duced displacement of hydrogen is a strong function of the irradiating temperature, being higher at higher temperatures, and (3) hydrogen is displaced more readily than deuterium.

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Measurement of the Velocity Autocorrelation Time in a Two-Dimensional Electron Liquid

C. L. Zipfel, T. R. Brown, and C. C. Grimes
Bell Laboratories, Murray Hill, New Jersey 07974
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The linewidth for electronic transitions in the image-induced potential well at the surface of liquid helium with a magnetic field applied parallel to the surface is observed to decrease as electron areal density increases. The interpretation is that the lines are motionally narrowed. The electron-velocity autocorrelation time is deduced from the data.

We describe a novel experiment that yields the first measurement of the velocity autocorrelation time for electrons in a particularly simple two-dimensional (2D) electron liquid. This system should thus provide a nearly ideal arena to test calculations of the properties of the 2D electron liquid. Calculations on such systems are notoriously difficult because for the liquid there is no small expansion parameter, in contrast to the situation in the low-density electron gas or the high-density electron crystal.

The two-dimensional electron system that we have studied consists of a sheet of electrons in surface states above a free surface of liquid helium. The electrons are trapped at the surface in a potential well formed by the attractive image potential and the repulsive barrier to penetration into the liquid which arises from the exclusion principle.[1-3] To study the electronic motion we have applied a magnetic field B parallel to the surface and measured the resulting broadening of spectroscopic transitions between bound states in the well. Application of B couples the electronic motion parallel to the surface to the spectroscopic transitions through the Lorentz force. A study of line broadening as a function of electron areal density then reveals how the electron-velocity autocorrelation time decreases with increasing electron density.

In making models for electron motion parallel to the surface, an important parameter is the ratio of the interelectron potential energy to the kinetic energy, defined as $\gamma = \pi^{1/2} n^{1/2} e^2/kT$, where n is the electron areal number density. The data presented here cover the range from $\gamma = 9$ to 36 as shown in Table I. For $\gamma < 1$ the electrons are expected to behave as a classical free-electron gas (FEG). As γ increases the electronic motion

TABLE I. Experimental parameters n, γ, $\Gamma(0)$, and α. The estimated uncertainty in n is $\pm 10\%$. The velocity autocorrelation time τ_c is derived from the data. The quantity ω_0^{-1} is the reciprocal of the harmonic oscillator frequency in a 2-d triangular lattice.

n (10^8 cm^{-2})	γ	$\Gamma(0)$ (GHz)	α (kG^{-2})	τ_c (10^{-11} sec)	ω_0^{-1} (10^{-11} sec)
0.15	9	1.5	0.57	9.9	12.6
0.20	11	1.4	0.51	7.2	10.1
0.51	17	1.4	0.39	4.8	5.0
1.4	29	1.5	0.19	2.1	2.4
2.2	36	1.7	0.12	1.4	1.7

is expected to become more and more correlated. There have been predictions that at high enough γ the electrons will condense into the 2D classical analog of a Wigner lattice.[4-6] A molecular dynamics simulation using 10^4 classical electrons exhibits a transition from a liquidlike phase to a triangular lattice at $\gamma = 95$.[6]

Recent experiments have measured Stark-shifted energies for transitions from the ground state to excited states in the image-induced potential well at the liquid helium surface.[3] The single-electron Hamiltonian is

