Frequency Dependence of the ac Josephson Effect in Small Dayem Bridges

T. D. Clark and P. E. Lindelof
Physics Laboratory I, H. C. Ørsted Institute, University of Copenhagen, Copenhagen, Denmark
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The current-voltage characteristics of superconducting thin-film indium microbridges (Dayem bridges) and their microwave-induced constant-voltage steps have been investigated at several frequencies. The results are well described by a resistively shunted Josephson junction model, which differs from the usual resistively shunted Josephson junction model only in having a current-phase relation of the form $I = I_0(\dot{\varphi}) \sin\varphi + I_1(\dot{\varphi}) \times \cos\varphi + I_2(\dot{\varphi})$, where the functions I_0, I_1, and I_2 are derived from time-dependent Ginzburg-Landau theory.

Superconducting thin-film microbridges (Dayem bridges) and point contacts are generally considered to be adequately described by the so-called resistively shunted Josephson junction (RSJ) model. The RSJ model is based on a two-fluid picture of the electrodynamics of the superconducting contact:

$$I = I_s(\varphi) + V/R_N, \tag{1}$$

where R_N is the normal resistance of the contact, $V = (\hbar/2e)d\varphi/dt$ is the voltage across the contact, and $I_s(\varphi)$ is the current-phase relation of the Cooper pairs which is assumed to be sinusoidal, i.e., $I_s(\varphi) = I_0 \sin\varphi$.

Interest has earlier been directed towards the condition for the validity of a static sinusoidal current-phase relation. Essentially two conditions must be fulfilled in order to have a (nearly) sinusoidal current-phase relation: (1) The size of the bridge must not exceed the temperature-dependent coherence length, and (2) the critical current of the bridge must only have a negligible influence on the superconductors on each side of the bridge. A large variety of weak links have been devised in order to fulfill these conditions. For Dayem bridges the rapid change of the cross sectional area perpendicular to the bridge current serves these purposes.[1]

In a dynamic situation, it is normally tacitly assumed that the phase and the superfluid density can follow any time dependence prescribed by Eq. (1). It is clear, however, that an irreversible process involving normal electronic excitations in time-varying electric fields cannot proceed infinitely fast. In earlier work[2,3] a time-dependent Ginzburg-Landau approach was used in order to remedy this. A particularly simple model was suggested in Ref. 3. According to this model the superfluid part of the current in Eq. (1) is of the form[3]

$$I_s(\varphi) = I_0(\theta) \sin\varphi + I_1(\theta) \cos\varphi + I_2(\theta), \tag{2}$$

where $\theta = (2eV\tau/\hbar)(l/\xi)^2 = \dot{\varphi}\tau(l/\xi)^2$, and I_0, I_1, and I_2 are universal functions of θ, which in turn contains the voltage V across the contact, the length l of the contact, the coherence length ξ, and the relaxation time τ which enters into the time-dependent Ginzburg-Landau theory. When $\tau = 0$, Eqs. (1) and (2) revert to the RSJ model. We shall refer to the two-fluid model given by Eqs. (1) and (2) as the RSJτ model. The applicability of the time-dependent Ginzburg-Landau equations to superconductors with an energy gap is far from clear. The simplicity of these equations and the fact that they introduce only one parameter more than the time-independent equations make them attractive. Most important for our purpose is the suggestion they give to a new practical model (RSJτ) which agrees well with experiments on microbridges and which may be of considerable practical use.

There are a number of ways in which the frequency and voltage dependence of the ac Josephson effect in weak links can be studied experimentally. Current-biased weak links always have an excess supercurrent for finite voltages due to the self-coupling of the ac Josephson oscillation via the resistive shunt of the normal excitations. The RSJ model yields an excess current $[(\langle V\rangle/R_N)^2 + I_0^2]^{1/2} - \langle V\rangle/R_N$, where $\langle V\rangle$ is the mean value of the voltage across the weak link. One way of detecting a voltage-dependent Josephson effect is to study this excess current. In Fig. 1 the I-V characteristics of an indium thin-film microbridge measured at three different temperatures below T_c are compared with the I-V curve of the RSJ model. R_N in the RSJ model is chosen so that the slopes dV/dI at high voltages are the same as for the experimental curve when meas-

