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Runs Tests as Predictors of Intermediate Structure in the Continuum*

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Chance coherences between S-matrix elements at neighboring compound-nucleus excitation energies in the continuum are shown to generate spurious intermediate structures and statistical correlations in synthetic analyzing-power excitation functions. Hence, certain runs tests may be unreliable predictors of intermediate structure.

In a recent Letter,[1] Glashausser *et al.* adduce the results of runs tests[2-4] as "evidence for intermediate structure in the inelastic scattering of polarized protons from ^{26}Mg and ^{27}Al."[1] Their analysis assumes implicitly that (1) intermediate structure (IS) is the only type of broad structure appearing in the data and (2) smoothing the raw data and selecting smoothed data at fixed intervals yields a sample whose members are independent and vary randomly in the absence of IS. If these assumptions hold, it is concluded that (3) the runs tests used can differentiate between the presence or absence of IS. The runs statistics used in Ref. 1 are derived for independent points, and test only for the presence or absence of correlations in the data, irrespective of the source of the correlations. Hence, assumptions (1) and (2) *must* hold in order to sustain conclusion (3).

However, we shall show that these assumptions are violated, because of the following: (1) Chance coherences of random nuclear amplitudes from overlapping continuum states produce *spurious* IS, as seen in Monte Carlo–generated analyzing-power excitation functions. (2) The smoothing and sampling procedure of Ref. 1 fails to remove the correlations between the sample points. We also find that (3) runs tests used in Ref. 1 fail to distinguish synthetic analyzing-power data with simulated IS from those without IS.

We therefore conclude that the analysis of Ref. 1 is not decisive evidence for IS. We also suggest modifications of the runs tests of Ref. 1 to increase their statistical efficiency for this application.

In the Ericson theory of statistical fluctuations in nuclear reactions,[5] the S-matrix element between channels c and c' at energy E is

$$S_{cc'}(E) = \bar{S}_{cc'}(E) - 2i(P_c P_{c'})^{1/2} \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E - E_\lambda + i\Gamma/2}, \tag{1}$$

where $\bar{S}_{cc'}(E)$ varies smoothly with energy, E_λ is a continuum energy level, Γ is the compound-nucleus (CN) coherence width, P_c and $P_{c'}$ are penetrabilities, and $\gamma_{\lambda c}$ and $\gamma_{\lambda c'}$ are random, complex reduced-width amplitudes.

In the continuum region of CN excitation energies, Γ is much larger than the average level spacing $\langle D_\lambda \rangle$. Hence, the S-matrix elements $S_{cc'}(E_1)$ and $S_{cc'}(E_2)$ are strongly correlated for $|E_1 - E_2| \sim \pi\Gamma$.[6] Because the cross section σ and the analyzing power A_y depend on bilinear products of $S_{cc'}$, σ and A_y are also correlated as functions of E—*despite the randomness of the reduced-width amplitudes* $\gamma_{\lambda c}$. Singh, Hoffman-Pinther, and Lang[7] have shown that these chance correlations produce spurious IS in synthetic cross sections. We have found similar effects in Monte Carlo–generated analyzing powers.

We calculated analyzing powers for the elastic scattering of spin-$\frac{1}{2}$ projectiles from a spin-0 target in the presence of overlapping CN resonances. The CN amplitudes were derived from a resonance-pole expansion of the collision matrix,[8] with $\gamma_{\lambda c}$ generated by a shift-register sequence algorithm.[9] The E_λ were computed from the semiempirical level-density formula of Gilbert and Cameron.[10] Direct-reaction amplitudes calculated from spin-orbit-coupled optical-model potentials and Coulomb-scattering amplitudes were included.

For comparison with Ref. 1, synthetic excitation functions for ^{26}Mg(p,p_0)^{26}Mg without IS (set A) were calculated at several angles for the energy interval 5.5 to 9.5 MeV in steps of 50 keV. The set-A analyzing powers included fluctuating CN and optical-model amplitudes, with optical-model parameters and Γ (50 keV) taken from analyses by Häusser *et al.*[11] for ^{26}Mg(p,p_0)^{26}Mg in the same energy range. At each angle, the average CN cross sections were about 12% of the differential cross section, in accord with Ref. 11.

The unsmoothed set-A data (Fig. 1) show pronounced fluctuations about the mean value of A_y. When these data are smoothed over an interval of 3Γ, as in Ref. 1, large structures of width 6 to 10 times Γ appear (Fig. 1), even though the computed S-matrix elements include only statistical fluctuations as in Eq. (1). (The $\gamma_{\lambda c}$ were subjected to runs tests,[3] and found to be random to a confidence level <1%.) Since the $\overline{S}_{cc'}(E)$ are, by construction, smoothly varying functions of E, it

