dent) nuclear levels are efficient predictors of IS, and are relatively easy to use.[2] However, in the continuum region, runs tests cannot be used to test for IS without first compensating for the effect of smoothing and for statistical correlations due to overlapping CN levels. Smoothing the data helps to reduce statistical fluctuations, but induces correlations which can and must be accounted for in the desired runs statistics.[20] Alternatively, one might dispense with smoothing altogether, model the correlation structure of the raw data, and then derive the appropriate runs statistics. Bateman, for example, has given a solution for runs above and below the median for one- and two-point Markoff processes.[21]

We are currently investigating modifications of runs tests which would make them more reliable as predictors of IS. However, our present results suggest the wisdom of a cautions approach, since statistical tests seldom speak unequivocally.

*Work performed in part under the auspices of the U. S. Energy Research and Development Administration.

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Possibility of Self-Consistent Long-Range Order in Nuclear Matter

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(Received 19 April 1976)

Hartree-Fock energies are calculated in nuclear matter for Overhauser-like orbitals and Skyrme forces as parametrized by Vautherin and Brink and others. In all cases considered, the plane-wave state is found to be more strongly bound.

The unproved assumption, implicitly made in practically all nuclear matter calculations, that plane-wave orbitals are the lowest-energy solutions to the Hartree-Fock equations in the thermodynamic limit, was critically examined by Overhauser in an early Letter.[1] Since this pioneering effort, a large number of papers[2] have appeared which analyze the stability, relative to plane waves, of non-plane-wave solutions implying some kind of long-range order (e.g., a spatially periodic single-particle density).

At least two important physical consequences of this new class of states can be mentioned: (a) the possibility of neutron-star matter crystallization under realistic forces,[3] probably at supra-nuclear densities, and (b) the apparent likelihood[4] of periodic α-cluster formation at the nuclear surface, i.e., at subnuclear densities. (The latter bears a striking analogy to some indications[5] that a system of classical random-close-packed hard spheres has a crystallinelike structure at its free surface.)

Overhauser's original study[1] involved a calculation yielding a so-called "density-ripple" state with 4 MeV *more* binding energy than the plane-wave state for nuclear matter at saturation density. This was done using a Karplus and Watson[6] nuclear force, which saturates nuclear matter, and which can be considered an early version of contemporary density-dependent, *effective*, Skyrme-type[7] interactions.

We consider the investigation of Ref. 1 to be of such relevance as to justify a similar study employing more modern versions of such a force. For this purpose we have used the Skyrme interaction with the set of parameters determined by Vautherin and Brink.[8] A periodic nuclear density resembling that of Overhauser was obtained by constructing a Slater determinant with the N single-particle wave functions, orthogonal in a cubic volume V and fourfold degenerate in spin and isospin,

$$\varphi_{\vec{k}}(\vec{r}) = \phi_{k_x}(x)\phi_{k_y}(y)\phi_{k_z}(z), \qquad (1)$$

$$\phi_{k_x}(x) \equiv D\{\exp(ik_x x) + \alpha \exp[i(k_x+q)x]\},$$

and the same for y and z,

$$-k_0 < k_x, k_y, k_z, < +k_0, \quad q \geq 2k_0,$$

$$D \equiv [(1+\alpha^2)V^{1/3}]^{-1/2},$$

where α is a real (variational) parameter, and $2k_0$ the side of the Fermi cube being filled in k space. These give rise to a global density ρ and a local "density ripple" $\rho(\vec{r})$ given, respectively, by

$$\rho = N/V = 4(k_0/\pi)^3,$$

$$\rho(\vec{r}) = \rho f(x) f(y) f(z), \qquad (2)$$

$$f(x) = 1 + [2\alpha/(1+\alpha^2)]\cos qx.$$

The set of orbitals defined by Eq. (1) were shown[9] to satisfy explicitly the full Hartree-Fock equations, for occupied states, for all α and $q \geq 2k_0$. The Hartree-Fock energy per particle for these orbitals and the above mentioned interaction can be shown to be

$$\frac{E(q,\beta;\rho)}{N} = \frac{\pi^2\hbar^2}{2\times 4^{2/3}m}\rho^{2/3}\left[1+12\left(\frac{q}{2k_0}\right)^2\frac{\beta}{1+\beta}\right] + \frac{3}{8}t_0\rho\left[1+\frac{2\beta}{(1+\beta)^2}\right]^3 + \frac{1}{16}t_3\rho^2\left[1+\frac{6\beta}{(1+\beta)^2}\right]^3$$
$$+ \frac{\pi^2}{4^{2/3}\times 16}(3t_1+5t_2)\rho^{5/3}\left[1+\frac{2\beta}{(1+\beta)^2}\right]^2\left[1+12\left(\frac{q}{2k_0}\right)^2\frac{\beta}{1+\beta}+\frac{2\beta}{(1+\beta)^2}\right] \qquad (3)$$
$$+ \frac{3\pi^2}{8\times 4^{2/3}}(9t_1-5t_2)\rho^{5/3}\left(\frac{q}{2k_0}\right)^2\frac{\beta}{(1+\beta)^2}\left[1+\frac{2\beta}{(1+\beta)^2}\right]^2,$$

where $\beta \equiv \alpha^2$, with parameters t_0, t_1, t_2, and t_3 given in Ref. 8. The coefficients in Eq. (3) which multiply terms containing $q/2k_0$ are non-negative (cf. Ref. 8); i.e., the energy increases monotically in $q/2k_0 \geq 1$. It is therefore minimized for Overhauser's choice $q = 2k_0$.