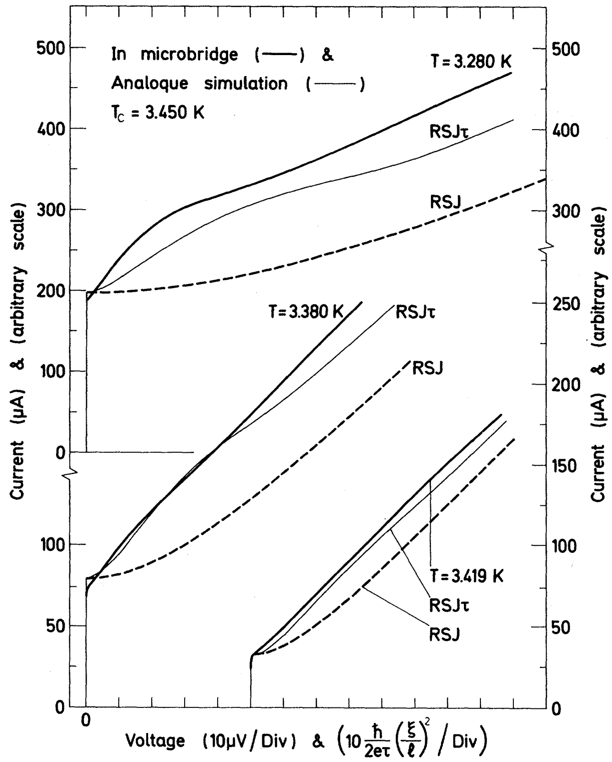

FIG. 1. Current-voltage characteristics of a Dayem bridge (film thickness, 0.13 μm; width, 0.45 μm; length, 0.25 μm) displayed at three different temperatures below T_c. The curves calculated on the basis of the RSJ model are dashed. R_N is chosen to be the high-voltage slope at $T = 3.419$ K. The three RSJτ curves have been calculated on the basis of the same parameters as used for the RSJ calculation plus the relaxation time $\tau = (10^{-10}\text{ sec})/(1 - T/T_c)$. From top to bottom the three RSJτ curves have $\Gamma = (2eR_N I_0\tau/\hbar)(l/\xi)^2 = 90$, 35, and 15.

ured close to T_c. The agreement is obviously rather poor and justifies our search for an alternative model. The indium microbridge investigated here was made by a cross-scratch method described in Ref. 1. The length and width of the bridge were 0.25 and 0.45 μm, respectively, as measured by a scanning electron microscope. The film thickness was 0.13 μm. This size of the bridge was around the minimum that could be routinely made and all bridges with these dimensions had I-V curves very similar to those displayed in Fig. 1.

The two most striking differences between the RSJ model and the experiment are the much smaller dV/dI of the experimental curve at low voltages (less than 20 μV) and the excess current at higher voltages which is of the order of I_0 in the experiment but disappears in the RSJ model. It is a virtue of the RSJτ model that it accounts for both deviations.

To what extent the RSJ and the RSJτ model give different I-V characteristics depends only on one parameter which in dimensionless form can be written $\Gamma = (2eR_N I_0\tau/\hbar)(l/\xi)^2$. The three RSJ$\tau$ curves in Fig. 1 have $\Gamma = 15$, 35, 90. In Fig. 1 we show three I-V characteristics calculated on the basis of the RSJτ model with R_N and I_0 chosen as for the RSJ curves and with τ chosen such that $(\hbar/2e\tau)(\xi/l)^2 = 1$ μV. This choice implies a temperature dependence for τ which is $\tau = \tau_0/(1-t)$, with $\tau_0 = 10^{-10}$ sec [$l = 0.25$ μm, $\xi = 0.13$ μm$/(1-t)^{1/2}$]. Perhaps the simplest implication of this temperature dependence (for the RSJτ model) is that the voltage at which the I-V characteristic has maximum negative curvature becomes independent of temperature. This is indeed found experimentally for all small Dayem bridges tested as far down in temperature as this part of the I-V characteristic could be studied. The value of τ_0 found for all these bridges was the same within a factor of 2. For temperatures lower than those displayed in Fig. 1 the I-V characteristic develops hysteresis first around 40 μV and later at the top of the supercurrent. We have noticed that only a slight change of the functions $I_0(\theta)$, $I_1(\theta)$, and $I_2(\theta)$ is required to generate instabilities very similar to the experiment. We have chosen, however, to use the universal functions given in Ref. 3 in order to retain the transparency of the model.

