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Deuteron Form Factor and the Short-Distance Behavior of the Nuclear Force*

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(Received 24 May 1976)

The large-q^2 falloff of the elastic deuteron form factor is used to relate meson exchange to underlying quark currents, to relate the deuteron form factor to large-angle n-p scattering, to demonstrate scale-invariant behavior of the nucleon-nucleon potential, and to determine the deuteron wave function at small n-p separation.

In this Letter we present several new results and predictions on the connection between nuclear and particle physics which follow from a study of the implications of the deuteron having an underlying six-quark constituent structure.[1,2]

The deuteron form factor $F_D(q^2)$ provides an ideal illustration of the continuity between nuclear and particle physics at the microscopic level. At low momentum transfers, where the nucleons can be treated as pointlike objects and are the essential degrees of freedom, the usual effective-potential Schrödinger theory is appropriate, and meson-exchange effects provide the framework for the nuclear interaction. However, at large momentum transfers where the nucleon form factor differs significantly from its $-q^2=0$ value, hadronic substructure comes into play and the electromagnetic interaction begins to resolve an elementary fermion current. The quark degrees of freedom then become appropriate. The elastic form factor of the nucleus is equivalent to the probability amplitude to rearrange n elementary constituents; the dimensional counting prediction,

$$(q^2)^{n-1}F_n \rightarrow \text{const}, \tag{1}$$

then follows assuming a scale-invariant theory.[2] The power law in Eq. (1) is to be contrasted with bootstrap or continuum models with a uniform current distribution. These theories imply an infinite-composite hadronic structure and exponentially damped form factors. Experiment appears to be consistent with Eq. (1) for the pion ($n=2$) and the proton ($n=3$) for $q^2 \gg M^2$. The pion form-factor data extend from 9 GeV^2 in the timelike region ($q^2>0$) to -4 GeV^2 in the spacelike region and the proton data extend to -33 GeV^2. The recent measurement[1] of the deuteron elastic form factor out to $-q^2=6$ GeV^2 appears to be consistent with an underlying six-quark structure for this elementary nucleus.

The deuteron form factor has been investigated using the simplest quark diagrams, in particular a democratic-chain model and a constituent-interchange model. These models are constrained

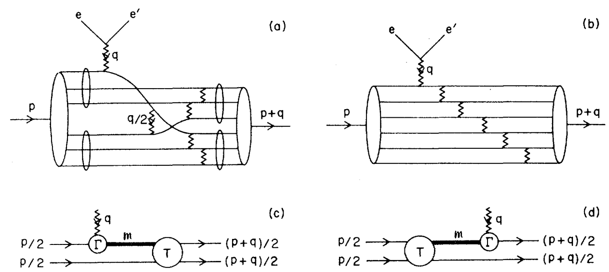

FIG. 1. Three views of the deuteron form factor at large q^2 from (a) constituent-interchange model, (b) democratic-chain model, and (c) off-shell nucleon-nucleon scattering at fixed $\theta_{c.m.}$.

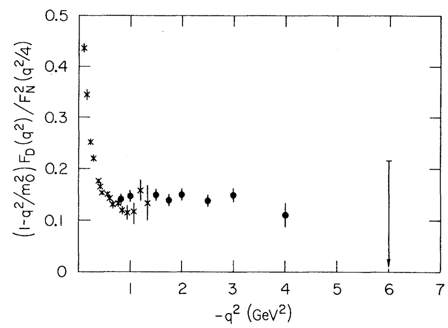

FIG. 2. The deuteron-form-factor data compared to the quark-model prediction, $(1-q^2/m_0^2)f_D(q^2)\to$ const with $m_0^2=\frac{6}{5}\beta^2=0.28$ GeV^2.

