≈ 0.06] step variation of M at the diffusion region (besides the variation in K_1), we calculate that the intensities for $\vec{H}_a \parallel \langle 100\rangle$ axis are a factor of 100 below the intensity values calculated for no variation of M. However, for $\vec{H}_a \parallel \langle 110\rangle$, $I_1/I_0 \simeq 1.2\times10^{-2}$, $I_2/I_0 \simeq 7\times10^{-4}$, and $I_3/I_0 \simeq 1.1\times10^{-4}$. The significant point here is that even for small changes in the anisotropy and demagnetizing fields at the diffusion region from their respective bulk values, it may be possible to induce *large* angular variations in the spin-wave-mode intensities. Clearly, both K_1 and M must change in this region. However, this model is too simplistic to take seriously when comparing with the data in totality. We believe the model contains the essential features required to explain the remarkable angular variations of the intensities observed in $\{100\}$ and $\{110\}$ films.

In conclusion, the strong angular variation of the spin-wave-mode intensities is explained in terms of a nonuniform anisotropy field localized in a diffusion region rather than by an artificial surface field. We would like to thank Dr. J. J. Krebs for helpful discussions, Dr. J. Murday for the Auger-spectroscopy measurements, and Dr. R. Henry for kindly providing some of the LPE YIG films.

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COMMENTS

Coulomb Dissociation of Relativistic ^{12}C and ^{16}O Nuclei*

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The dissociation of relativistic ^{12}C and ^{16}O nuclei by the Coulomb fields of target nuclei has been inferred from the systematics of cross-section data. Coulomb contributions to the total fragmentation cross sections are interpreted by the Weizsäcker-Williams method. The minimum-impact parameters deduced by this method are characterized by radial overlap distances comparable to the charge-skin thicknesses of the interacting nuclei, compatible with the effects of nuclear absorption.

We report in this Letter experimental evidence for the dissociation of Bevatron/Bevalac beams of ^{12}C and ^{16}O in the nuclear Coulomb fields of target nuclei. This evidence comes from experiments on the target dependence of the isotopic production cross sections for secondary nuclei produced by the fragmentation of ^{12}C and ^{16}O beam nuclei at energies $E = 1.05$ GeV/n (^{12}C) and 2.1 GeV/n (^{12}C and ^{16}O).[1] By use of photonuclear cross-section data and the Weizsäcker-Williams (WW) method of virtual quanta,[2,3] we are able to account for the deduced cross sections and to determine the minimum impact parameters for Coulomb dissociation of heavy-ion projectiles.

Lindstrom *et al.*[1] have measured the isotopic production cross section σ_{BT}^{F} for the single-particle inclusive reaction $B + T \to F + \ldots$, where B, T, and F are the beam, target, and fragment nuclei, respectively. Essential to our analysis is that the cross sections $\sigma_{BT}{}^F$ are factorable, i.e., $\sigma_{BT}{}^F = \gamma_B{}^F \bar{\gamma}_T$, where $\gamma_B{}^F$ is dependent on B and F only, and $\bar{\gamma}_T$ is the target factor. Given in Ref. 1 are the measured cross sections $\sigma_{BT}{}^F$ and the factored quantities $\gamma_B{}^F$ and $\bar{\gamma}_T$ for all isotopes produced by the fragmentation of ^{12}C and ^{16}O projectiles in H, Be, C, Al, Cu, Ag, and Pb targets. Plotted in Fig. 1 are the target factors $\gamma_T = \sigma_{BT}{}^F/\gamma_B{}^F$ versus target mass A_T (amu). For fragment nuclei with mass $A_F \leq A_B - 2$, i.e., at least two nucleons are removed from the beam projectile, all isotopic production cross sections, for a given target, are interrelated by a unique target factor, $\bar{\gamma}_T$. Striking deviations of γ_T from $\bar{\gamma}_T$, up to 30% in Pb, are observed for those fragmentation cross sections that involve the loss of one nucleon from the projectile. The differences between the observed values of γ_T and $\bar{\gamma}_T$ increase approximately as $Z_T{}^2$ of the target, indicative of a Coulomb effect. We therefore attribute the target factors $\bar{\gamma}_T$ to nuclear fragmentation and the Z_T-dependent differences between γ_T and $\bar{\gamma}_T$ for fragments with mass $A_F = A_B - 1$ to Coulomb dissociation. The experimental Coulomb-dissociation cross sections are therefore defined as $\sigma_{WW}(\text{expt}) = \sigma_{BT}{}^F - \gamma_B{}^F\bar{\gamma}_T$, the difference between the measured and factored cross sections.

