modulation $\tilde{n}/n$ of approximately 5×10^{-3} and a wavelength of 1.5 mm at the laser interaction volume located 135° around the torus from the wave guide. We scanned wavelength from 1 mm to 1 cm at positions of l from the center of the plasma to the limiter radius and did not observe the driven wave. The estimated sensitivity was greater than the expected level of fluctuations by a factor of 5.

In the ATC, the thermal ion feature in the scattered spectrum is expected to be peaked near 400 MHz for a 1-mm-wavelength fluctuation. Scattering was not observed from this feature and this implies the absence of an ion-acoustic wave turbulence during the ATC discharge at levels larger than ten times the thermal level.

We wish to acknowledge the collaboration of M. Porkolab in developing the CO_2 scattering diagnostic for plasma research, of J. Cecchi in interfacing the present apparatus to the ATC, and D. R. Moler in the design and operation of this experiment. We also are indebted to many members of the staff at the Princeton Plasma Physics Laboratory for helpful discussions and for their encouragement, and to the members of the ATC crew for their technical assistance. We also wish to acknowledge helpful conversations with A. Hasegawa, W. Horton, and H. Ikezi.

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[5]Hasegawa has called to our attention that at values of $\tilde{n}/n$ of 10^{-2} the nonlinear coupling of radial and poloidal modes through the $\vec{E}(k,\omega)\times\vec{B}_T$ drift of ions can produce an effective frequency shift comparable to the drift-wave frequency [A. Hasegawa, Phys. Lett. 75A, 143 (1976)]. This might explain both the observed frequency spread at fixed wave vector and the isotropic nature of the turbulence in the plane perpendicular to $\vec{B}_T$.

Resonances in Binary Charged-Particle Collisions in a Uniform Magnetic Field

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I have considered the two-body problem in a uniform magnetic field. For the case of like-particle collisions, a resonance between the velocities transverse to the magnetic field of the test and field particles reduces the problem to that without a magnetic field for the relative-velocity scattering angle and scattering cross section. This resonance condition results in large changes in the test-particle collisional parameters. A secondary resonance for like-particle and resonances for unlike-particle collisions have also been found.

The binary collisional scattering and the related collisional parameters of a plasma in a magnetic field are important for the transport,[1-3] heating,[4-6] and magnetic confinement[7,8] of thermonuclear plasmas in open-ended as well as toroidal configurations. The transport of intense relativistic electron beams, radially confined by their self-pinch magnetic fields, through gas-plasma media over long distances,[9] as well as the interaction of intense electron beams with virtual cathodes in the presence of externally applied magnetic fields[10] also depend on the binary collisional scattering. Previous treatments[11,12] of the binary collision problem in a magnetic field have been approximate, to the extent that ultimately only the maximum impact parameter is altered from the Debye length λ_D to an appropriate average Larmor radius ρ_a whenever $\rho_a<\lambda_D$, thus resulting only in small changes in the value of the Coulomb logarithm.

I start with a test particle of mass m_t, charge q_t, velocity $\vec{v}_t$, and position $\vec{r}_t$ and a field particle of corresponding parameters m_f, q_f, $\vec{v}_f$, and $\vec{r}_f$. I define the reduced-mass and center-of-mass variables:

$$\vec{r}_r = \vec{r}_t - \vec{r}_f, \quad \vec{v}_r = \vec{v}_t - \vec{v}_f, \quad m_r = m_t m_f / m_s, \tag{1}$$

$$\vec{r}_s = (m_t \vec{r}_t + m_f \vec{r}_f)/m_s, \quad \vec{v}_s = (m_t \vec{v}_t + m_f \vec{v}_f)/m_s, \quad m_s = m_t + m_f. \tag{2}$$

