangian for a surface wave, the last term for a ^{4}He atom, and the middle term represents the coupling between them with

$$\varphi_{\vec{q}}(\vec{R}) = -(6\lambda/\pi)\int d^2r_{\parallel}\Omega^{-1/2}\cos(\vec{q}\cdot\vec{r})[|\vec{R}_{\parallel} - \vec{r}_{\parallel}|^2 + Z^2]^{-3}. \tag{5}$$

We have assumed a sharp termination of density at the surface.

To solve the problem quantum mechanically we invoke the Feynman-Hibbs[5] path-integral method

to write the probability amplitude for elastic reflection as a sum over trajectories. The Feynman propagator is written

$$F(x_2, x_1) = \int_{x_1}^{x_2} \exp[iS(x_2, x_1)/\hbar]\, \mathfrak{D}x(t), \quad (6)$$

where $x(t)$ represents values of the coordinates (both $\vec{R}$ and $A_{\vec{q}}$) at time t; the integral is taken to be a weighted sum over all possible classical trajectories between x_1 and x_2, and the action for a given trajectory is

$$S = \int_{t_1}^{t_2} \mathcal{L}(\dot{x}, x, t)\, dt. \quad (7)$$

We are interested only in the probability amplitude for propagating from x_1 to x_2 without exciting a ripplon; hence we define

$$\tilde{F}_1(\vec{R}_2, \vec{R}_1) = \langle \Psi_0 | F(x_2, x_1) | \Psi_0 \rangle = \int_{\vec{R}_1}^{\vec{R}_2} \exp[iS_1(\vec{R})/\hbar]\, \mathfrak{D}\vec{R}(t) \times \gamma(\vec{R}), \quad (8)$$

where $|\Psi_0\rangle$ is the ground state of the ripplon field, and S_1 is the action for the atom uncoupled from the ripplons, interacting only with the Van der Waals potential.

We now introduce the semiclassical approximations that as far as calculation of γ is concerned the dominant trajectories are those in which the particle moves in a straight line to and from the point of closest approach to the surface, Z_0, and that the particle traverses the region of interaction rapidly compared with the ripplon frequency. A further mathematically well-founded approximation in the integrations gives the result that γ and hence the final reflectivity depend only on K_z, the momentum of the particle normal to the surface:

$$\gamma = \exp(-\delta\epsilon/K_z^2 Z_0^{15/2}), \quad (9)$$

$$\delta = 9\lambda^2 m^2/4\pi\hbar^3(\rho T)^{1/2}, \quad (10)$$

$$\epsilon = \int_0^\infty dx\, x^{13/2} \int_x^\infty dy\, K_2(y)/y^2. \quad (11)$$

K_2 denotes the second-order modified Bessel function of the second kine, and ϵ was evaluated by numerical integration.

The remaining expression for $\tilde{F}_1$ was then solved exactly to give the reflectivity. Note that the calculation is not affected by the indistinguishability of target and projectile atoms: Exchange is only important if wave functions of target and particle overlap and we have assumed that the incident particle has very small probability of touching the liquid because otherwise the hard-core forces would enter the problem.

In Fig. 1 we compare the reflectivities calculated in this model with experiment. Although the results follow the correct trend they are too large by typically an order of magnitude.

Of the approximations mentioned above, we can see only one of them decreasing the reflectivity by an order of magnitude. Such considerations as interaction with bulk modes and modified dispersion relationships for the surface waves are all largely frustrated in their attempts to depress reflectivity by the very sharp cutting on of interaction which is an inevitable consequence of geometry combined with the Van der Waals force law. Only by making the surface diffuse can the reflectivities be sufficiently depressed. This has the effect of smoothing the $-\lambda Z^{-3}$ variation of the elastic potential outside the surface, and greatly reduces its reflecting powers. Changes made to the ripplon coupling are relatively minor in effect and are neglected.

How diffuse is the surface? Recent computer calculations[6] imply that the density goes to zero over a range of about 5 Å and we have taken a linear variation of density over this distance. The modified reflectivity curves are also shown in Fig. 1 and show much improved comparison with experiment. Being a little high they imply that the surface is perhaps even more diffuse, but we feel that the present level of agreement is within what can be expected of our model in any case.

The reproduction of the experimental results over two orders of magnitude variation makes us confident that the dominant ingredients in the problem are Van der Waals forces and surface-tension waves, while, for good measure, providing confirmation through experiment of the diffuseness of liquid ^{4}He surfaces.

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[1]D. O. Edwards, P. Fatouros, G. G. Ihas, P. Mrozinski, S. Y. Shen, F. M. Gasparini, and C. P. Tam, Phys. Rev. Lett. 34, 1153 (1975).

[2]P. W. Anderson, Phys. Lett. 29A, 563 (1968).

[3]K. R. Atkins, Can. J. Phys. 31, 1165 (1953).

[4]D. O. Edwards, J. R. Eckardt, and F. M. Gasparini, Phys. Rev. A 9, 2070 (1974).

[5]R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[6]K. S. Liu, M. H. Kalos, and G. V. Chester, Phys. Rev. B 12, 1715 (1975).