We have performed a numerical search in the remaining parameter β for fixed ρ, comparing $E(1,\beta;\rho)/N$ with the plane-wave value,

$$E_{\rm PW}(\rho)/N \equiv E(1,0;\rho)/N. \qquad (4)$$

Three sets of potential parameters were employed: (i) force I, (ii) force II, both from Ref. 8, and (iii) a simplified (less repulsive) version of force I where $t_1 = t_2 = 0$, which saturates nuclear matter for plane waves at $\rho = 0.164$ with a binding energy of -16.34 MeV; however, it overbinds the α particle by almost 100%.

For ρ we took the values given by

$$g\rho = 1,\ 2,\ 4,\ 8\ \text{fm}^{-3}, \qquad (5)$$

$$g = 10^4,\ 10^3,\ 10^2,\ 10,\ 1.$$

We sought extremal values β_0 given by

$$[\partial E(1,\beta;\rho)/\partial\beta]_{\beta=\beta_0} = 0 \qquad (6)$$

for densities given by Eq. (5). For ρ smaller than 10^{-2} fm^{-3} no extremum was found; for higher ρ, extrema appeared but corresponded to energies *exceeding* the associated plane-wave value $\beta = 0$ by more than 50 MeV.

The present negative results, however, can by no means be interpreted as ruling out the presence of self-consistent periodic states for nuclear matter which are more stable than the plane-wave one. For certain conditions and forces their existence is already unambiguously established by variational calculations[3,4] which produce lower energy states with long-range order.

We thus believe that, in the spirit of Overhauser's early work,[1] the search for different periodic solutions in nuclear matter should be continued.

*Work supported in part by Instituto Nacional de Energía Nuclear, México

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Comments on Primordial Superheavy Elements*

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(Received 25 June 1976)

A Woods-Saxon potential which reproduces the single particle levels in the lead region provides the basis for a discussion of the stability properties of nuclei in the superheavy region. A closed $N=228$ neutron shell is associated with the recent observation of nuclei with $Z=124$ and 126.

Strong evidence has recently been obtained[1] for the existence of naturally occurring superheavy elements. In particular, proton-induced x-ray analysis of monazite inclusions in biotite mica, characterized by giant halos,[2] indicates the presence of at least three superheavy elements with $Z=116$, 124, and 126. It is the main purpose of this note to show that a qualitative description of the major features of the observations can be obtained within the framework of conventional theory. Specifically, (a) we give arguments for the existence of adequately stable nuclei ($\tau_{1/2}\sim 10^6$–10^9 yr) with $Z\sim 114$, $Z\sim 126$, and $Z\sim 164$; (b) we speculate that the 10–15-MeV α radiation, presumably responsible for the formation of the giant halos,[2] may be due to the decay of elements with $Z\sim 164$; and (c) we discuss the chemical compatibility of these elements with the host material. A few suggestions are also given for the further study of elements in the superheavy region.

The stabilities of nuclei with respect to α decay, β decay, and fission can be discussed by standard methods. We consider first the single particle spectrum generated by a Woods-Saxon potential with parameters optimized for extrapolation into the region of superheavy nuclei. The parameters of our potential were obtained from a fit to the particle levels in ^{209}Pb and ^{209}Bi carried out by Rost.[3] The potential has a standard form[3] with depth given by (upper sign for protons)

$$V_0=51.6[1\pm 0.73(N-Z)/A]\ \mathrm{MeV}. \qquad (1)$$

The remaining parameters were held constant and are as given by Rost, namely $r_0=1.262$ fm, $r_{so}=0.908$ fm, $a=0.70$ fm, $\lambda=17.5$ for protons; $r_0=1.295$ fm, $r_{so}=1.194$ fm, $a=0.70$ fm, $\lambda=28.2$ for neutrons. Our approach differs from that of Rost in that Eq. (1) is used for scaling the potential depths into the superheavy region. This scaling procedure is consistent with the hypothesis of charge independence and also yields reasonable fits to observed particle levels in ^{120}Sn and ^{138}Ba. We also use a pairing correction term similar to that introduced by Blomqvist and Wahlborn[4] which improves the predicted energies of occupied levels in ^{208}Pb, ^{120}Sn, and ^{138}Ba.

We have calculated single particle levels for a variety of nuclei in the region $Z=108$–168 and $N=127$–312 using the above prescription. Strong shell closures are obtained for $N=184$, 228, and 308. Weaker proton shell closures occur in approximate correspondence with these neutron closures for $Z=114$, 126, and 164, respectively. Results for these particular cases are shown in Fig. 1. Other authors[5-7] have observed some of these same features but they have not discussed the important combination $Z=126$ and $N=228$. In