The relaxation time which we use is two orders of magnitude longer than that (τ_{GL}) normally assumed in time-dependent Ginzburg-Landau theory.[2] Other types of experiments give similarly long relaxation time.[4] For superconductors with an energy gap the relaxation process is, however, considerably more complex than in a gapless situation normally assumed, because it involves the rather slow relaxation of the imbalance in the distribution between the different branches of the dispersion curves for the excitations out of the ground state. It is for the time being not clear how such a nonequilibrium can be treated within a Ginzburg-Landau approach and our use of the time-dependent Ginzburg-Landau equations must be considered entirely phenomenological.

It is interesting to compare the RSJτ model with a flux-flow model valid for larger bridges.[5] A moving vortex picture makes predictions about the I-V characteristic which for small voltages (single vortex region) are essentially indistinguishable from those reported here. In Ref. 5 τ_{GL} was used to describe the vortex friction. In order to bring these calculations to a similar type of agreement with our experimental results as the RSJτ model it is, however, again necessary to insert a relaxation time which is two orders of magnitude larger than τ_{GL}. The similarities of these two approaches may possibly explain why the RSJτ model seems to be more widely applicable than the assumptions on which it is based would indicate.

To gain more insight into the frequency dependence of the ac Josephson effect in Dayem bridges we studied the I-V characteristic when the bridges were irradiated with electromagnetic radiation in the microwave frequency region. Earlier investigations of this type[1] were conducted to check the validity of the RSJ model. It was concluded that the RSJ model forms a very good basis for the understanding of Dayem bridges. However, deviations from this simple theory were noted. We have studied the microwave-induced steps at 1, 4.8, 8.3, 16, 37, and 70 GHz and find the supercurrent and the microwave-induced steps in the I-V characteristic vary with the microwave field amplitude in a Bessel-function-like manner similar to the RSJ model at low frequencies, but that this behavior fades out at microwave frequencies beyond 10 GHz, independent of temperature. Thus at higher frequencies the step heights just increased to a maximum and then gradually decreased to zero with increasing microwave power. At the higher frequencies a large number of subharmonic steps appear. The Bessel-function-like behavior could be studied at 1 and 4.8 GHz. We have compared our results to the predictions of the RSJ model by taking the ratio of the microwave field intensities at the first and the second minima of the critical current.[6] This allowed us to determine from experiment the parameter $\hbar\omega/2eR_NI_0$. For temperatures close to T_c ($T = 3.419$ K, Fig. 1), the experimental value is in good agreement with the value found by simply inserting ω, R_N, and I_0. However, at lower temperatures the value of $\hbar\omega/2eR_NI_0$ is too high suggesting that a lower value of R_N must be inserted here. By simulating the microwave-induced steps on the analog computer using the RSJτ model all these features were in fact observed.

In order to display more quantitatively the comparison between experiments and the RSJτ model we have looked for a parameter we could plot as a function of frequency. What is required is a measurable quantity which is characteristic of a certain frequency and temperature and which can be defined by varying the microwave power alone.

The maximum step height of the first step at $V = \hbar\omega/2e$ is such a quantity; it can be maximized without knowledge of the microwave field strength. To get an overall view of the variation of the maximum step height with frequency and temperature we have plotted the experimentally determined ratio of the maximum first step and the supercurrent at zero microwave power versus the parameter $\hbar\omega/2eR_NI_0$ in Fig. 2 at three different temperatures each for the six different frequencies we had available. For clarity we have plotted the data for each temperature separately. For the RSJ model such a plot gives the universal curve shown in Fig. 2. As can be seen at high frequencies the step heights are smaller than predicted by the RSJ model, while at low frequencies they are larger. Only at temperatures close to T_c do the low-frequency step heights lie on the RSJ curve. On the other hand, the curves in Fig.

