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Direct Evidence for the Bose-Einstein Effect in Inclusive Two-Particle Reaction Correlations*

N. N. Biswas, J. M. Bishop, N. M. Cason, V. P. Kenney, and W. D. Shephard
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556

and

L. R. Fortney, A. T. Goshaw, J. W. Lamsa, J. S. Loos, W. J. Robertson, and W. D. Walker
Duke University, Durham, North Carolina 27706

and

G. Levman, V. A. Sreedhar, and T. S. Yoon
University of Toronto, Toronto, Ontario, Canada

and

G. Hartner and P. M. Patel
McGill University, Montreal, Quebec, Canada
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Two-particle correlations are studied as a function of t_{12}, the square of the difference of the four-momentum of the particles. We observe that the $\pi^-\pi^-$ correlation differs from the $\pi^-\pi^+$ correlation only at low values of t_{12}. This difference can quantitatively be understood as a consequence of Bose-Einstein statistics.

Short-range correlations for particles produced in high-energy inclusive reactions are known to exist. Various models for multiparticle-production processes, e.g., clustering,[1] have been considered to account for the observed correlations. These models yield correlations arising from dynamical production processes which have a range of 1–2 units of rapidity and are essentially independent of the azimuthal angle between the transverse momenta. Both of these features are observed for correlations involving unlike pions; when the pion pairs have like charges, however, a striking narrow correlation (width ~ 0.4 units of rapidity) for small azimuthal-angle separation is observed.[2,3] It has been suggested that this narrow correlation may be a Bose-Einstein (BE) symmetry effect,[4] rather than a consequence of charge-structure dynamics in particle production. Experimental evidence supporting this explanation has so far been lacking. We present here an analysis which shows evidence for the presence of both dynamical and BE symmetry effects for like charges.

The data presented here are obtained from π^-p interactions at 200 GeV/c using the Fermilab 30-in. bubble-chamber, wide-gap spark-chamber hybrid system. Experimental details have been reported elsewhere.[2] The data sample consists of ~ 17 000 events of all topologies.

The normalized two-particle correlation function is given by

$$R(y_1,y_2\varphi,Q_1,Q_2)=\frac{\rho_2(y_1,y_2,\varphi,Q_1,Q_2)}{\rho_1(y_1,Q_1)\rho_1(y_2,Q_2)}-1. \qquad (1)$$

Here ρ_2 and ρ_1 are the two-particle and single-particle densities, respectively, y is the center-of-mass-system rapidity, Q is the magnitude of the transverse momentum, and the azimuthal angle φ is defined by $\cos\varphi=\vec{Q}_1\cdot\vec{Q}_2/Q_1Q_2$. We integrate Eq. (1) over y_1 and consider the resulting $R(\Delta y,\varphi,Q_1,Q_2)$, where $\Delta y=y_1-y_2$. We denote the correlations for $\pi^-\pi^-$ (like charges) and for $\pi^-\pi^+$ (unlike charges) as $R(--)$ and $R(-+)$, respectively.

In Fig. 1, we present $R(--)$ and $R(-+)$ as a function of Δy for various choices of φ and Q. The values of R, summed over all Q_1 and Q_2, are shown for three regions of φ (0°–45°, 45°–135°, and 135°–180°) in Figs. 1(a) (like charges) and 1(b) (unlike charges). As noted in Ref. 2, $R(--)$ and $R(-+)$ manifest different characteristics in the φ dependence of the rapidity correla-