Jackson[2] presents a classical development of the WW method of virtual quanta for point charges

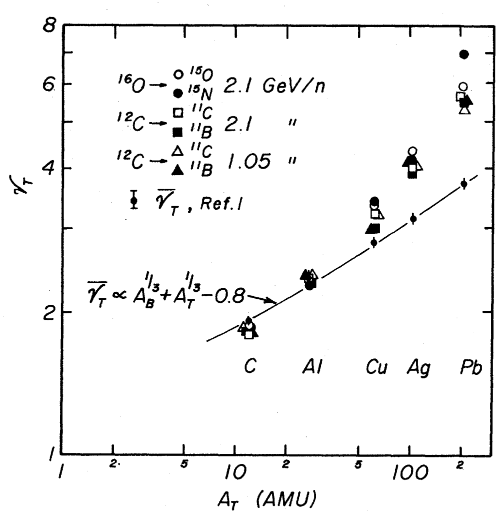

FIG. 1. Target factors γ_T plotted versus target mass A_T (amu), from Lindstrom *et al.* (Ref. 1). Individual values of γ_T are shown for the single-nucleon-loss cross sections indicated. The curve $\bar{\gamma}_T \propto A_B{}^{1/3} + A_T{}^{1/3} - 0.8$ is drawn through the mean target factors, shown with error bars, for all cross sections $\sigma_{BT}{}^F$ where $A_F \leq A_B - 2$.

moving at relativistic velocities. Jäckle and Pilkuhn[3] have extended the validity of the WW formula to nonrelativistic energies, and have incorporated nuclear absorption and charge form factors in the theory. The present analysis refines the work of Artru and Yodh,[4] who applied Jackson's treatment of the WW method to estimate the cross sections for Coulomb dissociation of relativistic nuclei.

To the extent that $N(\omega)$, the equivalent number of virtual photons per MeV, is the same for all electric and magnetic multipoles,[5] the WW cross section for the dissociation of a nucleus, at velocity β, by the Coulomb field of a target nucleus, atomic number Z, is given by

$$\sigma_{WW} = \int_{\omega_0} \sigma_\nu(\omega) N(\omega)\, d\omega, \tag{1}$$

where $\sigma_\nu(\omega)$ is the measured photonuclear cross section at photon energy ω. The number density of virtual photons has the functional form $N(\omega) = (Z^2/\omega\beta^2)F(\beta, \omega b_{\min}/\beta\gamma)$, where $b_{\min}$, the minimum-impact parameter, is the only adjustable parameter in σ_{WW}.

References to the photoneutron and photoproton cross sections we used to compute σ_{WW} are, for ^{12}C, $\sigma(\gamma, n)$,[6] $\sigma(\gamma, p)$[6,7]; and for ^{16}O, $\sigma(\gamma, n)$,[8] $\sigma(\gamma, p)$.[9] The cross section $\sigma(\gamma, p)$ for ^{12}C was obtained from the difference between $\sigma(\gamma, \text{total})$[7] and $\sigma(\gamma, n)$.[6] The cross-section data given by Fultz *et al.*,[10] Cook *et al.*,[11] Taran and Gorbunov,[12] Cook *et al.*,[13] and Gorbunov and Osipova[14] were used to extrapolate $\sigma_\nu(\omega)$ to higher values of $\omega_{\max}$ (to 65 MeV for ^{12}C and to 62 MeV for ^{16}O). Because the shape of the high-energy tail of $\sigma_\nu(\omega)$ has little effect on σ_{WW}, we have taken the extrapolated values of the cross sections to be constant.