$$H = (1/2m)\vec{p}^2 + V(z) + e\mathcal{E}z, \qquad (1)$$

where z is the perpendicular distance from the surface and $\mathcal{E}$ is an electric field applied in the z direction. For $z<0$, the potential $V(z)$ is about 1 eV. For $z>0$, the image potential is $V(z) = -Ze^2/z$, where $Z = \frac{1}{4}(\epsilon-1)/(\epsilon+1) = 7\times 10^{-3}$. Here we have taken the dielectric constant of liquid helium to be $\epsilon = 1.057$. The barrier at the surface is sufficiently large that the solutions to (1) with $\mathcal{E}=0$ are fairly well approximated by $zR_{\mathfrak{N}}(z)$, where $R_{\mathfrak{N}}(z)$ is the s-state wave function for the hydrogen atom with principal quantum number $\mathfrak{N}$. The expectation values of the wave functions for $\mathfrak{N}=1$ and $\mathfrak{N}=2$ are 114 and 456 Å, respectively. The eigenvalues in this approximation are given by $E_{\mathfrak{N}} = -mZ^2e^4/2\hbar^2\mathfrak{N}^2$. This predicts a transition energy from $\mathfrak{N}=1$ to $\mathfrak{N}=2$ of 119 GHz, while 125.9 GHz is actually measured. The effect of the electric field $\mathcal{E}$ is to increase the transition energies and to decrease the expectation values.

For energies near 125 GHz it is most convenient experimentally to work at fixed frequency and to use $\mathcal{E}$ to produce Stark tuning through resonance.[3] The experimental cell is formed by plates 2.2 cm long, 1.3 cm wide, and spaced 0.188 cm apart. The walls of the cell are grounded and $\mathcal{E}$ is applied between the plates. Radiation is coupled into and out of the cell by wave guides at either end. The cell is half-filled with liquid helium, and electrons are put on the helium surface at the beginning of a run by briefly heating a tungsten filament. The field $\mathcal{E}$ is modulated at 100 kHz and the component of absorbed power synchronous with this is detected. This yields the derivative of the absorption line with respect to $\mathcal{E}$.

The top plate potential V_T is always negative or zero, the bottom plate potential V_B is positive, and the filament and walls of the cell are at ground potential (zero). Electrons from the filament will be drawn to the helium surface until it is at the same potential as the filament. If the cell is half full, Gauss's law gives the electron areal number density $n = (\frac{1}{2})(V_B - |V_T|)/\pi he$, where h is the spacing between the plates, and where we have made the approximation $\epsilon = 1$.

The experiment has been performed at temperatures around 1.2 K. The absorption line in zero magnetic field is a Lorentzian and its width $\Gamma(0)$ (taken as the peak-to-peak width of the derivative signal) varies linearly with helium-vapor atom density. This linewidth has been attributed to two mechanisms: direct scattering of the electron by helium atoms in the vapor, and smearing of the electron energy levels caused by fluctuations in the number of helium atoms the electron interacts with as it moves along the surface.[3]

Figures 1 and 2 show the effect of the magnetic field B on the linewidth for transitions from $\mathfrak{N}=1$ to $\mathfrak{N}=2$ at 255 GHz. The linewidths were measured in units of electric field and then converted to frequency units by using $d\nu/d\mathcal{E} = 0.38$ GHz/(V/cm), the Stark-tuning rate at 255 GHz. The linewidth as a function of magnetic field $\Gamma(B)$ has been normalized to unity at $B=0$ to give the normalized linewidth $\tilde{\Gamma} = \Gamma(B)/\Gamma(0)$. Experimental values of n, γ, and $\Gamma(0)$ are given in Table I. The

