Statistical Approach to Large-$p_\perp$ Phenomena in Meson-Nucleon and Nucleon-Nucleon Collisions*

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It is shown that the new features of large-$p_\perp$ phenomena observed in high-energy inclusive $\pi^{\pm}p$ and pp reactions can be understood in terms of a statistical model. Consequences of this model in other processes, for example, $K^{\pm}p$ and $\bar{p}p$, are also discussed.

In a recent paper,[1] Donaldson *et al.* have reported on an experiment on inclusive production of large-transverse-momentum ($p_\perp > 1$ GeV/c) neutral pions at large c.m. system angles ($\theta \approx 90°$) in $\pi^{\pm}p$ and pp collisions at laboratory energies ($E_{\rm lab}$) 100 and 200 GeV/c (c.m. system total energy $\sqrt{s} = 13.7$ and 19.4 GeV, respectively). It is observed that the ratio of inclusive cross sections

$$R(90°,p_\perp;s;p/\pi^{\pm}) = \frac{d\sigma(90°,p_\perp;s;pp \to \pi^0 X)/d^3p}{d\sigma(90°,p_\perp;s;\pi^{\pm}p \to \pi^0 X)/d^3p} \qquad (1)$$

at given s decreases with increasing $p_\perp$, while the corresponding ratio for $\pi^+p \to \pi^0 X$ to $\pi^-p \to \pi^0 X$, $R(90°,p_\perp;s,\pi^+/\pi^-)$ is approximately equal to unity. Furthermore, the s dependence of the ratio given in Eq. (1) is such that R can be expressed as a function of a single variable of the form $p_\perp s^{-h}$, where h is a constant. Since this measurement is the first experiment on inclusive large-$p_\perp$ particle production in meson-nucleon collision processes, it is expected that the result may reveal some of the unknown features of the large-$p_\perp$ phenomena and will provide a crucial test for the various theoretical approaches. Comparison between the data and a class of models has already been made in Ref. 1. The purpose of this note is to show that the above-mentioned features can be understood in terms of the statistical model discussed by Meng[2] and Meng and Moeller.[3]

According to this model,[2,3] particles with large $p_\perp$ are produced in violent collision processes (we note that *not all* high-energy processes are violent[2,3]), the physical picture of which can be summarized as follows: (a) The high-energy colliding particles arrest each other and form a conglomerate. (b) The conglomerate expands and than decays when a critical volume is reached. (c) At the moment of decay, the conglomerate is considered to be in a state of statistical equilibrium. The critical volume, which is in general a function of all the quantum numbers (baryon number, isospin, etc.) of the total system (that is, of the conglomerate), is assumed to be independent of the incident energy.

It follows from this simple picture (and standard thermodynamics) that the temperature T of the conglomerate at the moment of decay depends on the total energy $\sqrt{s}$ and the critical volume V in the following way:

$$T \sim s^{1/8} V^{-1/4}. \qquad (2)$$

In the first-order approximation, the single-particle inclusive cross section at $\theta = 90°$ in the large transverse momentum region ($p_\perp \gg m$, m is the mass of the observed particle) can be written as $d\sigma/dp^3 \approx \exp(-p_\perp/C_T)$, and it follows that

$$\frac{d\sigma}{d^3p}(90°,p_\perp;s) = f(V)\exp\{-CV^{1/4}p_\perp s^{-1/8}\}, \qquad (3)$$

where C is a constant and f is an undetermined function of V. We obtain from Eqs. (1) and (3)

$$R(90°,p_\perp;s;p/\pi^{\pm}) \propto \exp\{-CV_{pp}^{1/4}[1-(V_{\pi^{\pm}p}/V_{pp})^{1/4}]p_\perp s^{-1/8}\}, \qquad (4)$$

where V_{pp} and $V_{\pi^{\pm}p}$ are the critical volumes of the conglomerates in violent pp and $\pi^{\pm}p$ collisions, respectively. Hence we expect the following:

(I) Both $d\sigma(90°,p_\perp;s;pp\to\pi^0X)/d^3p$ and $d\sigma(90°,p_\perp;s;\pi^\pm p\to\pi^0X)/d^3p$ are functions of the variable $Z_\perp \equiv p_\perp s^{-1/8}$ only. In first-order approximation, they are simple exponential functions of $Z_\perp$. Comparison with the data for $pp\to\pi^0X$ has already been made.[2,4] Comparison with the $\pi^+p\to\pi^0X$ data of Donaldson *et al.*[1] is shown in Fig. 1. The data show an exponential function of $Z_\perp$. The corresponding plot for $\pi^-p\to\pi^0X$ is not presented because of the empirical fact[1] that $R(\pi^+/\pi^-)\approx 1$.