The equations of motion for the reduced mass and center of mass are found from the single-particle equations of motion:

$$\frac{d\vec{v}_r}{dt} = \frac{q_t}{m_r}\vec{E}_r + \left\{\vec{v}_s\left(\frac{q_t}{m_t} - \frac{q_f}{m_f}\right) + \vec{v}_r\left(q_f\frac{m_r}{m_f^2} + \frac{q_t}{m_t} - \frac{q_t}{m_s}\right)\right\} \times \vec{B}, \tag{3}$$

$$m_s\frac{d\vec{v}_s}{dt} = \left\{\vec{v}_s(q_t + q_f) + \vec{v}_r\left(q_t - q_t\frac{m_t}{m_s} - q_f\frac{m_t}{m_s}\right)\right\} \times \vec{B}, \tag{4}$$

$$q_t\vec{E}_r = -\nabla_r U(|\vec{r}_r|); \; U = q_t q_f / 4\pi\epsilon_0 r_r; \; \vec{B} = \hat{\imath}_z B, \tag{5}$$

where B is the applied magnetic field and ϵ_0 is the vacuum permitivity in mks units. I have solved Eqs. (3) and (4) numerically on the computer. Certain properties of the complete solution can be predicted from the Lagrangian of the two-particle system and the canonical angular momentum,

$$L = \tfrac{1}{2}m_r v_r^2 + \tfrac{1}{2}m_s v_s^2 - U(|\vec{r}_r|) - \tfrac{1}{2}\{q_t(m_f/m_s)^2 + q_f(m_t/m_s)^2\}(\vec{r}_r \times \vec{B})\cdot\vec{v}_r$$
$$- \tfrac{1}{2}\{q_t m_f/m_s - q_f m_t/m_s\}\{(\vec{r}_r \times \vec{B})\cdot\vec{v}_s + (\vec{r}_s \times \vec{B})\cdot\vec{v}_r\} - \tfrac{1}{2}\{q_t + q_f\}(\vec{r}_s \times \vec{B})\cdot\vec{v}_s. \tag{6}$$

I use cylindrical coordinates $(\rho, \theta, z, \hat{\imath}_\rho, \hat{\imath}_\theta, \hat{\imath}_z)$ for both the reduced mass and center of mass, with appropriate subscripts, to find the canonical angular momentum. For the case of like-particle collisions, the reduced and center-of-mass motion are decoupled and I find

$$P_{\theta_r} = m_r \rho_r^2 \dot{\theta}_r + q_t B \rho_r^2/4 = \text{const}, \quad P_{\theta_s} = m_s \rho_s^2 \dot{\theta}_s + q_t B \rho_s^2 = \text{const}. \tag{7}$$

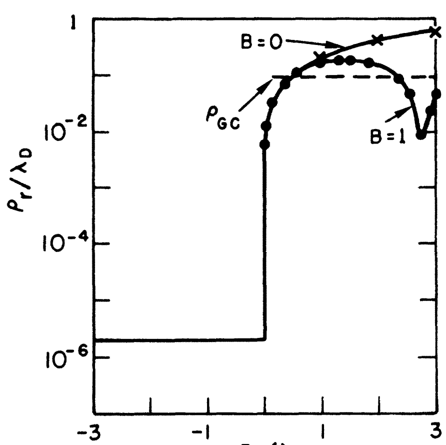

FIG. 1. Reduced-mass trajectory in the ρ-z plane. The scattering center is located at the origin, $\rho_r = z_r = 0$. The trajectory for the case of zero magnetic field is indicated by the solid line for $z \leq 0$ and the cross (×) points for $z > 0$. The impact parameter is $b = 10b_0$ and the scattering angle is 11.42°. The trajectory with the same initial conditions but for $B = 1$ T is given by the curve with dots. The guiding-center trajectory, after scattering, is shown by the dashed curve (ρ_{GC}). The azimuthal plane of ρ_{GC} has advanced from $\theta = 0$ before to $\theta = \pi/2$ after the collision. The parameters chosen to find the Debye length λ_D are electron density 10^{14}/cm^3 and electron temperature 100 eV.

