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COMMENTS

Anomalous Angular Distributions and the Unique Structure of the $l = 1$ Transition Amplitude*

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The anomalous $l = 1$ angular distributions found in heavy-ion-induced single-particle transfer reactions have been studied. The existence of a simple but special form of the $l = 1$ transition amplitude is pointed out. The sensitivity of this transition amplitude to the geometry of the exit-channel distorting potential is demonstrated and a possible explanation of the observed anomalies in terms of this sensitivity is discussed.

Recently several experimental angular distributions observed in heavy-ion reactions for transitions from the $1p$ orbit in projectiles to $\frac{1}{2}^+$ states in residual nuclei have been reported[1-4] where the data showed oscillations which were completely out of phase with theoretical predictions. In some instances, similar transitions to $\frac{3}{2}^+$ states have also been observed.[5] These anomalies in all cases correspond to the unique $l = 1$ transfer at certain incident energies or for the predominant $l = 1$ transfer in the case of Ref. 5 if the recoil effect could be neglected. Attempts have been made to explain them in terms of coupled-inelastic-channel effects[6] for specific cases. However, it is not clear that these effects are the sole cause of the anomalies in all cases reported. In particular, in the mass-13 nuclei, the core-polarization effect with ^{12}C in the 2^+ configuration should contribute more to the ground state than the $\frac{1}{2}^+$ state in question,[7] while no evidence of anomalies was observed in the ground-state transitions.[1-3,8] The coupling in the exit channel is also difficult to justify, because the transitions carry, at most, single-particle transition strengths. It may be, however, different for the transitions; for example, in the ^{40}Ca region core-polarization effects are known to be very important. In fact, a coupled-channel Born-approximation calculation has shown a promising indication of such an effect.[6]

In this short Comment, we would like (1) to point out the particular form of the exact full-recoil distorted-wave Born-approximation (EFR-DWBA) amplitude for $l = 1$ transfer which tells us that the $l = 1$ angular distribution can be very sensitive to the DWBA parameters, and (2) to demonstrate in specific instances how this sensitivity can result in drastic phase shifts in the calculated angular distributions.

On the left-hand side of Fig. 1, some of the $l = 1$ transitions observed are shown. The reactions $^{12}C(^7Li,^6Li)^{13}C(\frac{1}{2}^+)$ at 36 MeV[8] and $^{26}Mg(^{11}B,^{10}Be)^{27}Al(\frac{1}{2}^+)$ at 114 MeV[9] are reproduced well by the EFR-DWBA calculation[10] (shown by the heavy solid curves). Their wave numbers are $k_a = 2.2$ and $5.4\ fm^{-1}$, respectively, for the incident channel. The other four cases[1-4] show anomaly and their wave numbers are between these for the above two reactions. The dashed and thin solid curves show partial differential cross sections, $\sigma_{m=0}$ and $\sigma_{|m|=1} \equiv \sigma_{m=1} + \sigma_{m=-1}$, respectively. Here

m is defined as the z component of the transferred angular momentum l, where the z axis is specified to be parallel to the momentum direction of the incoming particle. It can be seen on the left-hand side of Fig. 1 that the $|m|=1$ component plays a predominant role particularly at the forward angles in all cases, whereas the $m=0$ part is suppressed and shows a rather in-phase oscillation with angular distributions observed. This relative suppression of the $m=0$ component in general is rather unexpected. In heavy-ion transfer reactions it can be expected that transfer components with lower m values give a comparable contribution to the total cross section, since the expectation value of m, which is also the z-axis projection of l_b, is very large. In light-ion transfer reactions, the highest m component always plays a predominant role,[11] since the partial waves contributing are rather low. Consequently, it is interesting to analyze the characteristic features of the transition amplitude that produce the predominance of this $|m|=1$ component.

The EFR-DWBA cross section is expressed by the incoherent sum of the following reduced amplitude over the m components, after the coherent sum over l_b (Ref. 10):

$$\beta_{l_b}{}^{lm}(\theta)=\sum_{l_a}(-1)^{l_b+m}(2l_b+1)[(2l_a+1)/4\pi]^{1/2} \times (l_a 0 l_b m | lm) I_{l_b l_a}{}^{l} Y_{l_b m}(\theta,\varphi=0). \quad (1)$$

The radial overlap integral $I_{l_b l_a}{}^{l}$ contains initial and final distorted waves, $\chi_{l_a}(r_a)$ and $\chi_{l_b}(r_b)$, and the radial factor $F_{l_b l_a}{}^{l}(r_b, r_a)$. For the $l=1$ transition, two interesting cases can be pointed out, depending on whether l_1+l_2 is even or odd, where l_1 and l_2 are the bound orbital angular momenta of the transferred nucleon in the residual and projectile nuclei, respectively. When l_1+l_2 is even, there appears only the diagonal element $I_{l_b, l_a=l_b}$ because of the parity conservation rule that $l_1+l_2+l_a+l_b$ must be even. The possible m transfers are restricted to ± 1 since l_a+l_b+l must be an odd number [see also Eq. (1)]. This first case corresponds, for example, to the $^{12}C(^{14}N, ^{13}N)^{13}C$(g.s. $\frac{1}{2}^-$) transition. On the other hand, if l_1+l_2 is odd, only the nondiagonal components, $I_{l_b, l_a=l_b\pm 1}$, appear and hence all three $m=0, \pm 1$ transfers are allowed since l_a+l_b+l is now an even number. An example for this situation is the $^{12}C(^{14}N, ^{13}N)^{13}C$(3.09 MeV, $\frac{1}{2}^+$) transition.