(II) $R(90°,p_\perp;s;p/\pi^\pm)$ is also a function of the variable $Z_\perp$ only and, in first-order approximation, a simple exponential function of $Z_\perp$. Since the factor $CV_{pp}^{1/4}$ is nothing else but the constant b, which has already been determined[2,4] by the $pp\to\pi^0X$ data, this function can be calculated, provided that the ratio $V_{pp}/V_{\pi^\pm p}$ is known. Now, in first-order approximation, the critical volume V can be considered as a sphere of radius r, while the total inelastic cross section is[5] proportional to r^2. Thus, we have in this approximation

$$V_{pp}/V_{\pi^\pm p}=[\sigma_{\text{in}}(pp)/\sigma_{\text{in}}(\pi^\pm p)]^{3/2}. \tag{5}$$

From Eqs. (4) and (5) we obtain

$$R(90°,p_\perp;s;p/\pi^\pm)\propto \exp\left\{-b\left[1-\left(\frac{\sigma_{\text{in}}(\pi^\pm p)}{\sigma_{\text{in}}(pp)}\right)^{3/8}\right]p_\perp s^{-1/8}\right\}. \tag{6}$$

Comparison of this expression with the data of Ref. 1 is given in Fig. 2. The slope of the straight line is calculated from Eq. (6) with $b=7.6$ which has been determined from the pp collision data in Refs. 2 and 4. It agrees with the data as shown in Fig. 2.

(III) $R(90°,p_\perp;s;\pi^+/\pi^-)$ can be expressed in terms of $\sigma_{\text{in}}(\pi^\pm p)$ in a similar way. From the empirical fact that at high energies

$$\sigma_{\text{in}}(\pi^+p)\approx\sigma_{\text{in}}(\pi^-p), \tag{7}$$

we obtain $V_{\pi^+p}\approx V_{\pi^-p}$, and hence,

$$R(90°,p_\perp;s;\pi^+/\pi^-)\approx 1. \tag{8}$$

This result is also in good agreement with experimental data.[1]

(IV) The arguments given in (I), (II), and (III) can be applied to other hadron-hadron (for example, $K^\pm p$, $\bar{p}p$) collision processes.[6] This means in particular that the single-particle inclusive cross sections $d\sigma/d^3p$ in the central region ($\theta\approx 90°$) in these cases are also simple exponential

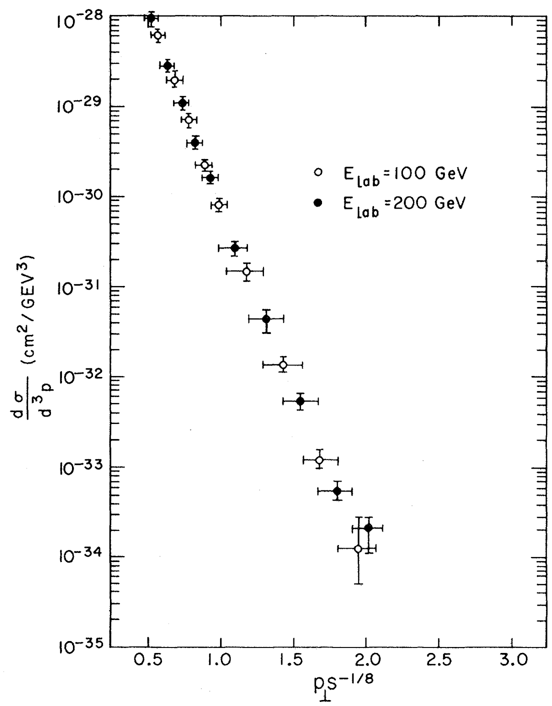

FIG. 1. Single-particle inclusive cross sections $d\sigma/d^3p$ for π^0 at c.m. system angle $\theta\approx 90°$ in the π^+p collision versus $z_\perp\equiv p_\perp s^{-1/8}$, where $p_\perp$ is the transverse momentum (in GeV/c) of the observed π^0 and $\sqrt{s}$ is the total c.m. system energy (in GeV). $d\sigma/d^3p$ is given in cm^2/GeV^3. Data are taken from Ref. 1.