In the absence of the magnetic field the equations simplify substantially and reduce to the classical two-body problem of Kepler. For easy later comparison I summarize the results in Fig. 1 and below. The reduced-mass and the center-of-mass motion are completely decoupled. The reduced-mass particle remains on its initial θ plane. The scattering angle ψ, in terms of the impact parameter b, and the Rutherford scattering cross

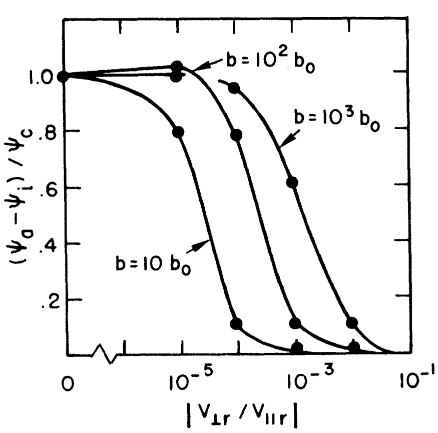

FIG. 2. Resonance curves for electron-electron collisions for $B = 1$ T.

section σ are given by

$$\psi = 2\cot^{-1}(b/b_0),\quad b_0 = q_f q_t/4\pi\epsilon_0 m_r v_r^2,\quad \sigma(\psi, v_r) = b_0^2/(1-\cos\psi)^2. \tag{8}$$

For the case of like-particle collisions ($q_t = q_f$, $m_t = m_f$) the reduced-mass and the center-of-mass motion are decoupled but their respective equations of motion include the Lorentz force. Numerical solution of Eq. (3) together with $d\vec{r}_r/dt = \vec{v}_r$ shows that when initially the component of velocity perpendicular to the magnetic field is zero, $\vec{v}_{\perp r} = 0$, $\vec{v}_r = \hat{\imath}_z v_0$, then after the collision the reduced-mass particle has velocity components

$$\vec{v}_{\parallel r}' = \hat{\imath}_z v_0 \cos\psi,\quad |\vec{v}_{\perp r}'| = v_0 \sin\psi, \tag{9}$$

in agreement with the values predicted by Eq. (8). Figure 1 shows details of a typical trajectory. When initially $v_{\perp r} \neq 0$ then the scattering angle in the collision quickly goes to zero, as shown in Fig. 2. In Fig. 2 there is plotted the normalized scattering angle $(\psi_a - \psi_i)/\psi_c$ versus the initial $|v_{\perp r}/v_{\parallel r}|$, where ψ_i is the initial angle and ψ_a is the final angle of the velocity vector as per Eq. (9), and ψ_c is the Rutherford scattering angle. Off-resonance scattering becomes so small, on account of the dominance of the Lorentz force, that a double-precision code would be necessary for its study. I have concluded, therefore, that in order for a binary collision to occur, the resonance condition

$$\vec{v}_{\perp r} = \vec{v}_{\perp t} - \vec{v}_{\perp f} = 0 \tag{10}$$

has to be satisfied. The scattering cross section is then given by the Rutherford cross section of Eq. (8). From Eq. (7) and conservation of energy I find

$$\dot{\theta}_r = \tfrac{1}{2}\omega_{cr}\left[\frac{b^2}{\rho_r^2} - 1\right],\quad \omega_{cr} = q_t B/2m_r, \tag{11}$$

$$\dot{z}_r^2 = \left\{v_0^2\left[1 - \frac{2b_0}{(\rho_r^2 + z_r^2)^{1/2}}\right] - \tfrac{1}{4}\omega_{cr}^2\rho_r^2\left[\frac{b^2}{\rho_r^2} - 1\right]^2\right\}\left\{1 + \left(\frac{d\rho_r}{dz_r}\right)^2\right\}^{-1}, \tag{12}$$

which reduce to the classical trajectory when $B \to 0$. The resonance condition of Eq. (10) also eliminates the Larmor radius as a relevant maximum impact parameter, and hence the Debye length λ_D is retained as a maximum impact parameter. In the curve for $b = 10^2 b_0$ in Fig. 2 and at $|v_{\perp r}/v_{\parallel r}| = 10^{-5}$, there is evident the existence of a secondary resonance peak superimposed on the primary resonance curve. This secondary resonance occurs when the Larmor radius, $\rho_L = v_{\perp r}/\omega_{cr}$, equals the impact parameter. For modest magnetic fields, $B \lesssim 1$ T, the secondary resonance appears as a small perturbation on the primary resonance while for large magnetic fields, $B = 10$ T, it provides a more substantial (15% higher peak) secondary resonance peak to the primary resonance.[13] The scattering angle in the secondary resonance exceeds that predicted by Rutherford[13] and the related scattering cross section is being investigated.