For this second case, it can be shown that the reduced transition amplitude $\beta_{l_b}{}^{m}(\theta)$ has a partic-

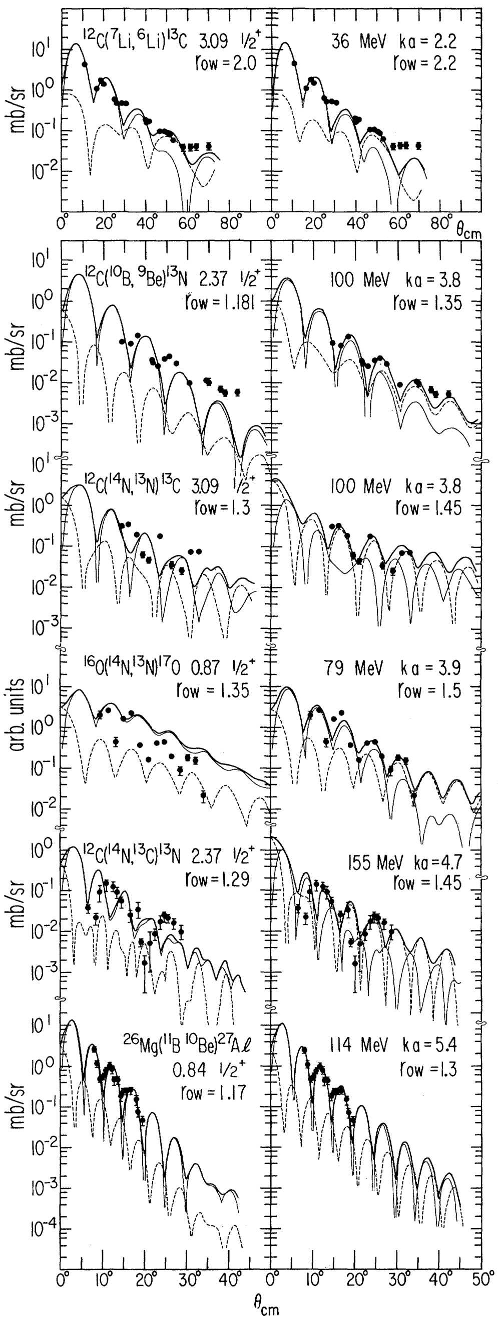

FIG. 1. The $l=1$ angular distributions. The data observed (Refs. 1–4, 8, 9) are shown by solid circles. The EFR-DWBA results obtained using the same and different imaginary radius parameters r_{0w} for the exit channel are shown on the left- and right-hand side, respectively. Other parameters are the same as used in the original works. The dashed and thin solid curves show the partial $m=0$ and $|m|=1$ cross sections, respectively, and the heavy solid curves show the sum of these two components.

ular form. Corresponding to the $m=0$ or $m=\pm 1$ transfer, it is expressed as

$$\beta_{l_b}{}^{0}(\theta)=C_{l_b}\left[I_+-\left(\frac{l_b}{l_b+1}\right)^{1/2}I_-\right]Y_{l_b 0}(\theta,0), \quad (2a)$$

$$\beta_{l_b}{}^{\pm 1}(\theta)=\mp C_{l_b}\frac{1}{\sqrt{2}}\left[\left(\frac{l_b}{l_b+1}\right)^{1/2}I_+ + I_-\right] \times Y_{l_b \pm 1}(\theta,0), \quad (2b)$$

where $C_{l_b}=[3(l_b+1)(2l_b+1)/4\pi]^{1/2}$ and $I_\pm$ represents $I_{l_b, l_a=l_b\pm 1}$. From these expressions, we can see that there is a unique but very simple interference form of overlap integrals $I_\pm$ corresponding to different m transfers. This property has already been pointed out by Bond *et al.*[5] for the special case of no-recoil, normal-parity transfer.

In the upper part of Fig. 2, the values of the overlap integrals are shown as functions of l_b for the reaction $^{12}C(^{14}N,^{13}N)^{13}C(\frac{1}{2}^+)$ at 100 MeV. Since each real (imaginary) part of I_+ and I_- components has the same sign[12] for every angular momentum l_b, the $|m|=1$ partial amplitude [Eq. (2b)] has a constructive form, whereas the $m=0$ amplitude [Eq. (2a)] has a destructive form. A similar situation prevails for other cases where the $|m|=1$ component is predominant thereby producing anomalies as can be seen on the left-hand side of Fig. 1. Since it is obvious that it is the enhancement of the $m=0$ component which could reproduce the observed angular distribution in cases where there are anomalies, we decided to test the sensitivity of the sign of the radial overlap integral to DWBA parameters. It was found that the results are quite sensitive to the radius parameter r_{0w} of the imaginary distorting potential in the exit channel especially for cases which exhibited an anomaly, where conventionally the same parameters as those in the incident channel have been employed for the sake of simplicity. One typical example is shown in the bottom of Fig. 2, where the radius r_{0w}(exit) was increased by 10% from that shown at the upper part for the same reaction. ImI_- is affected dramatically by this change around the most important partial angular momenta, $l_b=20-26$, whereas the ImI_+ is hardly altered.

