Hadron Multiplicity in Color-Gauge-Theory Models

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The universality of the rise of hadron multiplicities in high-energy hadron and lepton collisions is attributed to the necessity of confining quark quantum numbers in a color-gauge-theory model.

The most straightforward applications of the quark-parton model are to short-distance phenomena for which scaling and power-law behaviors of cross sections can be derived. Predictions concerning the detailed structure of the final state are far more difficult, being closely related to the problems of quark confinement and large-distance phenomena. Most approaches have been at a heuristic or descriptive level in terms of jet structure, short-range rapidity correlation, etc. In principle, however, it should be possible to compute final-state structure for any given underlying theory.

One of the few examples of computation from first principles occurs in quantum electrodynamics (QED) for which the multiplicity of soft photons emitted in charged-particle scattering is well known.[1] The salient features of the results are that (a) soft photons arise via bremsstrahlung from initial or final charged-particle lines. Neutral particles do not radiate. (b) The average multiplicity consists of a sum over charged-particle pairs, each contribution depending only on the product of charges times a function which increases with the relative rapidity of the pair.

It is the purpose of this Letter to discuss the implications of the analogous picture in a color-gauge model such as quantum chromodynamics.[2] In this case, charge is replaced by color, and the hadrons, which are color singlets, do not radiate. Radiation of colored gluons occurs only when two colored objects (e.g., quarks) are separated in momentum so that their relative rapidity is nonzero. In addition, there is a natural infrared cutoff determined by the size, $R \sim 1$ F, of the confinement region for color. The soft-gluon calculations should be valid in a region where $E_{\text{c.m.}} \gg |\vec{k}| \gg R^{-1}$. We presume that the radiated color gluons eventually materialize[3] as hadrons in such a way that the hadron multiplicity is a direct, monotonic (possibly linear) function of the rising gluon multiplicity, $\langle n_h \rangle = F(\langle n_g \rangle)$, and hence only depends on the separating color currents, i.e., the underlying quark configuration. Two processes with the same initial color-current configuration will thus produce the same multiplicity in the final hadronic state. Further, the gluon radiation is independent of the quark flavor for a given relative rapidity. The principal effect of quark flavor will be to influence the *quantum numbers* of the leading hadrons and to make changes of order 1 in the hadron multiplicity. Our underlying assumption is not unlike that made in computing widths of the ψ resonances in quantum chromodynamics[4]; in this case one identifies the gluon final states with the sum over all physical hadronic states. Thus the separation of color combined with the necessity of confining color naturally leads to a rising hadron multiplicity.

We will first consider the general implications of this picture for hadronic multiplicity in e^+e^- annihilation and then relate it to deep inelastic scattering and hadron-hadron scattering. Our principal result is that because of the color group structure, it is natural for the plateaus in e^+e^- annihilation and deep inelastic scattering (as discussed, for instance, by Bjorken, Ref. 3) to have a common structure. In addition the empirical fact noted by Albini *et al.*[5] that the hadron multiplicity has a universal parametrization for *all* of the above processes has a simple interpretation in our approach.

In $e^+e^- \to X$, each event begins with the creation of separating 3 and $\overline{3}$ currents [see Fig. 1(a)], the radiation of gluons, and the eventual creation of the hadron multiplicity. Thus the average multiplicity $\langle n \rangle_{e^+e^- \to X} = n_{3\overline{3}}(s)$. In analogy with quantum electrodynamics, the function $n_{3\overline{3}}(s)$ depends on the gluon coupling strength $\alpha_{3\overline{3}}$ to the $3\overline{3}$ system, on the masses of the 3 and $\overline{3}$, and on the infrared

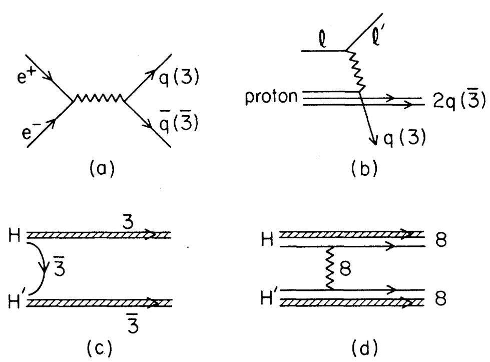

FIG. 1. Separation of 3 and $\bar{3}$ of SU(3) color in (a) e^+e^- annihilation, (b) $lp \to l'X$, and (c) hadron-hadron collisions. (d) indicates the separation of right and left moving color octets following the exchange of a colored gluon.

cutoff R^{-1}, and increases monotonically with the initial separation in rapidity $y_{3\bar{3}}$.