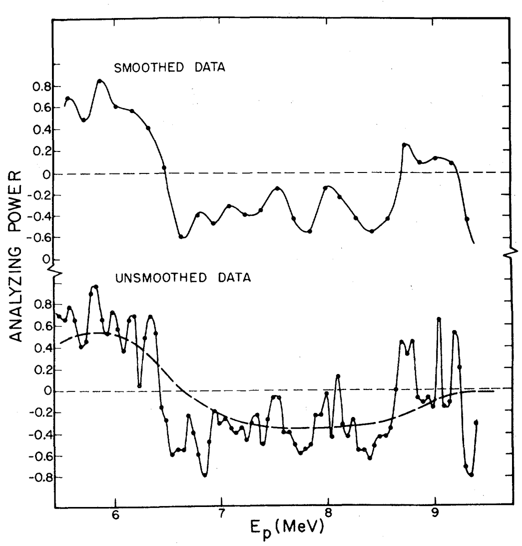

FIG. 1. Set-A analyzing powers computed as a function of proton lab energies E_p for ^{26}Mg(p,p_0)^{26}Mg(0^+ g.s.) at 140° c.m. without IS. The lower curve simulates raw data taken at intervals of 50 keV with a 50-keV-thick target. The upper curve is derived from the lower by averaging over an interval of 3Γ (150 keV) and selecting points 3Γ apart. The dashed curve shows the optical-model background, and solid curves are guides to the eye. We remark the similarity to Fig. 1(a), Ref. 1.

is clear that these large structures are not real IS, but arise from chance correlations between S-matrix elements at neighboring energies.

Hence, as pointed out by Singh *et al.*,[7] "widths alone... are not reliable signatures of the real intermediate structures" in analyzing powers as well as in cross sections. Since the runs tests used in Ref. 1 are a measure of the width of structures in the smoothed excitation function, those tests may also be unreliable indicators of IS.

The runs tests of Ref. 1 are derived for *independent* random data. Hence, they cannot be applied to raw continuum excitation-function data for σ or A_y because of the correlations due to the finite coherence width of the overlapping CN levels. In Ref. 1, "values of A_γ averaged over 2 to 3 times Γ and separated by 2 to 3 times Γ were considered independent."[1] However, as we now demonstrate, this smoothing and sampling technique fails to remove the CN correlations from the sample points.

The statistical correlation among σ or A_y data is described by the autocorrelation function of energy lag ϵ. In the Ericson theory, with infinite sample size and in the absence of a direct-reaction trend, this function is[12,13]

$$\rho(\epsilon/\Gamma)=[1+(\epsilon/\Gamma)^2]^{-1}. \tag{2}$$

For finite sample size, or when a trend is present in the data, the values of ρ tend to lie above this limit.[14]

If, as in Ref. 1, the sample points $A_y(k)$ are spaced at intervals roughly equal to Γ, and if only forward nearest-neighbor correlations are considered, the unsmoothed data form a one-point Markoff sequence,[15] in which the nth point is

$$A_\gamma(n)=\delta A_y(n)+\rho(1)A_y(n-1), \tag{3}$$

where $\delta A_y(n)$ is a random number [Eq. (1) and Refs. 5 and 13]. Following Ref. 1, we then average $A_y(n)$ with its nearest neighbors to obtain

$$\overline{A}_y(n)=\delta\overline{A}_y(n)+\rho(1)[A_y(n)+A_y(n-1)+A_y(n-2)]/3, \tag{4}$$

where the bar denotes an average quantity. The same procedure applied to $A_y(n-3)$ shows that $A_y(n-2)$ contributes with weight $\rho(1)$ to both $\overline{A}_y(n)$ and $\overline{A}_y(n-3)$.

The Markoff model, of course, underestimates the correlations in the unsmoothed excitation functions. In the Ericson theory, the CN fluctuations are correlated both forward and backward, and even next-next-nearest-neighbor correlations have a magnitude of 0.1 [Eq. (2)]. However, even the simpler Markoff picture shows that the $\bar{A}_y(k)$ used for runs tests in Ref. 1 are not independent. This lack of independence translates into runs statistics which show a definite correlation in the sample, although the correlation is only an artifact of the smoothing.

We also tested the efficiency of the runs tests used in Ref. 1 in revealing *known* intermediate structure. Data points from set A were smoothed over intervals of 3Γ, as in Ref. 1. Tests for the total number of runs above and below an arbitrary cut[3] (Test 1 in Ref. 1) and a test based on the length of the longest run[4] (Test 2 in Ref. 1) were then applied to this sample. The results of these tests, given in Table I (set A), indicate that the probability for the sampled data to be random is typically less than 1%.

A second set of synthetic data, denoted set B, used a single-level Breit-Wigner resonance[16] to simulate IS. Set-B calculations have CN fluctuation and optical-model parameters identical to those of set A, but include in the $p_{3/2}$ scattering amplitude a resonance with parameters $E_R = 7.22$ MeV, $\Gamma_{IS} = 1.2$ MeV, and $\Gamma_p = 0.7$ MeV (c.m. values). The total width Γ_{IS} was chosen as the conjectured width of the IS.[17] The ratio Γ_p/Γ_{IS} equals the largest possible value which does not violate unitarity in the S matrix, and is a gross overestimate because neutron channels are open at these energies. Figure 2 shows the resulting excitation functions, and Table I (set B) gives the results of runs tests, made as for set A. Although these tests typically indicate less than a 0.5% chance that the data are random, the results do not differ significantly from those for set A, in which there is no IS. Calculations with other choices of Γ_{IS} and different backgrounds[18] show that the results of runs tests with and without IS remain practically indistinguishable.