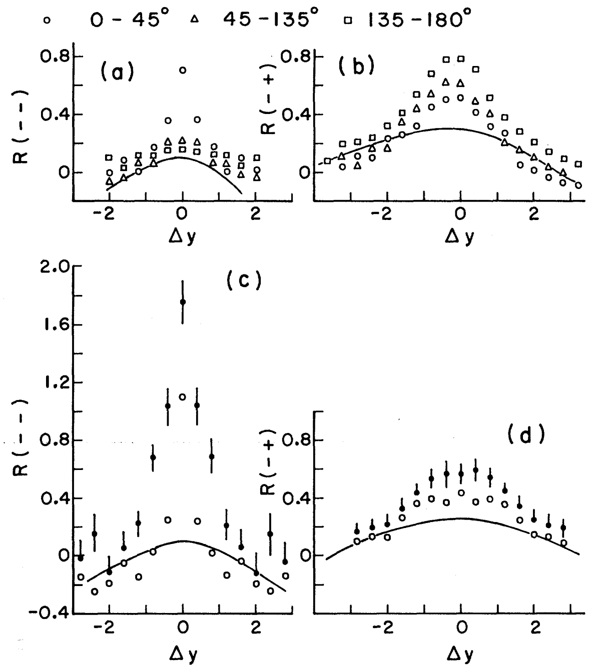

FIG. 1. R as a function of Δy. (a) $R(--)$ for all Q, but three ranges of φ; (b) $R(-+)$ for all Q, but three ranges of φ; (c) $R(--)$ for $\varphi<45°$, but for $0<Q_1=Q_2<0.15$ GeV/c; and (d) $R(-+)$ for $\varphi<45°$, but for 0.25 GeV/$c<Q_1=Q_2<0.5$ GeV/c. The curves correspond to a Monte Carlo calculation (see text).

tion length. Studying the $0°\leqslant\varphi\leqslant45°$ data for subsamples chosen with Q_1 and Q_2 small and equal ($0<Q_1,Q_2<0.15$ GeV/c, closed circles) and with Q_1 and Q_2 larger and equal (0.25 GeV/$c<Q_1,Q_2<0.5$ GeV/c, open circles), shown in Figs. 1(c) and 1(d), we observe that the $\Delta y=0$ enhancement in $R(--)$ is even more pronounced than in Fig. 1(a), and Q dependent. By contrast, the $R(-+)$ distributions are similar to the distributions of Fig. 1(b). Correlations for both like and unlike charges are in excess of the predictions (solid curves) of a Monte Carlo calculation generating events which reproduce the multiplicity and single-particle momentum distributions. This Monte Carlo calculation includes no explicit dynamical correlations, but does exhibit the kinematic consequences of energy-momentum conservation.

In Fig. 2 we present corresponding distributions for data subsets with $Q_1\neq Q_2$ ($0<Q_1<0.15$ GeV/c; 0.5 GeV/$c<Q_2<1.0$ GeV/c) for the three ranges of azimuth angle. It is evident that the strong, short-correlation-length enhancement in $R(--)$ at $\Delta y=0$ for $\varphi<45°$ essentially disappears. The $R(--)$ distributions for the three ranges of azimuth angle are rather similar, when $Q_1\neq Q_2$. The

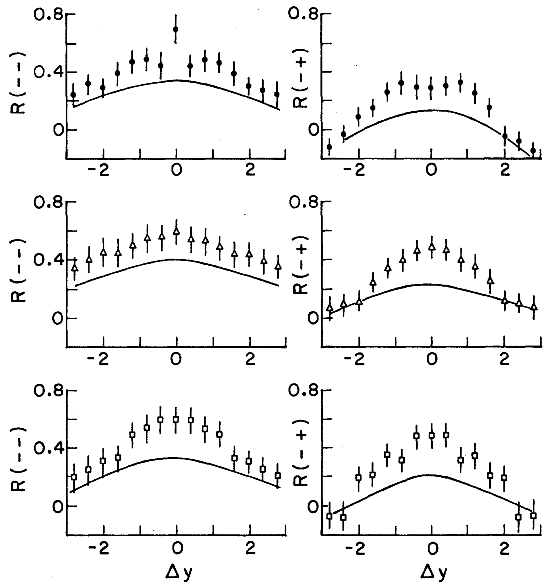

FIG. 2. R as a function of Δy for $0<Q_1<0.15$ GeV/c and 0.5 GeV/$c<Q_2<1.0$ GeV/c but for three ranges of φ: (a), (b) $R(--)$ and $R(-+)$ for $\varphi<45°$; (c), (d) $R(--)$ and $R(-+)$ for $45°<\varphi<135°$; (e), (f) $R(--)$ and $R(-+)$ for $135°<\varphi<180°$. The curves correspond to a Monte Carlo calculation (see text).