to agree with dimensional counting and Bjorken scaling. In particular, the structure of F_D in a relativistic theory can be understood in some detail from Fig. 1(a). If one neglects the nuclear binding, then the calculation of the form factor requires each nucleon to absorb momentum transfer $\sim\frac{1}{2}q^\mu$. Thus it is natural to define the "reduced" form factor of the deuteron,

$$f_D(q^2)\equiv F_D(q^2)/F_N^2(\tfrac{1}{4}q^2), \qquad (2)$$

where the two powers of the nucleon form factor remove in a minimal way the effects of nucleon structure. Using dimensional counting, Eq. (1), we predict for large q^2

$$(q^2-m_0^2)f_D(q^2)\to \text{const}. \qquad (3)$$

A comparison of this prediction with the data of Arnold *et al.*,[1] Elias *et al.*,[3] and Galster *et al.*[3] is shown in Fig. 2. (The value $m_0^2=0.28$ GeV^2 is predicted from the parametrization of the pion form factor.) The approximately constant behavior of $(q^2-m_0^2)f_D(q^2)$ in Fig. 2 for $-q^2 \gtrsim 0.7$ GeV^2 appears to be a striking success for the quark-counting approach. This result is insensitive to the precise value of m_0^2. Furthermore this premature onset of scaling appears to be a general feature of electromagnetic interactions of hadrons.

The diagram in Fig. 1(a) can be regarded as the prototype for meson-exchange currents. At lower q^2 where coherent exchanges of gluon interactions can bind the quark lines to form virtual-meson states, the quark approach merges with the conventional calculations of the meson-exchange currents. Previous calculations using the meson degrees of freedom, however, have predicted a deuteron form factor at large q^2 far in excess of experiment.[4] In common with Fig. 1(a), these calculations explore mechanisms which share the transferred momentum q^μ equally to the deuteron's two nucleons. The meson-exchange-current calculations have not, however, included the off-shell vertex form factors at the meson-nucleon vertices.[5] These must diminish at least as fast as $F_N(q^2/4)$ as is immediately evident in the quark calculations. Thus the short-distance behavior of nucleon interactions as dictated by the quark model supplies the missing constraints of the previous hadronic-level calculations. The deuteron form factor must reflect the underlying five powers of q^2.

For a democratic-chain model as in Fig. 1(b), we have the result,

$$F_n(q^2)\sim\prod_{j=1}^{n-1}[\beta^2-j(j+1)q^2/n^2]^{-1}, \qquad (4)$$

where we have assumed a universal mean-square quark momentum (proportional to β^2) and have neglected spin effects. To order β^2/q^2 Eq. (4) is equivalent to

$$F_n(q^2)=C_n\left[\frac{1}{1-q^2/m_n^2}\right]^{n-1}, \qquad (5)$$

where $m_n^2\equiv n\beta^2$. Setting the value of β^2 from the pion form factor $F_\pi=[1-q^2/(0.471\pm0.010)]^{-1}$, one obtains $F_N(q^2)\simeq C_N(1-q^2/0.71)^{-2}$ for the nucleon, and $F_D(q^2)\simeq C_D(1-q^2/1.41)^{-5}$ for the deuteron. We do not attempt to calculate the value of C_N and C_D, which depend on a much more detailed parametrization of the binding and interaction strength. Comparison of the data with Eqs. (4) or (5) shows that scaling sets in roughly 2 GeV^2 lower in q^2 than when using Eq. (1) for the pion, proton, and deuteron.

Three especially interesting results follow from the intimate connection of n-p and p-p elastic scattering at large t and fixed angle to the deuteron form factor. If the deuteron wave function $\psi_D(x)$ is finite at $x_\mu \to 0$, then the calculation of the large-q^2 limit of $F_D(q^2)$ is equivalent to the calculation of the amplitude for the process

$$\gamma_V(q^2) + p + n \to p' + n',$$

where the initial nucleons each have four-momentum $\sim p/2$ and the final nucleons have four-momentum $\sim (p+q)/2$. Thus as seen in Fig. 1(c), F_D has the structure

$$F_D(q^2) \sim 2\Gamma(q^2)\Delta(q^2/2)T(q^2)\psi_D^2(0), \tag{6}$$

where the coupling of the nucleon to an off-shell state with mass squared $\mathfrak{M}^2 = q^2/2$ is given by the vertex function $\Gamma(q^2)$, the off-shell nucleon propagator is $\Delta(\mathfrak{M}^2)$, and

$$T(q^2) = T(t = u = q^2/4, \mathfrak{M}^2 = q^2/2) \tag{7}$$

is the connected n-p scattering amplitude for 90° scattering with one nucleon leg off-shell.