The failure of these particular runs tests as efficient predictors of IS is traceable to the statistical correlations discussed above. These correlations also persist in inelastic scattering, which differs from elastic scattering primarily in the direct reaction strength and in the angular momentum coupling. In either case, coherent addition of S-matrix elements from overlapping CN levels produces structures of large width in σ or A_y excitation functions, which masquerade as IS.

We have shown that spurious IS can be produced by the chance coherence of random amplitudes from overlapping CN resonances. We have also demonstrated that the smoothing and selection procedure of Ref. 1 fails to remove statistical correlations between data points, so that the data are not independent. For either or both of these reasons, we find that runs tests applied as in Ref. 1 fail to differentiate excitation functions with IS from those without IS.

The simplicity of runs tests makes them intrinsically attractive.[19] In the region of isolated resonances, runs tests applied to the width and spacing distributions of isolated (hence indepen-

TABLE I. Probablity that the synthetic excitation-function data are random, computed from statistical tests described in the text. Set-A calculations include CN fluctuations superimposed on optical-model background amplitudes; set-B amplitudes are identical to set A except for a simulated IS in the $p_{3/2}$ channel.

		Angle	
$^{26}Mg(p,p_0)$	120°	140°	160°
Set A			
Test 1	0.005	< 0.005	< 0.005
Test 2	0.25	0.01	0.1
Set B			
Test 1	< 0.005	< 0.005	< 0.005
Test 2	< 0.005	0.1	0.1

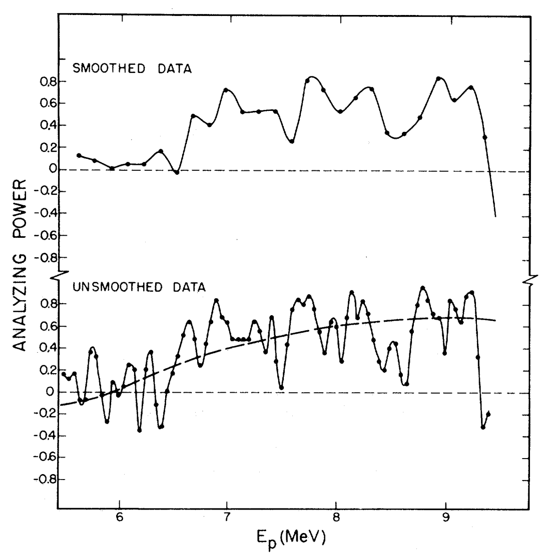

FIG. 2. Set-B analyzing powers computed as a function of proton lab energies E_p for $^{26}Mg(p,p_0)^{26}Mg(0^+$ g.s.) at 140° c.m., with a $p_{3/2}$ resonance to simulate IS. The upper curve is derived from the lower using the averaging procedure followed for set-A data. The dashed curve indicates the optical-model and resonance background; solid curves are guides to the eye.

dent) nuclear levels are efficient predictors of IS, and are relatively easy to use.[2] However, in the continuum region, runs tests cannot be used to test for IS without first compensating for the effect of smoothing and for statistical correlations due to overlapping CN levels. Smoothing the data helps to reduce statistical fluctuations, but induces correlations which can and must be accounted for in the desired runs statistics.[20] Alternatively, one might dispense with smoothing altogether, model the correlation structure of the raw data, and then derive the appropriate runs statistics. Bateman, for example, has given a solution for runs above and below the median for one- and two-point Markoff processes.[21]

We are currently investigating modifications of runs tests which would make them more reliable as predictors of IS. However, our present results suggest the wisdom of a cautions approach, since statistical tests seldom speak unequivocally.

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Possibility of Self-Consistent Long-Range Order in Nuclear Matter

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Hartree-Fock energies are calculated in nuclear matter for Overhauser-like orbitals and Skyrme forces as parametrized by Vautherin and Brink and others. In all cases considered, the plane-wave state is found to be more strongly bound.

The unproved assumption, implicitly made in practically all nuclear matter calculations, that plane-wave orbitals are the lowest-energy solutions to the Hartree-Fock equations in the thermodynamic limit, was critically examined by Overhauser in an early Letter.[1] Since this pioneering effort, a large number of papers[2] have appeared which analyze the stability, relative to plane waves, of non-plane-wave solutions implying some kind of long-range order (e.g., a spatially periodic single-particle density).

At least two important physical consequences of this new class of states can be mentioned: (a) the possibility of neutron-star matter crystallization under realistic forces,[3] probably at supra-nuclear densities, and (b) the apparent likelihood[4] of periodic α-cluster formation at the nuclear surface, i.e., at subnuclear densities. (The latter bears a striking analogy to some indications[5] that a system of classical random–close-packed hard spheres has a crystallinelike structure at its free surface.)

Overhauser's original study[1] involved a calculation yielding a so-called "density-ripple" state