ress (Academic, New York, 1961).
[12]P. L. Kapitsa, *High-Power Microwave Electronics* (Pergamon, New York, 1964).
[13]An excellent review of the power levels, efficiencies, etc., of the various devices is given by V. L. Granatstein, R. K. Parker, and P. Sprangle, in *Proceedings of the International Topical Conference on Electron Beam Research and Technology, Albuquerque, New Mexico, 1975,* edited by Gerold Yonas (U. S. Department of Commerce, Washington, D. C., 1976), Vol. II, p. 401.
[14]S. Silver, *Microwave Antennas Theory and Design* (McGraw-Hill, New York, 1949).
[15]Matheson Company Inc. furnished a filtered and dried sample of ordinary air. The same results were obtained with a sample of 80% nitrogen, 20% oxygen, both spectroscopically pure, obtained from AIRCO.
[16]L. Gould, "Handbook on Breakdown of Air in Waveguide Systems," Microwave Associates Report, April 1956 (unpublished); P. Felsenthal and J. M. Proud, Rome Air Development Center Report No. RADO-TR-65-142, 1965 (unpublished); A. D. McDonald, *Microwave Breakdown in Gases* (Wiley, New York, 1966).
[17]E. Ott and R. V. Lovelace, Appl. Phys. Lett. 27, 378 (1975). We note that these calculations are for a planar magnetron and therefore are applicable only approximately to our cylindrical device.

## COMMENTS

# Relativistic Particle Dynamics for an $N$-Body Interacting System

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Claims in a recent Comment by Liou to find new forms for the three-body interactions to order $1/c^2$ in an $N$-body relativistic interacting system are shown to arise from a misunderstanding. The proposed forms are encompassed by the solution to this problem obtained previously by the present authors. Several other clarifications are also made.

In a recent Comment in this journal[1] the discovery of "new particular solutions" to order $1/c^2$ for the three-body interactions in an $N$-body relativistic system is claimed; "new" has reference relative to the results previously reported in earlier publications by the present authors.[2,3] Regretfully, we must dispute this claim and in this Comment attempt to clarify the proper relation between the two sets of results. It appears to us that a misunderstanding is the principal reason for the disputed claim, this misunderstanding arising perhaps from the way in which the term "particular solution" is used in Ref. 1.

References 1, 2, and 3 are concerned with the solution of commutation relations which, in this particular context, is analogous to the problem of solving a system of linear inhomogeneous partial differential equations. It is well known that (a) the general solution of such a system is the sum of *any particular solution* and the general solution of the associated set of homogeneous equations (the complementary function); (b) the difference between two particular solutions is a specific complementary function; (c) conversely, the sum of any particular solution and a specific complementary function is another particular solution. Hence if one knows *a* particular solution and the complementary function there is no more to be determined beyond the satisfaction of boundary conditions.

If we examine the question of determining one quantity of interest, $V^{(2)}$ in the terminology of Ref. 1, we find that it must satisfy[4] Eqs. (I.6a) and (I.6b) where in the latter the first two terms on the right-hand side are dropped because of the assumption that $V^{(0)}$ and $\vec{W}^{(2)}$ are zero.[5] A set of solutions of these equations are presented in Eq. (I.10). Each member of the set is a particular solution.[5] The difference between any two is a solution of (I.6a) and (I.6b) with the right-hand side of the latter now zero. The general solution of the last set of these equations is any rotationally invariant function of internal variables only; this is indicated obliquely in the text following Eq. (I.10).

If one examines Ref. 3 one finds in its Eq. (III.31) one of the members of the set given in Eq. (I.10), and indeed this is remarked in Ref. 1.

On the other hand, in the first sentence following Eq. (III.40) we point out that we obtain a more general solution[7] by including in our $U^{(1)}$ (which corresponds to $V^{(2)}$ of Ref. 1) separable rotationally invariant functions of relevant internal variables only. Thus the two solution sets of Refs. 1 and 3 are in fact identical, and hence the "new particular solutions" claimed to be found in Ref. 1 are encompassed by the solutions we have previously obtained in Ref. 3. While in the long run it is possible that one particular solution may ultimately be found to have an advantage over another, there is no basis for such a choice at the present time.

The essential difference between the approach of Refs. 1 and 3 is that in the former one starts from the Bakamjian-Thomas[8] solution for two-particle systems and then expands in powers of $c^{-1}$ while in our Refs. 2 and 3 we start with arbitrary generators satisfying the Poincaré Lie algebra and expand similarly. If the former method has any advantages, some of these are substantially offset by the possibility of unitary transformations which leave both the Bakamjian-Thomas form and the separability of solutions invariant but change the two-body interactions to phase-shift–equivalent ones as described in Ref. 3. Referance 1 inadvertently[9] omits to mention that Ref. 3 contains a proposal for integrating the required commutation relations to all orders in $c^{-1}$ and describes the procedure in considerable detail relative to its executability.

Finally we note that Eqs. (I.9) are not correct expressions of separability except in the two special cases: where the system consists of two particles only, or, if it contains $N>2$ particles, where there is no interaction at all or there is only an $N$-body interaction. The correct form of Eq. (I.9a) should read

$$\lim_{a\to\infty} \| [\exp(-i\vec{a}\cdot\vec{P}_A)V^{(2m)}\exp(i\vec{a}\cdot\vec{P}_A) - V_A{}^{(2m)} - V_B{}^{(2m)}]\varphi_v \| = 0,$$

where $V_A$ and $V_B$ are the internal interactions within subsystems $A$ and $B$ and the superscripts $(2m)$ represent the part of these interactions of order $c^{-2m}$. A similar equation, *mutatis mutandis*, should replace Eq. (I.9b).

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[1]M. K. Liou, Phys. Rev. Lett. 36, 1255 (1976).

[2]L. L. Foldy and R. A. Krajcik, Phys. Rev. Lett. 32, 1025 (1974).

[3]L. L. Foldy and R. A. Krajcik, Phys. Rev. D 12, 1700 (1975).

[4]Equations from Refs. 1 and 3 will be referred to as, for example, Eq. (I.6a) and Eq. (III.31), respectively.

[5]We shall use the term "particular solution" here and below in the conventional sense. In Ref. 1 it is used to indicate that what is there called $V^{(0)}$ and $\vec{W}^{(2)}$ are assumed to be zero. Incidentally, we are not clear, however, as to what the first sentence on p. 1258 of Ref. 1 is intended to convey since we are not clear as to what is meant by a many-body interaction which does not vanish for a two-particle system unless "many-body" means "two-body."

[6]See remark in footnote 5 above.

[7]We use the term "a more general solution" rather than "the general solution" since in the treatment of Ref. 3 this is more aptly applied to the general solution without imposing the conditions $V^{(0)}=\vec{W}^{(2)}=0$ as described in footnote 5 above. The general solution without these restrictions is in fact found in Ref. 3. The fact that Liou's "particular" solutions are encompassed by ours was independently communicated to us by Dr. G. E. Bohannon.

[8]B. Bakamjian and L. H. Thomas, Phys. Rev. 92, 1300 (1953).

[9]M. K. Liou, private communication.