$R(-+)$ distributions similarly show no pronounced φ dependence. Both $R(--)$ and $R(-+)$ distributions again indicate correlations in excess of the predictions (solid curve) of the Monte Carlo calculation reproducing energy-momentum conservation effects.

The enhanced $R(--)$ correlation is observed for $\Delta y=0$ (longitudinal momenta are approximately equal), φ small, and Q_1 and Q_2 small and equal (transverse momentum vectors are approximately parallel and equal), which corresponds to low values of the square of the difference in four-momentum of the particles $t_{12}=(p_1-p_2)^2$. This suggests that R is a function of t_{12} rather than of the complete set of inclusive variables. In Table I we present values[5] of $R(--)$ and $R(-+)$ for $\Delta y=0$, $\varphi<45°$, for various ranges of Q_1 and Q_2. We also show the average values of t_{12} for the particle pair for each Q_1,Q_2 region.

The dependence of R on t_{12} is readily apparent in Fig. 3, where we have plotted the data of Table I. A striking increase in $R(--)$ compared to $R(-+)$ is seen for small t_{12} values. The data points for $R(-+)$ can be empirically parametrized to a linear expression.

$$R_s=(0.56\pm0.07)+(2.8\pm1.1)t_{12}, \tag{2}$$

as is shown as the dashed line in Fig. 3. As pre-

TABLE I. Values of R for $-0.2<\Delta y<0.2$ and $\varphi \leq 45°$, as a function of Q_1 and Q_2. The numbers in parentheses correspond to the values of $-\bar{t}_{12}$ (GeV^2).

(a) Values of $R(--)$

Q_2 \ Q_1 GeV/c	0.0 - 0.15	0.15 - 0.25	0.25 - 0.5	0.5 - 1.0
0.0 - 0.15	1.68 ± 0.10 (0.005)			
0.15 - 0.25	1.64 ± 0.08 (0.012)	1.50 ± 0.07 (0.013)		
0.25 - 0.5	0.89 ± 0.10 (0.045)	1.15 ± 0.07 (0.032)	1.05 ± 0.10 (0.032)	
0.5 - 1.0	0.71 ± 0.07 (0.089)	0.88 ± 0.06 (0.063)	0.95 ± 0.07 (0.070)	0.74 ± 0.10 (0.085)

(b) Values of $R(-+)$

Q_2 \ Q_1 GeV/c	0.0 - 0.15	0.15 - 0.25	0.25 - 0.5	0.5 - 1.0
0.0 - 0.15	0.65 ± 0.10 (0.006)	0.53 ± 0.08 (0.021)	0.38 ± 0.10 (0.044)	0.36 ± 0.10 (0.090)
0.15 - 0.25	0.54 ± 0.10 (0.013)	0.54 ± 0.07 (0.017)	0.54 ± 0.10 (0.034)	0.35 ± 0.10 (0.067)
0.25 - 0.5	0.25 ± 0.10 (0.046)	0.39 ± 0.07 (0.036)	0.39 ± 0.10 (0.046)	0.32 ± 0.10 (0.083)
0.5 - 1.0	0.28 ± 0.10 (0.088)	0.40 ± 0.08 (0.066)	0.31 ± 0.10 (0.082)	0.29 ± 0.10 (0.127)

viously noted, the $R(-+)$ correlations are much larger than what would be expected from energy-momentum conservation. R_s can thus be expected to represent a correlation of predominantly dynamical nature.

To explain the particular dependence of $R(--)$ on t_{12}, the influence of Bose-Einstein statistics on the two-particle correlation must be taken into account.[6] This effect has been theoretically formulated by Goldhaber, Goldhaber, Lee, and Pais (GGLP).[7] The relativistic form of such a correlation is given by

$$E_{\mathrm{BE}} = 1 + R_{\mathrm{BE}} = 1 + \exp[(\rho/2.15\mu)^2 t_{12}] \qquad (3)$$