In fact, we can argue from the observation of Bjorken scaling in deep inelastic scattering or $e^+e^- \to N + X$ that $\Gamma(q^2)\Delta(q^2/2)$ is scale invariant at large q^2. For simplicity we ignore spin here; inclusion of spin factors leads to the same results. Thus

$$F_D(q^2) \sim 2(q^2/2)^{-1}T(q^2)\psi_D^2(0), \tag{8}$$

and the asymptotic behavior of $F_D(q^2)$ is controlled directly by the large $t=u$ behavior of the off-shell n-p scattering amplitude.

In the case of *on-shell* p-p and n-p scattering the fixed $\theta_{\text{c.m.}}$ cross section fits the form

$$d\sigma/dt = (1/16\pi s^2)|T|^2 = f(\theta_{\text{c.m.}})/s^n, \tag{9}$$

with $n = 9.7 \pm 0.5$ for $pp \to pp$, $|t|, |u| \gtrsim 2$ GeV^2. (The dimensional counting prediction is $n = 10$.[2]) Thus at 90°, $T \propto s^{-3.85 \pm 0.25}$. For general sets of field-theory graphs, it is straightforward to show[2] that the scaling behavior $T(q^2) \propto (q^2)^{-n}$ for $t = u = q^2/4$ is unchanged by the extrapolation from on-shell to spacelike $\mathfrak{M}^2 = q^2/2$. Thus we can predict directly from Bjorken scaling of deep-inelastic scattering and the observed fixed-angle scaling behavior of nucleon-nucleon scattering the large-q^2 result

$$F_D = \text{const}/(q^2)^{4.85 \pm 0.25}, \tag{10}$$

in agreement with Eq. (1) for $n = 6$ and Eq. (3) as in Fig. 2.

It is of interest to see whether we can understand the order-of-magnitude constants entering in the form-factor calculation. The large-t on-shell 90° nucleon-nucleon scattering amplitude fits the approximate form $T \sim (5 \times 10^3\ \text{GeV}^8)/t^4$. For the off-shell continuation, we shall assume the form

$$T(q^2) \sim (5 \times 10^3\ \text{GeV}^8)/t^2(t + \mathfrak{M}^2)^2, \tag{11}$$

as suggested by the extrapolation of off-shell form factors. Using Eq. (8), we then have

$$\psi_{NR}^2(0) \cong 2M_p\psi_D^2(0) = M_p q^2 F_D(q^2)/2T(q^2). \tag{12}$$

Taking $(q^2)^5 F_D(q^2) \sim 1\ \text{GeV}^{10}$ gives

$$u'(0) = (4\pi)^{1/2}\psi_{NR}(0) \sim 0.1\, m_\pi^{3/2}, \tag{13}$$

which is of the order of magnitude of the s-wave wave function [normalized to $\int_0^\infty u^2(r)\,dr = 1$] obtained for the soft-core potentials.[6]

Notice also that the consistency between the asymptotic scaling laws for $T(q^2)$ and $F_D(q^2)$ requires $u'(r)$ to be nonzero at $r \to 0$. Thus there is no "hard core" in the effective nucleon-nucleon potential, at least for the range $r \gtrsim 1/Q_{\max} \sim 0.06/m_\pi \sim 0.08\,F$ probed thus far by the deuteron-form-factor measurements.

The third result from the connection of N-N elastic scattering and the deuteron form factor relates to $V_{\text{eff}}(q^2) \equiv 2M_p T(q^2)$, the effective nucleon-nucleon potential in two-body Schrödinger theory. As we have seen, the asymptotic decline of $T(q^2)$ is consistent with the $(q^2)^{-4}$ behavior of $F_N^2(q^2/4)$. Thus the entire falloff of the effective potential can be understood to be due to just the dynamical structure of the nucleons themselves, with no additional falloff from the exchange force. The scaling behavior for the reduced amplitude

$$t(q^2) = T(q^2)/F_N^2(\tfrac{1}{4}q^2) \sim \text{const} \tag{14}$$

is in fact (modulo logarithmic) exactly what is expected in underlying theories which are scale-invariant at short distances, including quantum electrodynamics (in perturbation theory) and gauge theories with asymptotic freedom.