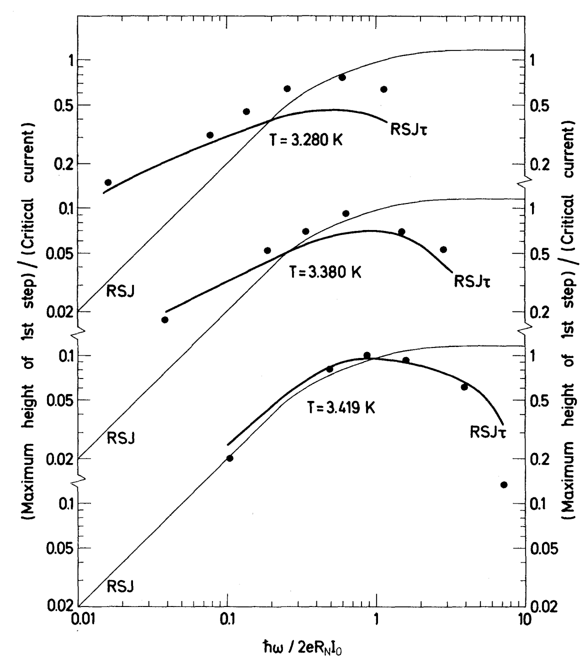

FIG. 2. Maximum height of the first microwave-induced step divided by the critical current, plotted versus normalized frequency $\hbar\omega/2eR_NI_0$ for $\omega/2\pi = 1$, 4.8, 8.3, 16, 37, and 70 GHz. Experimentally the parameter $\hbar\omega/2eR_NI_0$ is determined from the ratio of the microwave field intensities at the first and second minima at low microwave frequencies and close to T_c and extrapolated to lower temperatures as explained in the text. This plot is shown for three different temperatures (corresponding to those of Fig. 1). The sample is the same as used in Fig. 1. The RSJ model yields the u universal curve shown as a thin line whereas the RSJτ model with $\tau = 10^{-10}/(1-t)$ sec gives the solid curves.

2 based on the RSJτ model, with τ chosen as before, qualitatively reproduce all the features in the experimental results. Although we cannot claim any exact correspondence of our experiments with the RSJτ model using the functions $I_0(\theta)$, $I_1(\theta)$, and $I_2(\theta)$ of Ref. 3 and only one adjustable parameter τ, this model is indeed very powerful in bearing out the main features of the properties of indium Dayem bridges over the entire frequency range in which they show the Josephson effect. At $T = 3.419$ K the photon energy at 70 GHz exceeds the pair-breaking limit which may explain the very small step observed. Investigations of indium bridges similar to the one described above irradiated by a 525-GHz laser did not yield any steps at any temperature, only heating effects.[7]

In the future the RSJτ model may well prove very useful in order to get a more realistic picture of the possible practical applications of the Josephson effect. Its roots in the time-dependent Ginzburg-Landau equations also emphasize the possible application of this phenomenological theory to nonequilibrium superconductors with an energy gap.

Some of the most promising applications of Dayem bridges (and point contacts) are in the millimeter and submillimeter wavelength region. But possible applications are based on the assumption that the RSJ model is valid at such high frequencies. Bridges fabricated using our cross-scratch method do not exhibit phaselocking of the ac Josephson effect to a submillimeter radiation[7] and if the RSJτ model is to be taken literally we estimate that bridges an order of magnitude smaller are in fact needed. However, such small bridges cannot be fabricated by our presently available techniques.

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Theory of Surface Spin Waves in Itinerant-Electron Ferromagnets*

A. Griffin and G. Gumbs
Department of Physics, University of Toronto, Toronto, Canada M5S 1A7
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Using a simple model for the boundary scattering, it is shown that the Stoner model of an itinerant electron gas with a finite spin polarization exhibits a well-defined surface-spin-wave mode. This mode always lies above the maximum frequency of the corresponding bulk-spin-wave modes and its degree of localization strongly increases with the bulk exchange splitting.

The basis of much of our understanding of itinerant-electron ferromagnets at low temperatures is the Stoner model. In connection with the transverse spin susceptibility χ_{+-}, this model treats the short-range exchange interaction between the d electrons in the self-consistent-field approximation [or random-phase approximation (RPA)]. As is well known, $\chi_{+-}(\vec{q}, \omega)$ for a bulk itinerant-electron ferromagnet exhibits an isolated spin-wave pole which is separated off from the Stoner (particle-hole) excitation spectrum. In this Letter, we show for the first time that this model also exhibits a well-defined surface-spin-wave mode. We assume a very simple form for boundary scattering of the electrons, namely, specular scattering without any spin flips. The appearance