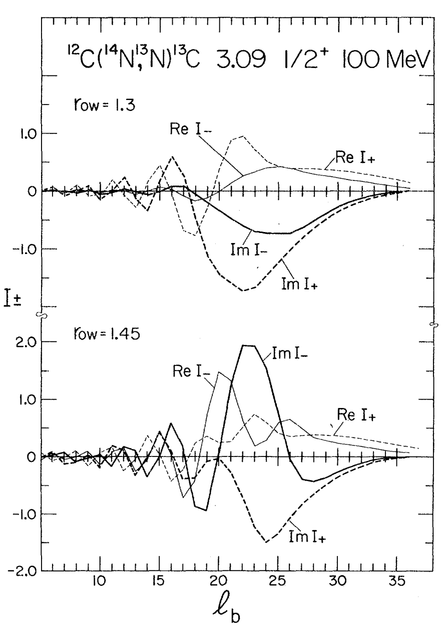

FIG. 2. The radial overlap integrals as a function of l_b for exit channel with r_{0w} unchanged (upper part) and after change (lower part).

In the right-hand side of Fig. 1, the results obtained after r_{0w}(exit) was increased by about 10% are shown. Several features can be noticed: (1) For the normal transitions, i.e. $^{12}C(^7Li,^6Li)^{13}C$ at 36 MeV and $^{26}Mg(^{11}B,^{10}Be)^{27}Al$ at 114 MeV, the changes are insignificant; (2) for anomalous cases, the angular distributions are shifted to the forward direction; and (3) the $m=0$ partial cross section is enhanced thereby reproducing the correct phase of the oscillations as well as the slope of the angular distribution. Particularly for the $^{12}C(^{14}N,^{13}N)^{13}C(\frac{1}{2}^+)$ case at 100 MeV, the $m=0$ component shows a predominance even for extremely forward angles and it was found that even a slight change of r_{0w}(exit) brings about an appreciable change of the $m=0$ partial cross section. The spectroscopic factor extracted for this reaction is in agreement within a factor of 2 with those previously known. For the unbound-state transitions (^{13}N, 2.37 MeV, $\frac{1}{2}^+$ residual state), the absolute values calculated are not reliable, since the bound-state approximation was employed in the present calculation.

The insensitivity mentioned first can be understood as follows: In the ($^7Li,^6Li$) reaction the wavelength of the contributing partial waves ($l_b \sim 12$) is very long at the nuclear surface due to the low incident energy and slow changing of the centrifugal potential. As to the ($^{11}B,^{10}Be$) case, such partial waves ($l_b \sim 39$) are almost out of the nuclear potential range and oscillating rapidly due to rapid changing of the centrifugal potential. Consequently, their distorted waves are rather insensitive to the change of potential parameters. The other four anomalous cases lie between these two extreme cases, and then the shape of the distorted waves as well as the amplitude seems more dependent on parameters.

The physical meaning of a favored long-range imaginary potential for the exit channel is not quite clear yet; nevertheless some reasons may be mentioned. Firstly, the scattering system in the exit channel obviously is different from that of the entrance channel and the valence nucleon is in the *sd* shell instead of in the *p* shell. Secondly, the present procedure may correspond to a manifestation of the higher-order effects in the exit channel which consists of loosely bound odd-mass nuclei. Therefore it appears reasonable to hope that more extended work, including higher-order effects as well as the elastic scattering study for the exit system wherever experimentally feasible, would give a complete interpretation for such anomalous phenomena.

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[12]It may be worthwhile to point out that such an in-phase relation in I_+ and I_- can be seen in the plane-wave-approximation limit, though such simplification should be taken with caution. The scattering waves are now $\chi_{l_b \pm 1} \sim j_{l_b \pm 1} \rightarrow \rho^{-1} \sin[\rho - \frac{1}{2}(l_b \pm 1)\pi]$ when $\rho = kr$ is large. Thus the matrix elements $I_\pm \sim \langle j_{l_b}{}' F j_{l_b \pm 1} \rangle$ have the same phase due to the opposite contribution of π from these Bessel functions and $i^{l_1 + l_2 - l_a - l_b}$ from F. Furthermore, the partial amplitudes for $|m| = 1$ and 0 can be written as $I_+ + I_- \sim 2 \langle j_{l_b}{}' F j_{l_b} \rangle$ and $I_+ - I_- \sim -2 \langle j_{l_b}{}' F (d/d\rho) j_{l_b} \rangle$, respectively, using the recursion relation of j_l's. It is noticed here that the first matrix element is insensitive to k (thus energy), while the second one has a strong dependence if there is an l localization.