Because of the flavor independence of the gluon coupling, the leading s dependence of the multiplicity in $e^+e^- \to$ hadrons is predicted to be the same below and well above the threshold for heavy quarks. When $y_{3\bar{3}}$ is small, there can be a temporary reduction in $\langle n\rangle_{e^+e^-\to X}$.[6] Note also that the multiplicity at $s = m_\psi^2$ will be continuous with the background if the ψ decays via the production of a pair of normal quarks.

It is interesting to also consider the multiplicity in $e^+e^- \to hX$ associated with the inclusive system X. For $z = 2p_h \cdot q/q^2$ near 1, it is clear that h should be considered as a fragment of one of the quarks. The recoil system begins with a separating 3 and $\bar{3}$ of SU(3) color with c.m. momenta $\vec{q}/2$ and $(1-z)\vec{q}/2$, thus leading to the multiplicity $\langle n\rangle_X = n_{3\bar{3}}(W^2)$, where $W^2 = (q-p_h)^2 = q^2 - 2p_h \cdot q + m_h^2$. Notice that the multiplicity is predicted to be independent of $2p_h \cdot q$ at fixed W^2. Physically we expect that for $z \to 0$, the associated multiplicity is just the total multiplicity minus 1. The fact that the form $\langle n\rangle_{e^+e^-\to hX} = n_{3\bar{3}}(W^2)$ satisfies this consistency check indicates that even in the "wee" $z \sim 0$ regime, this same underlying quark topology (with constant 3 and $\bar{3}$ masses) is applicable.

In deep inelastic lepton scattering $lh \to l'X$ (electromagnetic or weak), each event begins with the creation of separating 3 and $\bar{3}$ systems with momenta $(1-x)p$ and $xp+q$ [see Fig. 1(b)], where $x = -q^2/2p\cdot q$ is the momentum fraction carried by the struck quark $[x = (k_0+k_3)/(p_0+p_3)]$. In general the struck parton can be a quark in a nonvalence component of the Fock-space wave function of the target. The mass of the $(1-x)p$ system at the instant of the scattering should still be of order of the target mass even for the states appropriate to the "sea" quarks. Thus as in $e^+e^- \to hX$, we treat the effective masses of the 3 and $\bar{3}$ systems as constants.[7] The hadron multiplicity is thus predicted to be $\langle n\rangle_{lh\to l'X} = n_{3\bar{3}}(W^2)$ with $W^2 = (1-x)2p\cdot q = 2p\cdot q + q^2$. (Note that this is the natural continuation of the result obtained above for $e^+e^- \to hX$.) The connection between the multiplicity in e^+e^- annihilation and deep inelastic lapton scattering is immediate:

$$\langle n\rangle_{lh\to l'X} = n_{3\bar{3}}(W^2) = \langle n\rangle_{e^+e^-\to X}|_{s=W^2}. \qquad (1)$$

The prediction that the multiplicity for $lh \to l'X$ is independent of q^2 for fixed W^2 is strikingly confirmed by experiment.[8] This represents a derivation of the result long advocated by Feynman[9] and Bjorken.[10] In their language, the hadronic and $q\bar{q}$ "plateaus" are assumed to have the same height[11] [e.g., $C_h \ln(W^2/|q^2|) + C_{e^+e^-} \ln|q^2| = C_{e^+e^-} \times \ln W^2$, where $\langle n\rangle_{e^+e^-} \sim C_{e^+e^-} \ln s$]. In our derivation there is no need to break up the multiplicity into two components and moreover the multiplicity is not constrained to be exactly a single power of $\ln s$. We also note that if a color-octet part of the current becomes operative by changing either q^2 or W^2 then one would expect a corresponding change in the multiplicity. The relation $\langle n\rangle_{e^+e^-\to X} = \langle n\rangle_{lh\to l'X}(W^2 = s)$ is still maintained however.