The effect of the primary resonance alone (valid for $B \lesssim 1$ T) on the test-particle collision frequency for momentum transfer for a nonstreaming bi-Maxwellian ($v_{\perp 0}, v_{\parallel 0}$) distribution of field particles is found[13] from Eq. (16) of Siambis and Stitzer,[14] together with the resonance condition of Eq. (10) included as a δ function:

$$\begin{aligned}\langle \nu_m(v_t)\rangle_f &= 4\big(\langle \nu_m(v_{\parallel t})\rangle_f\big)_{\rm Sp} \exp[-(v_{\perp t}/v_{\perp 0})^2],\quad v_{\parallel t}/v_{\parallel 0} \leq 1,\\ \langle \nu_m(v_t)\rangle_f &= \pi^{-1}\big(\langle \nu_m(v_{\parallel t})\rangle_f\big)_{\rm Sp} \exp[-(v_{\perp t}/v_{\perp 0})^2],\quad v_{\parallel t}/v_{\parallel 0} \geq 3,\end{aligned} \tag{13}$$

where the subscript Sp stands for the Spitzer values.[1,14] Note that the "thermal" like-particle collision frequency is equal to 1.5 times the Spitzer value. The existence of resonances and resonance effects in like-particle binary collisions has been suggested by Longmire.[15]

For the case of $q_t = -q_f$, $m_t \neq m_f$, the resonance condition that initially decouples the reduced-mass motion from the center-of-mass motion is found to be

$$\vec{v}_{\perp s} = -\vec{v}_{\perp r} m_f/m_s. \tag{14}$$

This resonance condition gives the following values for the initial test- and field-particle velocities transverse to the magnetic field: $\vec{v}_{\perp t} = 0$, $\vec{v}_{\perp f} = -\vec{v}_{\perp r}$. Numerical solution of Eqs. (3) and (4) shows that when $v_{\perp f} = 0$ as well, then the scattering of the relative velocity is given by Eq. (9), where ψ is the classical angle and the Rutherford cross section applies and one obtains a primary resonance. When the impact parameter for the reduced-mass motion equals the Larmor radius of the field particle, then a secondary resonance is found superimposed on the primary resonance. For very large impact parameters, $\lambda_D > b \gg b_0$, and for very small values of ψ_i and from a small, nonexhaustive set of computed trajectories one finds that $\psi_a^2 \approx \psi_i^2 + \psi_c^2$.

For the case of $q_t = q_f$, $m_t \neq m_f$, the resonance condition that initially decouples the reduced-mass motion from the center-of-mass motion is found to be

$$\vec{v}_{\perp s} = \vec{v}_{\perp r} m_t / m_s. \quad (15)$$

This resonance condition gives the following values for the initial test- and field-particle velocities transverse to the magnetic field: $\vec{v}_{\perp t} = \vec{v}_{\perp r}$, $\vec{v}_{\perp f} = 0$. Numerical solution of Eqs. (3) and (4) shows that when $\vec{v}_{\perp t} = 0$ as well, then the scattering of the relative velocity vector is given by Eq. (9), where ψ is the classical angle and the Rutherford cross section applies and one obtains a primary resonance. When the impact parameter for the reduced-mass motion equals the Larmor radius of the test particle, then a secondary resonance is found superimposed on the primary resonance.

For the cases given by the resonances of Eqs. (14) and (15) I have also numerically computed trajectories whose initial coordinates do not satisfy the primary and secondary resonance conditions. For these cases I find that there is a very strong coupling between the reduced-mass and center-of-mass motion both in the *absence* and presence of the scattering potential. The equations are essentially driven by the "Lorentz force" that provides the coupling. This strong coupling makes the motion slowly varying or adiabatic and reduces the scattering effects, due to the scattering potential, to higher order in the coupled motion, as opposed to the cases of $B = 0$ and of like particles with $B \neq 0$, where the reduced and center-of-mass motions are decoupled and the effect of the scattering potential is of lower order in the reduced-mass trajectory. It appears, therefore, that a new reference frame will be needed, free of the strong coupling found in the center-of-mass–reduced-mass frames of reference for unlike-particle collisions, which will also permit study of the binary collisional interaction of test and field particles under conditions less restrictive than those given by Eqs. (14) and (15).

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