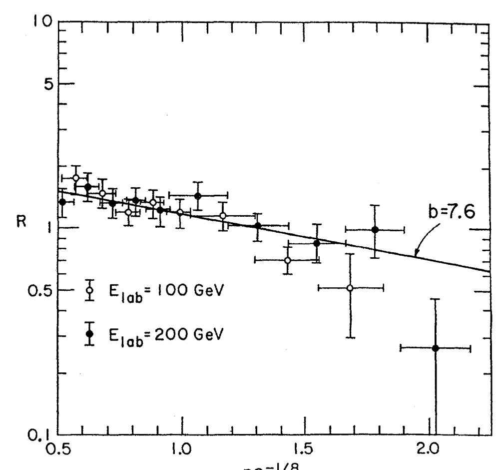

FIG. 2. $R(90°,p_\perp;s;p/\pi^+)$ versus $Z_\perp$. Data are taken from Ref. 1. The straight line is calculated from the model.

functions of $Z_\perp$. We have measured $d\sigma(K^\pm p \to \pi^0 X)/dp$ at 100 and 200 GeV/c as a function of $P_\perp s^{-1/8}$. They show similar scalings to the π data (they are not presented here). The $Z_\perp$ dependence of the ratio of $d\sigma/d^3p$ for different projectiles or targets can be calculated from the corresponding ratio of total inelastic cross sections.

(V) Similar results can be obtained for the general case where θ is arbitrary. In the first-order approximation we have

$$\frac{d\sigma}{d^3p}(\theta, p_\perp; s) \propto \exp\{-CV^{1/4} p s^{-1/8}\}, \qquad (9)$$

where $p = p_\perp/\sin\theta$ is the momentum of the observed particle. That is, $d\sigma/d^3p$ is a simple exponential function of the variable[7]

$$Z = p s^{-1/8} \qquad (10)$$

which reduces to $Z_\perp$ at $\theta = 90°$. Since, in obtaining Eq. (9), we have neglected all noncentral violent collisions,[2,3] the effect of which becomes stronger when $\Delta\theta \equiv |\theta - 90°|$ becomes larger,[8] the expression (9) can be a good approximation for small $\Delta\theta$ only. Nevertheless we expect that (9) and the corresponding equation (4) will provide a rough, qualitative description of the θ dependence in violent collision processes.

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[1]G. Donaldson *et al.*, Phys. Rev. Lett. 36, 1110 (1976). One of the parton models has not been considered, namely $q + (qq) \to M^* + B^*$ subprocess. The data are consistent with this subprocess only.

[2]Meng Ta-chung, Phys. Rev. D 9, 3062 (1974).

[3]Meng Ta-chung and E. Moeller, State University of New York at Stony Brook Report No. ITP-SB-75-53 Rev. (to be published).

[4]S. E. Ellis, in *Proceedings of the Seventeenth International Conference on High Energy Physics, London, England, 1974*, edited by J. R. Smith (Rutherford High Energy Laboratory, Didcot, Berkshire, England, 1975), p. V-23; P. V. Landshoff, *ibid.*, p. V-57.

[5]Recent experiments (see the papers cited in Refs. 2 and 3) seem to suggest the following simple physical picture: Particles with large transverse momentum and/or particles associated with large multiplicities (i.e., also those in high-multiplicity events which only consist of low-$p_\perp$ particles) are produced by the *same* kind of collision processes (violent collisions) which can be described by the model given in Refs. 2 and 3. This implies, among other things, that violent collisions contribute a large part to the total inelastic cross section.

[6]The simple picture for high-energy hadron-hadron collisions can also be applied to hadron-nucleus processes. See Meng Ta-chung, State University of New York at Stony Brook Report No. ITP-SB-76-7 (to be published).

[7]Here we only consider the large-$p_\perp$ region where the mass of the observed particle can be neglected.

[8]This point has already been discussed by E. Fermi, Phys. Rev. 81, 683 (1951).