If q^2 is taken to zero in $\langle n\rangle_{lh\to l'X} = n_{3\bar{3}}(W^2)$, then one predicts that the multiplicity for $\gamma h \to X$ at c.m. energy $\sqrt{s}$ is the same as in deep inelastic scattering at $W^2 = s$. This suggests then that the multiplicity for any hadron scattering process $\langle n\rangle_{hh'\to X}$ is also of the universal form $n_{3\bar{3}}(s)$. However, a closer examination reveals that this result depends upon assumptions on the underlying quark topology. In particular, we will show that multiplicity universality holds if hadron collisions are dominated by wee quark exchange of the type indicated in Fig. 1(c).

In Feynman's description[9] of hadron-hadron scattering the constancy of the total cross section arises from the exchange of wee partons with a dx/x spectrum in the Fock-space decomposition of the incoming hadron states. When we take the exchanged parton to be a quark, the interaction once again sets up a system with separating 3 and $\bar{3}$ color currents. As before, these spectator systems, which carry essentially all of the initial momenta, have a predetermined mass of order of the hadron masses. Thus we again obtain the universal form $\langle n\rangle_{h_1h_2\to X} = n_{3\bar{3}}(s)$. Similarly, the multiplicity in the recoil system for single-particle inclusive reactions is given by $\langle n\rangle_{h_1h_2\to h_3X} = n_{3\bar{3}}(\mathfrak{M}_X^2)$. ($h_3$ can be in the central or fragmentation region.)

In contrast to the above universal behavior, in the Low-Nussinov[12,13] model of the Pomeron, the initial interaction is via gluon exchange [see Fig. 1(d)], which sets up a system with separated color octets near the ends of the rapidity axis. In this case there is no reason to assume the gluon multiplicity is the same as in the $3\bar{3}$ case. In particular, the coupling constant α_{88} of the gluon to separating color octets is $\frac{9}{4}$ times the coupling constant to separating 3 and $\bar{3}$. Accordingly, one would also expect a significant rise at small $x = |q^2|/2M\nu$ in $\langle n\rangle_{eh\to eX}$ at fixed W^2, as the dominant interaction changes from quark exchange to gluon exchange in this model.

Similar effects also occur in standard multiperipheral-model calculations.[14] The multiplicity associated with the nonplanar Pomeron diagram is approximately twice that associated with planar dual diagrams appropriate to Reggeon exchange, e^+e^- annihilation, and valence-dominated deep inelastic scattering. The absence of any multiplicity rise at small x for fixed W^2 in the $ep\to eX$ data[8] is in dramatic conflict with both approaches to the Pomeron multiplicity.

One may clearly extend this general approach to the hard scattering models of high-p_T reactions.[15] Final-state multiplicities depend upon and thus can be used to discriminate between the different possible subprocesses.

It is interesting to speculate on the form for the gluon multiplicity in non-Abelian gauge theories, using quantum electrodynamics as a guide. In QED, the n-soft-photon cross section is[1] $\sigma_n = (2\alpha\tilde{B})^n \exp[-2\alpha B]/n!$ and hence $\langle n_\gamma\rangle = 2\alpha\tilde{B}(k_{\max})$, where

$$2\alpha\tilde{B}(k_{\max}) = -\sum_{i,j}\frac{\alpha}{4\pi^2}Q_iQ_j\eta_i\eta_j\int_{k_{\min}}^{k_{\max}}\frac{d^3k}{2k_0}\frac{p_i\cdot p_j}{p_i\cdot k\,p_j\cdot k}. \tag{2}$$

The sum in i and j is over all incoming ($\eta_i = +1$) and outgoing ($\eta_i = -1$) charged lines. The factor $e^{-2\alpha B}$ gives the effect of the virtual gluons.

