Let us come back to the optical-phonon model for the A_2 resonance. According to the reported measurements,[8] the A_2 resonance energy shifts from 9.5 meV for $x=0$ to 21 meV for $x=0.14$. Since as we go from HgTe to CdTe, the optical phonon frequency[16] only changes by about 10%, this model cannot explain the observed large shift. Besides, it has been learned[8] that whether a clear conductivity anomaly could be seen in a given sample or not depends on the heat treatment of the sample. This indicates that the anomaly is indeed connected with stoichiometric defects rather than with optical phonons. The present work thus lends strong support for the acceptor resonance model. In addition, our predictions about the movement of the acceptor state in the open-gap configuration should induce more experimental investigations in this field.

†Work supported by a grant from Centre National de la Recherche Scientifique–ATP "Transition Métal-Isolant".

*Fulbright Senior Research Scholar.

‡Permanent address.

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COMMENTS

Comment on Higher-Order Nonlinearities and Coupling Saturation of Parametric Decay Instabilities

D. F. DuBois and B. Bezzerides
Theoretical Division, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545
(Received 10 May 1976)

The effect of higher-order nonlinearties on the coupling-saturation mechanism for parametric decay instabilities is analyzed. In particular it is shown that cubic nonlinearities in the Poisson equation describing the unstable electrostatic fields play an important role. In the case of the $2\omega_{pe}$ decay instability these effects increase the importance of coupling saturation for $T_e \gg T_i$ and reduce the saturated wave energy by a factor of T_i/T_e.

In a recent Letter[1] we showed that a new nonlinear saturation mechanism for parametric decay instabilities, which we called coupling saturation (CS), arises from cross correlations between Fourier modes. In deriving the equations describing this effect we started from the nonlinear Poisson equation which included only quadratic nonlinearities in the electric field intensity. In this note we wish to point out that cubic nonlinearities may play an important role in this effect and, in fact, enhance the importance of CS in determining the saturated state of $2\omega_{pe}$ decay instability.

For this study we generalize Eq. (1) of I to include third-order and higher-order terms in E:

$$\epsilon(k)E(k)=\frac{1}{2!}\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}X_{(2)}(k,k',k'')E(k')E(k'')(2\pi)^4\delta^4(k-k'-k'')$$
$$+\frac{1}{3!}\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}\int\frac{d^4k'''}{(2\pi)^4}X_{(3)}(k,k',k'',k''')E(k')E(k'')E(k''')$$
$$\times(2\pi)^4\delta^4(k-k'-k''-k''')+O(E^4). \qquad (1)$$

The second-order susceptibility $X_{(2)}(k,k',k'')$ was called simply $\chi(k,k',k'')$ in I. Here we wish to assess the role of $X_{(3)}$ and higher-order terms. We proceed exactly as in I by substituting the *Ansatz*

$$E(k)=E_0(k)+\sum_{s=1}^{2}\sum_{n_s}(2\pi)^4[E_{n_s}\delta^4(k-k_{n_s})+E_{n_s}{}^*\delta^4(k+k_{n_s})]+(\vec{k}_{n_s}\to-\vec{k}_{n_s})+\delta E(k) \qquad (2)$$

into Eq. (1).

The equations for the nonresonant beat field δE [Eqs. (5a) and (5b) of I] are unchanged to terms of order $|E_{n_s}|^2$ since the $X_{(3)}$ nonlinearity cannot contribute to these beat fields. However, the equations for E_{n_1} and E_{n_2} now acquire a direct contribution of order $|E_{n_s}|^2$ from $X_{(3)}$:

$$\epsilon(k_{n_1})E_{n_1}=X_{(2)}(k_{n_1},k_0,-k_{n_2})E_0E_{n_2}{}^*$$
$$+[1/(2\pi)^4\delta^4(0)]\textstyle\sum_s\sum_{n_s'}\{X_{(2)}(k_{n_1},k_{n_s'},k_{n_1}-k_{n_s'})E_{n_s'}\delta E(k_{n_1}-k_{n_s'})+(\vec{k}_{n_s'}\to-\vec{k}_{n_s'})\}$$
$$+\textstyle\sum_s\sum_{n_s'}X_{(3)}(k_{n_1},k_{n_1},k_{n_s'},-k_{n_s'})E_{n_1}|E_{n_s'}|^2+\sum_{n'}X_{(3)}(k_{n_1},-k_{n_2},k_{n_1'},k_{n_2'})E_{n_2}{}^*E_{n_1'}E_{n_2'}. \qquad (3)$$