The giant dipole resonance dominates the photonuclear reaction in the photon-energy interval from about 15 MeV (threshold) to 30 MeV. The photodissociation of ^{12}C and ^{16}O proceeds mainly by single-nucleon emission. Furthermore, contributions to σ_{WW} from the higher-threshold multinucleon-loss photoreactions are suppressed by the ω^{-1} weighting [from $N(\omega)$] of $\sigma_\nu(\omega)$ in Eq. (1). The experimental observation that only the single-nucleon-loss fragmentation cross sections exhibit significant deviations from strict factorization in high-Z targets is thus in accord with the process of Coulomb excitation and dissociation.

By equating $\sigma_{WW}(\text{expt})$ to σ_{WW}, Eq. (1), we have determined the impact parameter $b_{\min}$ appropriate for each cross section. The minimum impact parameter is defined by the relation $b_{\min} = r_{0.1}{}^B + r_{0.1}{}^T - d$, where the $r_{0.1}$'s are the 10% charge-

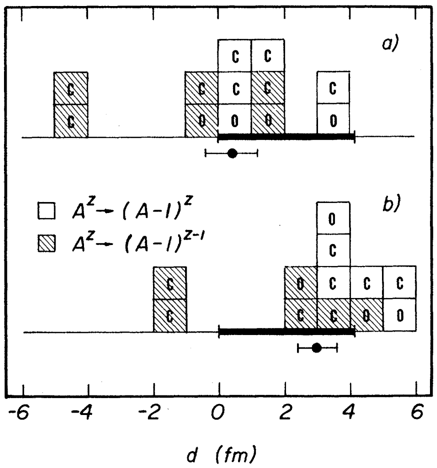

FIG. 2. Distributions of overlap distances $d(b_{\min})$, and their means, derived from σ_{WW}(expt) when fitted by the Weizsäcker-Williams cross sections σ_{WW}, as given (a) by Jackson (Ref. 2) and (b) by Jäckle and Pilkuhn (Ref. 3). The dark horizontal bar delineates the overlap region bounded by $0 \leq d \leq t_B + t_T$, the sum of the charge-skin thicknesses of the beam and target nuclei.

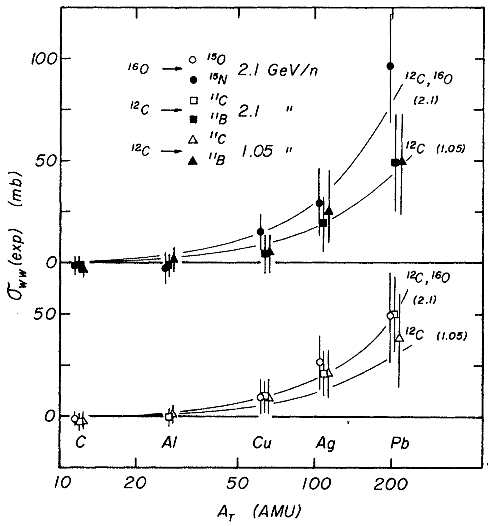

FIG. 3. Target dependence of the measured cross sections σ_{WW}(expt) for the Coulomb dissociated reactions indicated. The curves are computed using the Jäckle and Pilkuhn form of σ_{WW} with $\bar{d} = 3.0$ fm.

density radii of the beam and target nuclei,[15] and d is the radial-overlap distance. The values of $b_{\min}$ obtained in this experiment are, to within the accuracy of the data, confined to a limited range in d. Presented in Figs. 2(a) and 2(b), then, are histograms of the overlap distances d that account for the experimental cross sections σ_{WW}(expt) for ^{12}C and ^{16}O projectiles in Ag and Pb targets. Because of the differences in the theory for the spectra of virtual quanta, we present two distributions for d, each based upon the expressions for $N(\omega)$, hence σ_{WW}, given by Jackson[2] and by Jäckle and Pilkuhn.[3]

The standard deviations of the d distributions are compatible with the statistical errors in σ_{WW}(expt). Systematic variations in σ_{WW}(expt) are expected to be small, since the cross sections are obtained from quantities that are insensitive to errors in beam monitoring, background, focusing corrections, etc. Possible systematic errors in $d(b_{\min})$, other than those from the theoretical differences in σ_{WW}, are the photonuclear cross sections $\sigma_\nu(\omega)$ and those inherent in the method used to extract σ_{WW}(expt) from σ_{BT}^F. On the average, a 12% change in $\sigma_\nu(\omega)$, a typical uncertainty in the photonuclear cross-section data, leads to a 1-fm change in $d(b_{\min})$.

