Validity of the Adiabatic Cranking Model when Applied to Fission

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The cranking model is applied to a two-center, asymmetric, shell model plus a pairing interaction. The energy increase due to the enforced collective motion is determined up to fourth order in the collective velocity. For a motion across the second saddle of ^{236}U the higher order terms are so large that the validity of the adiabatic model as a first approximation to fission dynamics is questionable.

Although the validity of the adiabatic model has been questioned for some time,[1-5] and the cranking model has been applied in studies of fission,[6] the extension of the cranking-model perturbation expansion to higher order appears never to have been numerically carried out for realistic second-saddle shapes. We attack this problem in this work, carrying out the cranking model to the next higher order for protons in ^{236}U, with the aim of determining the extent to which low-energy fission is adiabatic or not adiabatic between saddle and scission.

Our starting point is a model Hamiltonian H which depends on particle coordinates $\vec{r}$, momenta $\vec{p}$, and a set of real coordinates α characterizing the shape of the nucleus. The eigenvectors $|j(\alpha)\rangle$ and eigenvalues of the Schrödinger equation

$$H(\vec{r},\vec{p},\alpha)|j(\alpha)\rangle = \epsilon_j(\alpha)|j(\alpha)\rangle , \tag{1}$$

are functions of the collective variables.

The operator

$$P_n = (\hbar/i)(\partial/\partial\alpha_n), \tag{2}$$

which generates a change of the variable α_n, is non-Hermitian because the right-hand side of the equation

$$P_n\langle a(\alpha)|b(\alpha)\rangle = -\langle P_n a|b\rangle + \langle a|P_n b\rangle \tag{3}$$

is nonzero in general for two arbitrary states a and b. For two orthornormalized eigenstates $|j\rangle$ and $|k\rangle$ of the Hamiltonian, however, the left-hand side of Eq. (3) is zero, and we may write

$$\langle j|P_n|k\rangle = \langle P_n j|k\rangle = \langle j|P_n k\rangle. \tag{4}$$

We choose the phases of our eigenstates so that the matrix elements of P_n are pure imaginary quantities. Consequently the diagonal elements are zero

$$\langle j|P_n|j\rangle = \langle j|P_n|j\rangle^* = 0. \tag{5}$$

We now apply the operators P_n as Lagrange constraints to generate a collective motion. In particular, we determine the ground-state vector $|\psi\rangle$ of the modified Hamiltonian

$$H' = H + V, \tag{6}$$

where

$$V = \sum_n \dot{\alpha}_n (\hbar/i)(\partial/\partial\alpha_n). \tag{7}$$

The real collective velocities $\dot{\alpha}_n$ play the role of Lagrange constraints. With the *Ansatz*

$$|\psi\rangle = |O(\alpha)\rangle + |\psi^{(1)}\rangle + |\psi^{(2)}\rangle + \dots , \tag{8}$$

where the vector $\psi^{(i)}$ is of ith order in the velocities and

$$\langle O(\alpha)|\psi^{(i)}\rangle = 0, \tag{9}$$

one obtains in standard Rayleigh-Schrödinger perturbation theory taking advantage of Eq. (5)

$$\begin{aligned} |\psi^{(1)}\rangle &= -|RVO\rangle , \\ |\psi^{(2)}\rangle &= -|RV\psi^{(1)}\rangle , \\ |\psi^{(3)}\rangle &= -|RV\psi^{(2)}\rangle + |R\psi^{(1)}\rangle\langle O|V\psi^{(1)}\rangle , \end{aligned} \tag{10}$$

where the operator R is defined by

$$R = \sum_{\substack{j \\ j\neq 0}} |j\rangle \frac{1}{\epsilon_j - \epsilon_0}\langle j| . \tag{11}$$

The energy increase above the ground-state energy ϵ_0 is given by the expression (since third-order terms in $\dot{\alpha}$ cancel out)

$$T = \frac{\langle\psi|(H-\epsilon_0)|\psi\rangle}{\langle\psi|\psi\rangle} \cong \langle\psi^{(1)}|(H-\epsilon_0)|\psi^{(1)}\rangle(1-\langle\psi^{(1)}|\psi^{(1)}\rangle) + \langle\psi^{(1)}|(H-\epsilon_0)|\psi^{(3)}\rangle + \langle\psi^{(2)}|(H-\epsilon_0)|\psi^{(2)}\rangle + \langle\psi^{(3)}|(H-\epsilon_0)|\psi^{(1)}\rangle \tag{12}$$

up to fourth order in the collective velocities. Using relation (3) this expression can be rewritten in a form where V operates only on eigenstates of the Hamiltonian and on matrix elements:

$$T = \langle V0|R|V0\rangle + 3\sum_{\substack{j\\ j\neq 0}} \frac{1}{\epsilon_j - \epsilon_0}[\langle V_j|R|V0\rangle]^2 - 3\langle V0|R|V0\rangle\langle V0|R^2|V0\rangle$$
$$+ 4\sum_{\substack{j\\ j\neq 0}} \langle V_j|R|V0\rangle \frac{1}{\epsilon_j - \epsilon_0} V\langle j|R|V0\rangle + \sum_{\substack{j\\ j\neq 0}} \frac{1}{\epsilon_j - \epsilon_0}[V\langle j|R|V0\rangle]^2$$
$$+ 2\sum_{\substack{j\\ j\neq 0}} \langle V0|R|j\rangle V\left[\frac{\langle V_j|R|V0\rangle}{\epsilon_j - \epsilon_0} + \frac{1}{\epsilon_j - \epsilon_0} V\langle j|R|V0\rangle\right]. \quad (13)$$

The first term on the right-hand side of Eq. (13) is the familiar cranking model expression:

$$\langle V0|R|V0\rangle = \sum_{\substack{j\\ j\neq 0}} \frac{|\langle 0|V|j\rangle|^2}{\epsilon_j - \epsilon_0}, \quad (14)$$

which is of second order in the collective velocities. The remaining fourth-order terms represent the first nonvanishing correction. The cranking model can be valid only up to velocities where the correction is small compared to the first term.