To obtain this we have used the invariance of $X_{(3)}(k,k',k'',k''')$ to permutations of the last three indices. There are six identical contributions which cancel the 3! in (1).

In addition to the terms already analyzed in I we see that $X_{(3)}$ contributions contain terms proportional to E_{n_1} which add to the renormalization of $\tilde{\epsilon}$ and terms proportional to $E_{n_2}{}^*$ which add to the renormalization of $\tilde{\chi}$. Thus the renormalized equations (6a) and (6b) still apply but the formulas (7), (8), and (9) of I for the renormalized quantities are altered. For example, Eq. (7) of I becomes

$$\tilde{\epsilon}(k_{n_1})=\epsilon(k_{n_1})-\sum_s\sum_{n_s'}\left\{\frac{X_{(2)}(k_{n_1},k_{n_s'},k_{n_1}-k_{n_s'})X_{(2)}(k_{n_1}-k_{n_s'},-k_{n_s'},k_{n_1})}{\epsilon(k_{n_s}-k_{n_s'})}\right.$$
$$\left.+(k_{n_1}\rightleftharpoons k_{n_2})+X_{(3)}(k_{n_1},k_{n_1},k_{n_s'},-k_{n_s'})\right\}|E_{n_s'}|^2, \qquad (4)$$

and Eq. (9) of I becomes

$$\tilde{\chi}(k_{n_1},k_0,-k_{n_2})=X_{(2)}(k_{n_1},k_0,-k_{n_2})$$
$$+\sum_{n_2'}\left\{\frac{(k_{n_1},k_{n_1'},k_{n_1}-k_{n_1'})X_{(2)}(k_{n_1}-k_{n_1'},k_{n_2'},-k_{n_2})}{\epsilon(k_{n_1}-k_{n_2'})}+(k_{n_2'}\rightleftharpoons k_{n_1'})\right.$$
$$\left.+X_{(3)}(k_{n_1},k_{n_2},k_{n_1'},k_{n_2'})\right\}\frac{\tilde{\chi}(k_{n_1'},k_0,-k_{n_2'})}{\tilde{\epsilon}(k_{n_1'})}|E_{n_2'}|^2. \qquad (5)$$

It is clear that to terms of order $|E_{n_s'}|^2$ the nonlinearities higher than the third make no contribution to these equations.

We briefly comment on the effect of the $X_{(3)}$ terms on the saturation of the $2\omega_{pe}$ decay instability. The $X_{(3)}$ correction to $\tilde{\epsilon}$ has the well-known effect of weakening the nonlinear-induced scattering of Langmuir waves from *electrons* but plays no role in the nonlinear-induced scattering from ions which was the only effect in $\mathrm{Im}\tilde{\epsilon}$ considered in I. The $X_{(3)}$ contribution to the renormalized $\tilde{\chi}$ appears to be quite important.