It is an open question whether there is a similar exponentiation in the non-Abelian theories.[16] Assuming there is we may calculate $\langle n_g\rangle$ from the lowest-order diagrams. The only change is that α becomes $\frac{4}{3}\alpha_s$ in quantum chromodynamics for the $3\bar{3}$ currents.[17] Equation (2) is formally infrared divergent in QED.[18] However, as assumed in the introduction, in the case of color confinement there is a maximum wavelength $1/k_{\min}$ of hadronic scale (defined in the $3\bar{3}$ c.m. system) beyond which the color gluon can only resolve an overall neutral state. Then (large $p_i\cdot p_j$)

$$\langle n_g\rangle \sim \frac{4}{3}\frac{\alpha_s}{\pi}\ln\frac{2p_i\cdot p_j}{m_3m_{\bar{3}}}\left[\left(\ln\frac{2p_i\cdot p_j}{k_{\min}^2}+c_1\right)+c_2\right]. \tag{3}$$

The maximum phase space for the ij pair, $E_{ij}^{\max} = \langle n_g\rangle k_{ij}^{\max}$, scales with the available energy, $(p_i\cdot p_j)^{1/2}$. The parameters c_1 and c_2 have at worst log-log variation in the kinematics. We thus obtain $\langle n_h\rangle \sim \langle n_g\rangle \sim \ln^2 s$, assuming the proportionality of hadron and gluon multiplicities. If instead we had used an effective gluon mass λ (of order of hadron masses), such as might be estimated by the confinement mechanism, then the result (3) is unchanged except that m_3, $m_{\bar{3}}$, and $k_{\min}$ depend on λ^2.

The result (3) is consistent with flat distribution in rapidity. The final-state multiplicity fills the rapidity gap because of the dk/k integration.[19] Further, the entire plateau dN/dy rises at any rapidity y because of the logarithmic angular integral which becomes increasingly singular due to the peaking (near the light cone) of the gluon distribution along the $3,\bar{3}$ jet axes.

Recent experimental results[20] suggest that the plateau height is rising in hadronic collisions. The universal form of Albini *et al.*[5] (written in terms of $\sqrt{s_a} = \sqrt{s} - M_a - M_b$, with GeV units),

$$\langle n_{\rm ch}\rangle^{ab\to\text{ hadrons}} = 2.50 + 0.28\ln\sqrt{s_a} + 0.53\ln^2\sqrt{s_a}, \tag{4}$$

is the best χ^2 fit to pp collisions and also fits $\pi p\to X$, $Kp\to X$, $\pi p\to p_{\rm slow}X$, $pp\to P_{\rm slow}X$, $e^+e^-\to X$, and $ep\to eX$ multiplicities. If $\langle n_h\rangle = \frac{3}{2}\langle n_{\rm ch}\rangle = \langle n_g\rangle$, then identifying the $\ln^2 s$ coefficient with that in Eq. (4) gives $\alpha_s = 0.46$ which is not dissimilar from other determinations.

In conclusion, a universal result for the multiplicities does arise in a picture where hadron production occurs as a result of kinematically necessary separation of a 3 and $\bar{3}$ of SU(3) color followed by gluon bremsstrahlung and hadron materialization. This picture also implies that the jet structure and associated hadron multiplicities in the central region are the same in e^+e^- annihilation, deep inelastic scattering, and forward hadron collisions. The crucial assumption in this explanation of universal rising multiplicity is that the materialization of hadrons only depends upon the SU(3) color topology initially established by the interaction. The specific form suggested by the gluon-emission structure of QED should be regarded more cautiously but nonetheless appears consistent with the data and with expectations concerning the size of α_s.

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[18]Except for Coulomb scattering $\sigma_{\rm tot}$ is always finite in QED. In fact $\sigma_{\rm tot}$ is even finite for $m_e\to 0$ when the incident particles are neutral. See Appelquist, Ref. 16, and references therein. However, the multiplicity still diverges for $m_e\to 0$.

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Two-Body Photodisintegration of ^{3}He and a New Test of Time-Reversal Invariance in the Electromagnetic Interaction

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We have measured the two-body photodisintegration process $\gamma^3\mathrm{He}\to pd$ in the energy region sensitive to intermediate photoexcitation of one nucleon to the isobar $\Delta(1238)$. We present angular distributions at center-of-mass angles from 30° to 150° for incoming photon energies 200–600 MeV. Magnetic dipole excitation appears to be suppressed.

We have studied the two-body photodisintegration of ^{3}He in the energy region spanning possible intermediate excitation of one nucleon to the $\Delta(1236)$ isobar with incident photon energies from 0.2 to 0.6 GeV. We sought to extract as complete and exact a set of angular distributions as experimentally feasible, as part of a new and independent test of time-reversal invariance in the elec-