The final results which are reported here generalize on the reduced form factor in Eq. (2). The underlying partition model as in Fig. 1 is based on the finiteness of the hadronic wave function at $x_\mu = 0$ relative separation.[2] Binding corrections can then be neglected at large q^2, and the calculation of the asymptotic form factor is equivalent to calculation of the amplitude $\mathcal{T}_n$ for rearranging n constituent quarks parallel to p^μ to the final direction $p^\mu + q^\mu$. The partition model also leads to simple predictions for nuclear targets or general systems with a series of scales of compositeness.

Thus, consider a composite of A constituents each with an on-shell form factor $F_i(q^2)$. In the limit where binding can be neglected, each constituent absorbs momentum $(m_i/m_A)q$. Thus it is natural to define the "reduced" form factor

$$f_A(q^2)=F_A(q^2)\left[\prod_{i=1}^{A} F_i\left(\frac{m_i^2}{m_A^2}\,q^2\right)\right]^{-1}, \tag{15}$$

which removes the minimal falloff of the form factor due to the constituents' structure. It is clear physically that $f_A(q^2)$ should be a decreasing function of q^2 since one still has to pay a penalty for keeping A intact. Using dimensional counting, i.e., an underlying scale-invariant theory, we have from Eq. (1)

$$f_A(q^2)\to \mathrm{const}/(q^2)^{A-1}. \tag{16}$$

Thus the reduced form factor is predicted to have the same falloff as a corresponding bound state of elementary constituents! In particular, $f_D(q^2)$ and $f_{^3\mathrm{He}}(q^2)$ are predicted to have the same monopole and dipole falloff as the mesons and baryons, respectively. The α particle reduced form factor is predicted to diminish as $(q^2)^{-3}$. The results for the deuteron form factor in Fig. 2 are interpreted as a confirmation of Eq. (16) for the $A=2$ bound state.

More generally for any nuclear reaction at large t, $aA\to aA$ where $a=e,\pi,p$, etc., it is useful to define the reduced scattering amplitudes

$$\tau(aA\to aA) \equiv \mathcal{T}(aA\to aA)\left[\prod_{i=1}^{A} F_{N_i}\left(\frac{m_i^2}{m_A^2}\,t\right)\right]^{-1}, \tag{17}$$

which removes the effect of the probability for keeping the nucleons intact. The reduced amplitude $\tau(aA\to aA)$ then reflects the nuclear aspects of the scattering. Further, the ratio

$$R(aA\to aA)=\mathcal{T}(aA\to aA)/\mathcal{T}(eA\to eA), \tag{18}$$

effectively removes the falloff of the amplitude due to keeping the nucleus intact, and is the most sensitive test of the specific interaction of the projectile. Therefore Eq. (18) is convenient for analyses of the validity of the impulse approximation at large momentum transfer.

The methods we have discussed can also be applied to inelastic electron scattering on nuclei, below the meson-production threshold. Following the parton-model analyses, one derives[2,7]

$$\frac{d\sigma(eA\to e'X)}{dq^2dx}=\sum_{i=1}^{A}\frac{d\sigma(eN_i\to eN_i)}{dq^2}G_{N_i/A}(x), \tag{19}$$

where $x=(-q^2/2p_A\cdot q)$, $d\sigma/dq^2$ is the elastic e-nucleon amplitude at $s'=xs$, and $G_{N_i/A}$ is the probability for the nucleon to have fractional momentum x in the infinite-momentum frame of the nucleus A. For $x\to 1$, and using quark counting rules, one obtains

$$G_{N/A}(x)=C(1-x)^{6(A-1)-1}, \tag{20}$$

where $3(A-1)$ is the number of quark spectators in the reaction. This result also provides continuity between the exclusive and inclusive limit and the prediction

$$d^2\sigma/dq^2dW^2=(d\sigma/dq^2)|_{\mathrm{elastic}}\,\rho(W^2), \tag{21}$$

that the inelastic and elastic cross sections fall uniformly in q^2 at fixed $W^2=(q+p_N)^2$. The prediction $G_{p/D}\sim(1-x)^5$ can be tested in the fragmentation of a deuteron in high-energy deuteron collisions.

The results and predictions presented in this Letter should help clarify the continuity between nuclear and particle physics which, in our view, exists at the microscopic level.

We are grateful to R. Arnold, R. Blankenbecler, G. Farrar, J. Gunion, I. Schmidt, and W. Schütz for helpful suggestions and discussions.

*Supported in part by the U. S. Energy Research and Development Administration and the National Science Foundation.

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