We have evaluated expression (13) numerically for a two-center oscillator Hamiltonian, i.e., a generalized Nilsson model which allows the nucleus to fission.[7] A pairing force is used as residual interaction. The variables α characterize the shape of the potential. The constraint operator V [Eq. (7)] is thus a one-body operator. When applied to an eigenstate of the Hamiltonian it changes the quasiparticle number of the state by zero, plus two, or minus two. Thus the excited states occurring in Eq. (13) are two-quasiparticle states except for the term

$$3\sum_{\substack{j\\ j\neq 0}} \frac{1}{\epsilon_j - \epsilon_0}[\langle Vj|R|V0\rangle]^2$$
$$= 3\sum_{\substack{j\\ j\neq 0}} \frac{1}{\epsilon_j - \epsilon_0}\left[\sum_{\substack{k\\ k\neq 0}} \frac{\langle 0|V|k\rangle\langle k|V|j\rangle}{\epsilon_k - \epsilon_0}\right]^2, \quad (15)$$

where j may also be a four-quasiparticle state.

The calculations were performed for protons in ^{236}U and one collective degree of freedom. This degree of freedom was chosen to describe the motion across the second saddle point, and consists essentially of a separation of the centers of the oscillators plus a slight constriction of the hyperbolic neck. We present the results in Table I, where m_0 and m_2 refer to the equation for T rewritten in the form

$$T = \tfrac{1}{2}(m_0\delta\alpha^2 + m_2\delta\alpha^4) = T_0\left(1 + \frac{2m_2}{(m_0)^2} T_0\right), \quad (16)$$

and

$$T_0 = \tfrac{1}{2}m_0\delta\alpha^2. \quad (17)$$

TABLE I. Mass parameter results for symmetric and asymmetric saddle points, for protons in ^{236}U.

Δ (MeV)	m_0 ($\hbar^2$/MeV)	m_2 ($\hbar^4$/MeV^3)	Four-quasiparticle intermediate state contribution	$\frac{^2m_2}{m_0^2}$ (MeV^{-1})
Symmetric				
1.07	39.9	2702	561	3.4
1.18	37.2	2376	458	3.4
1.35	32.2	1965	331	3.8
Asymmetric				
1.08	38.0	1956	532	2.7

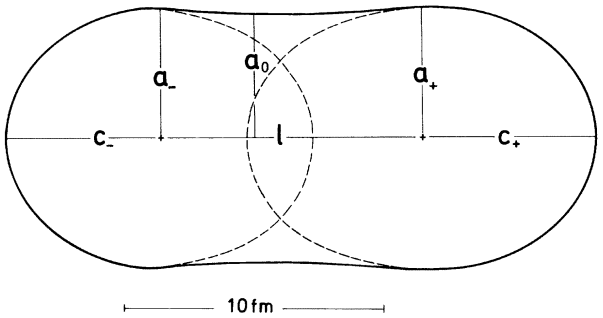

FIG. 1. The asymmetric saddle-point shape for ^{236}U is characterized by the dimensionless quantities: $x_1 = 1/2a_+ = 1$, $x_2 = -c_0^2/a_0^2 = 0.05$, $x_3 = (c_+ + c_-)/2a_+ = 1.25$, $x_4 = a_+^2 c_+/a_-^2 c_- = 1.2$, $x_5 = (c_+ - c_-)/2a_+ = 0.08$, where the quantities a and c are the principal axes of the two spheroids and the central hyperboloid. The one-dimensional motion across the second saddle as defined by the variations $\delta x_1 = \alpha$, $\delta x_2 = 0.6\alpha$. The variables x_3, x_4, and x_5, the center of mass position, and the volume of the nucleus are kept constant.

The first three rows show the results for three values of the pairing strength for the symmetric saddle point, which corresponds to two ellipsoids of the same volume and shape, joined by a hyperboloid shaped neck. Our asymmetric-saddle results are given in the fourth row, and correspond to two ellipsoids with a volume ratio of 1:2 joined by a smooth neck, as shown in Fig. 1. Column three of Table I gives the contribution of four-quasiparticle states $|j\rangle$ through the term given in Eq. (15). The excitation of four-quasiparticle states accounts for an appreciable through not the dominant part of the term m_2. Column four gives the ratio $r = 2m_2/(m_0)^2$ which occurs in Eq. (16) if we express the velocity $\dot{\alpha}$ in terms of the excitation energy T_0 in the adiabatic limit [Eq. (17)]. The large values of the ratio r tell us that the corrections to the adiabatic expression for the energy increase T caused by the generating operator for the collective velocity [Eq. (7)] are appreciable even if T_0 is only a few tenths of an MeV. This is a general result in the sense that it holds true also for larger deformations between the second saddle and the scission point. Since estimates of the prescission kinetic energy range between 7 and 30 MeV [8-11] the conclusion must be that the velocity $\dot{\alpha}$ cannot be used as an expansion parameter in a perturbation treatment, i.e., the cranking model when applied to fission is invalidated. A corresponding breakdown of the cranking model when applied to rotations is the so-called backbending effect. Finally we note that as the nucleus picks up collective velocity in sliding down from saddle to scission some of the available energy will go irreversibly into internal excitation.

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