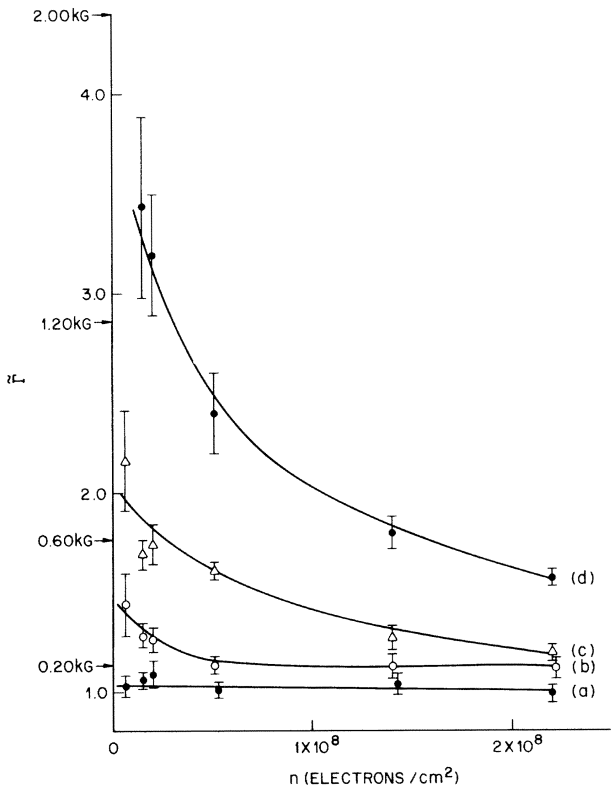

FIG. 1. Normalized linewidth $\tilde{\Gamma}$ vs charge density n for several values of the magnetic field B: (a) 0.20 kG, (b) 0.60 kG, (c) 1.2 kG, (d) 2.0 kG. The lines have no theoretical meaning. Predictions from the FEG model are shown by arrows for each of these values of B.

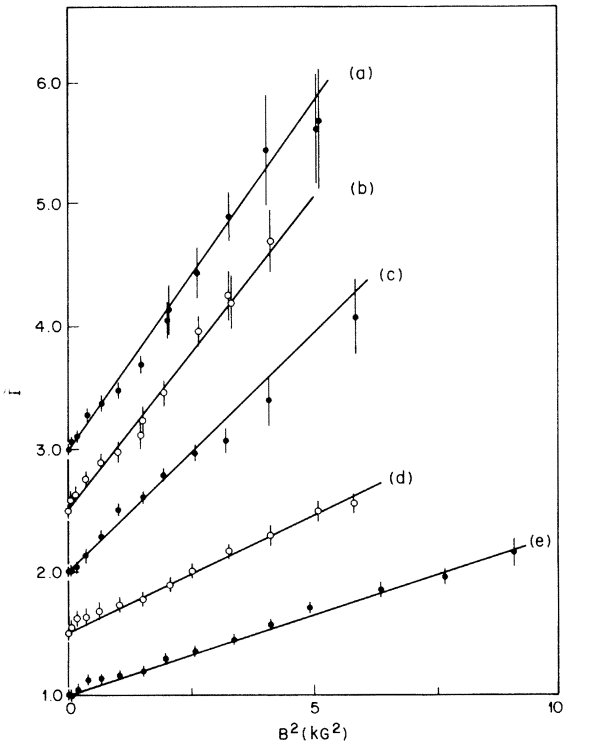

FIG. 2. Normalized linewidth $\tilde{\Gamma}$ vs B^2 for several values of charge density: (a) 0.15×10^8 cm^{-2}, (b) 0.20×10^8 cm^{-2}, (c) 0.51×10^8 cm^{-2}, (d) 1.4×10^8 cm^{-2}, (e) 2.2×10^8 cm^{-2}. For clarity each successive curve has been displaced upward by one half unit. The velocity autocorrelation time is deduced from the slope of each line.

variations in $\Gamma(0)$ were caused by small changes in temperature during measurements. Figure 1 shows that in a given magnetic field $\tilde{\Gamma}$ decreases sharply with increasing n and appears to be approaching unity for large n. Figure 2 shows $\tilde{\Gamma}$ at constant n vs B^2. The apparent quadratic dependence on B suggests that the data can be fitted by the expression $\tilde{\Gamma}=1+\alpha B^2$, where the value of α for each value of n is given in Table I.

In the following paragraphs we consider the effect of B on the absorption line. First we calculate what the linewidth would be if the electrons were free particles described by the FEG model. Second, we assume that at the electron densities used in this experiment the velocity autocorrelation time is so short that the line is motionally narrowed. We use the experimental linewidths to find the correlation time as a function of n.