[see Eqs. (14a), (14b) of Ref. 7] which depends solely on t_{12}. The parameter ρ describes the range of the BE correlation in units of pion Compton wavelength (1.41 fm). If the experimentally observed like-particle enhancement $E(--) \equiv 1 + R(--)$ is the resultant of the BE enhancement, E_{BE}, and of the dynamical enhancement $E_s \equiv 1 + R_s$, then $E(--)$ may be expressed as[8] $E(--) = E_{\mathrm{BE}}E_s$. In terms of R, we then have,

$$R(--) = R_{\mathrm{BE}}(1+R_s) + R_s. \qquad (4)$$

Using R_s as given by Eq. (2), we can reproduce

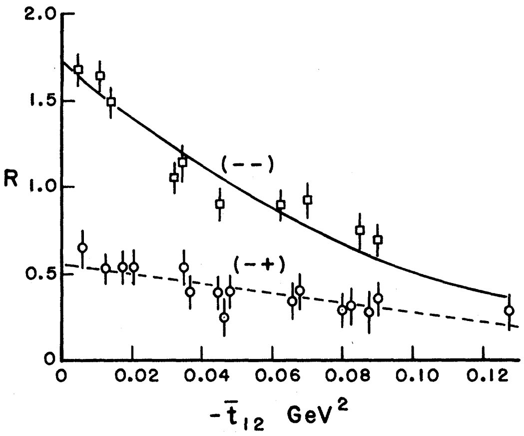

FIG. 3. R for $\Delta y = 0$ and $\varphi < 45°$ as a function of t_{12}. The curves correspond to best fits (see text).

the t_{12} dependence of $R(--)$ by parametrizing R_{BE} as

$$R_{\mathrm{BE}} = 0.80 \pm 0.10 \exp[(11.2 \pm 2.4) t_{12}]. \qquad (5)$$

The solid line in Fig. 3 shows $R(--)$ calculated from these fits. Comparing Eq. (5) with Eq. (3), we obtain $\rho = 1.00 \pm 0.25$. The R_s dynamical correlation need not be the same for $(--)$ and $(-+)$. Allowing R_s in Eq. (2) to vary within 2 standard deviations and fitting $R(--)$ we find $\rho = 1.00 \pm 0.45$. This value of ρ is consistent with that obtained for the BE effect (GGLP effect) for pions produced in $p\bar{p}$ annihilation and other reactions.[9]

In summary, we conclude that for the two-particle system, the Bose-Einstein symmetry effect leads to an additional enhancement, over and above dynamical correlations, for like charges with small t_{12}. This effect depends solely on t_{12} and is quantitatively consistent with the GGLP formulation.

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[5]Using four intervals for each Q, the table shows ten values of $R(--)$ and sixteen values of $R(-+)$. In computing $R(--)$ for $Q_1 \neq Q_2$, the data have been folded in since $R(--)$, as a function of Q_1 and Q_2, must be symmetric for $Q_1 \rightleftarrows Q_2$. This reduces the number of R values to ten for like charges from sixteen for unlike charges.

[6]Note that t_{12} is directly related to the di-pion mass, $t_{12} + M_{12}^2 = 4\mu^2$, and hence the data of Fig. 3 encompass an M_{12}^2 range of $0.08 - 0.12\ \mathrm{GeV}^2$, well below the masses of known resonances. It is difficult to envision a dynamical production and/or decay process which would so strongly enhance a severely limited region of phase space.

[7]G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. 120, 300 (1960).

[8]In this formulation, $E_s = 1$ and $E(--) = E_{\mathrm{BE}}$ if there is no dynamical enhancement. $E(--) = E_{\mathrm{BE}} E_s$ if the correlations due to BE statistics and production dynamics occur incoherently. Note that for unlike particles, $E_{\mathrm{BE}} = 1$ and hence $E(-+) = E_s$, the dynamical enhancement observed.

[9]In their original study of angular correlations of like particles in $\bar{p}p$ annihilation, GGLP (Ref. 7) found that the maximum effect of BE correlation occurs for values of ρ between 0.5 and 0.75. Similar estimates have also been made for lower energy $\pi^- p$ reactions: P. L. Bereyni *et al.*, Nucl. Phys. B37, 621 (1972).