The unweighted mean (and its statistical error) of the d distributions are $\bar{d} = 0.4 \pm 0.8$ fm (Jackson) and 3.0 ± 0.6 fm (Jäckle and Pilkuhn). These mean values are shown in Fig. 2. Also included in this figure is the interval of overlap distances bounded by $0 \leq d \leq t_B + t_T$, where t_B and t_T are the charge-skin thicknesses of the beam and target nuclei, which, in this experiment, range from 1.9 to 2.3 fm.[15]

Figure 3 presents the cross-section data from this experiment, $\sigma_{WW}(\text{expt}) = \sigma_{BT}^F - \gamma_B^F \bar{\gamma}_T$, plotted as a function of target mass. Superimposed on the data are curves of the computed cross sections σ_{WW} (Jäckle and Pilkuhn) evaluated for a constant overlap distance $\bar{d} = 3.0$ fm. [Curves of σ_{WW} (Jackson) versus A_T evaluated for $\bar{d} = 0.4$ fm are indistinguishable from those shown.]

Following Lindstrom *et al.*,[1] we find that $\bar{\gamma}_T \propto (A_B^{1/3} + A_T^{1/3} - 0.8)$ gives an excellent fit to the target factors of σ_{BT}^F for $A_T \geq 12$, as illustrated in Fig. 1. When expressed in terms of $r_{0.1}$, the target factor has the form of an impact parameter, $\bar{\gamma}_T \propto (r_{0.1}^B + r_{0.1}^T - 2.0)$, where $r_{0.1} = r_{0.5} + t/2$ and $r_{0.5} = 1.18A^{1/3} - 0.48$.[15] Thus, we find that the effective overlap distance in $\bar{\gamma}_T$ is $d' = 2.0$ fm, a value that agrees well with the $\bar{d}$'s (0.4 and 3.0 fm) obtained in this analysis.

To summarize our results, all the salient features of σ_{WW}(expt) are attributable to the fragmentation of projectile nuclei by the Coulomb field of the target nucleus. Irrespective of the theoretical model,[2,3] use of the WW method to interpret σ_{WW}(expt) correctly accounts for (i) the identification of those isotope-production cross sections that are significantly enhanced by Coulomb dissociation, (ii) the target dependence of σ_{WW}(expt), and (iii) the magnitudes of σ_{WW}(expt). The energy dependence of σ_{WW}(expt) is within the errors of this experiment and verification of this feature will have to await further experiments. The values of $b_{\min}$ derived from σ_{WW}(expt) limit the radial overlap, d, of the colliding nuclei to distances comparable to their charge-skin thicknesses t, a manisfestation of the effects of nuclear absorption. The Coulomb and nuclear fragmentation processes are related by the results that $\bar{d} \approx d'$, which shows that the maximum overlap distance that accounts for Coulomb dissociation is, in essence, tantamount to the nuclear overlap distance required to account for nuclear (direct-interaction) fragmentation.

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Two-Electron, One-Photon Transition Energies

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Wölfli's experiment about $K^{-2} \to L^{-2}$ two-electron, one-photon transitions was criticized by Nagel *et al.* in a recent paper. In the present Comment I discuss arguments of Nagel *et al.* and show that Wölfli's interpretation about cooperative x-ray transition is valid.

Nagel *et al.*[1] have recently published a paper about the experiment of Wölfli *et al.*[2] on cooperative ($K^{-2} \to L^{-2}$) x-ray emission observation, asserting that the energy of the line observed by Wölfli *et al.* has not the correct energy to be the $K^{-2} \to L^{-2}$ transition. I present a Comment giving a value of this energy deduced from our experiments and asserting that the Nagel calculation cannot invalidate the Wölfli interpretation.

Nagel *et al.* correctly assumed that the energy