In the FEG model the electrons are assumed to behave like particles in a classical ideal gas with a Maxwellian velocity distribution. When B is applied in the y direction, the Hamiltonian (1) becomes

$$H=\frac{1}{2m}\left(p_x-\frac{e}{c}Bz\right)^2+\frac{1}{2m}(p_y^2+p_z^2)+V(z)+e\mathcal{E}z,$$

where the gauge used is $\vec{A}=[zB,0,0]$. In this model, electron motion in the x and y directions is described by plane waves, and p_x and p_y are constants of the motion. Expanding the first term gives two perturbation terms $H'=(eB/mc)zp_x$ and $H''=(m/2)(eB/mc)^2z^2$. The perturbation H' adds to $e\mathcal{E}z$ and is equivalent to a shift in $\mathcal{E}$ of v_xB/c for an electron with velocity v_x.[7]

In the FEG model the probability of an electron having a particular value of v_x is proportional to $\exp(-mv_x^2/2kT)$. Therefore, an absorption line at $\mathcal{E}_0$ with zero width would be broadened into a Gaussian $A(\mathcal{E})=A_0\exp[-(\mathcal{E}-\mathcal{E}_0)^2/2\sigma^2]$ with $\sigma=(kT/m)^{1/2}B/c$. When expressed in gigahertz the width of the Gaussian is $2\sigma=3.0B$, where B is in kilogauss. However, since $\Gamma(0)$ is finite, the resulting line will be a convolution of the zero-field Lorentzian with the Gaussian. Predictions of the

FEG model are shown as arrows along the ordinate in Fig. 1. The experimental linewidth rises sharply as n decreases, but it is not clear whether it is approaching the FEG predictions. The FEG model predicts linewidths about twice as large as the measured linewidths at the lowest charge densities, and it fails badly at the highest charge densities.

To estimate the velocity autocorrelation time τ for the electrons, both electron-electron and electron–helium-atom interactions must be taken into account. We denote by τ_c the correlation time due to electron-electron interactions alone. The scattering time τ_s for an electron interacting with helium atoms in the vapor has been determined from cyclotron-resonance experiments, and at 1.2 K it is 1.1×10^{-10} sec.[8] If we take into account both types of interactions, τ will be $(\tau_s^{-1}+\tau_c^{-1})^{-1}$.

In the FEG model the change in the resonance frequency due to the magnetic field is $\Delta\omega_0 = 2\pi \times (v_x B/c)(d\nu/d\mathcal{E})$. If τ is less than $(\Delta\omega_0)^{-1}$ the line will be motionally narrowed.[9] For small values of $\Delta\omega_0\tau$ the half-width of the line will be $(\Delta\omega_0)^2\tau$, and we assume that the line will be Lorentzian. Its width will then add to the zero-field width to give $\Gamma(B) = \Gamma(0) + (2/\sqrt{3})(\Delta\omega_0)^2\tau$, where the widths are in radians/second and where the factor $2/\sqrt{3}$ enters in converting to the peak-to-peak width of the derivative. This can be written as $\tilde{\Gamma} = 1+\alpha B^2$, where $\alpha = (3.0\times 10^9\pi)^2/\Gamma(0)(\tau_s^{-1}+\tau_c^{-1})$. As pointed out earlier, the linewidth data can be fitted by an expression having this form. We have used the experimental values of α to calculate the correlation times τ_c given in Table I.

Crandall has calculated the potential seen by an electron undergoing small oscillations about its equilibrium position in a triangular lattice by summing the potentials due to all the remaining electrons on their lattice sites.[4] The harmonic-oscillator frequency for this well is $\omega_0 = 2.1(e^2/m)n^{3/4}$, where n is the electron density. Table I gives ω_0^{-1} at each electron density studied. The close correspondence shown between τ_c and ω_0^{-1} is striking. It suggests that the momentum relaxation time relevant is motional narrowing is the same as the momentum relaxation time for a collection of harmonic oscillators trapped at lattice sites by their mutual repulsive forces.

In summary, the broadening of the absorption line for surface-state electrons in a magnetic field provides important clues about the motion of the electrons parallel to the surface. The model presented here is only intended to suggest a possible starting point for a detailed theoretical analysis of the system in the region where $1<\gamma<95$.

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