We have computed $X_{(3)}$ by taking the third functional derivative of the density using the techniques of

DuBois.[2] The result is

$$X_{(3)}(k_{n_1}, -k_{n_2}, k_{n_1'}, k_{n_2'}) = \frac{e^2}{m^2\omega_{pe}^4}(\hat{k}_{n_1}\cdot\hat{k}_{n_1'})(\hat{k}_{n_2}\cdot\hat{k}_{n_2'})\chi_e(k_{n_1}-k_{n_1'})(\vec{k}_{n_1}-\vec{k}_{n_1'})^2$$
$$+(\hat{k}_{n_1}\cdot\hat{k}_{n_2'})(\hat{k}_{n_2}\cdot\hat{k}_{n_1'})\chi_e(k_{n_1}-k_{n_2'})(\vec{k}_{n_1}-\vec{k}_{n_2'})^2$$
$$+(\hat{k}_{n_1}\cdot\hat{k}_{n_2})(\hat{k}_{n_1'}\cdot\hat{k}_{n_2'})\chi_e(k_{n_1}+k_{n_2})(\vec{k}_{n_1}+\vec{k}_{n_2})^2, \tag{6}$$

where we have assumed $\omega_{pe} \approx \omega_{pe} \gg k_{n_s} v_e$, $\omega_{n_s'} \approx \omega_{pe} \gg k_{n_s'} v_e$ $(s = 1, 2)$. Here $\chi_e(k)$ is the familiar collisionless electron susceptibility. The terms for which the frequency arguments of χ_e are small dominate. For the $2\omega_{pe}$ instability we can have

$$\omega_{n_1} - \omega_{n_1'} \ll |\vec{k}_{n_1} - \vec{k}_{n_1'}| v_e, \quad \omega_{n_1} - \omega_{n_2} \ll |\vec{k}_{n_1} - \vec{k}_{n_2'}| v_e,$$

since $\omega_{n_1}, \omega_{n_2} \sim \omega_{pe}$, but in the last term $\omega_{n_1} + \omega_{n_2} \sim 2\omega_{pe} \gg |\vec{k}_{n_1} + \vec{k}_{n_2}| v_e$ and so this term is smaller by terms of order $|\vec{k}_{n_1} + \vec{k}_{n_2}|^2 / k_{De}^2$. The well-known result for $X_{(2)}$ can be written to leading terms as

$$X_{(2)}(k_{n_2}, k_{n_2'}, k_{n_2} - k_{n_2'}) = i\frac{e}{m\omega_{pe}^2}(\hat{k}_{n_2}\cdot\hat{k}_{n_2'})X_e(k_{n_2} - k_{n_2'})|\vec{k}_{n_2} - \vec{k}_{n_2'}|.$$

The curly-bracketed expression in (5) can then be written on substitution of (6) and (7) as

$$\left\{ -\frac{e^2}{m^2}\frac{(\hat{k}_{n_2}\cdot\hat{k}_{n_2'})(\hat{k}_{n_1}\cdot\hat{k}_{n_1'})}{\omega_{pe}^4}\frac{\chi_e(k_{n_1}-k_{n_2'})[\chi_i(k_{n_1}-k_{n_1'})+1]}{\epsilon(k_{n_1}-k_{n_1'})} + (k_{n_1} \rightleftarrows k_{n_2}) \right\},$$

where we have used $\epsilon(k) = 1 + \chi_e(k) + \chi_i(k)$ and $\chi_i(k)$ is the collisionless ion susceptibility. This result differs by a factor $-(\chi_i + 1)/\chi_e$ from that found where $X_{(3)}$ was neglected. In the static approximation used there this factor can be written as $-(T_e/T_i)$ (since $|\vec{k}_{n_1} - \vec{k}_{n_2'}|/k_{De} \ll 1$). Thus the coefficient of the integral in Eq. (15) of I is multiplied by $-(T_e/T_i)$ and the subsequent equations are altered by replacing W by $-W(T_e/T_i)$. The final result for the saturated wave energy of the $2\omega_{pe}$ decay instability which replaces Eq. (17) of I then becomes

$$W = 2(\gamma_l/\omega_{pe})(P-1)^{1/2}(1 + T_i/T_e). \tag{8}$$

When $T_e/T_i \gg 1$ we see that the inclusion of the $X_{(3)}$ terms makes a significant quantitative difference. The application of this theory to the electron-ion decay instability for $T_e \gg T_i$ will be considered elsewhere.[3]

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