COMMENTS

Vacuum Periodicity in a Yang-Mills Quantum Theory*

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We propose a description of the vacuum in Yang-Mills theory and arrive at a physical interpretation of the pseudoparticle solution and the attendant violation of symmetries. The existence of topologically inequivalent classical gauge fields gives rise to a family of quantum mechanical vacua, parametrized by a *CP*-nonconserving angle. The requirement of vacuum stability against gauge transformations renders the vacua chirally noninvariant.

A classical pseudoparticle solution to the SU(2) Yang-Mills theory in Euclidean four-dimensional space has been given by Belavin, Polyakov, Schwartz, and Tyupkin,[1] with the suggestion that it be used to dominate the functional integral which describes a quantum field theory continued to Euclidean space. 't Hooft[2] has shown that these nontrivial minima of the action give nonvanishing contributions to amplitudes which would be zero in the ordinary sector. Specifically in a theory of fermions coupled to Yang-Mills fields, with chiral U(1) and *CP* symmetries, symmetry-nonconserving effects are found through the pressence of the axial-vector-current anomaly.[3] Thus he provides a possible resolution of the long-standing U(1) problem[4] and an intriguing suggestion for the origin of *CP* nonconservation. The phenomena are $O(\exp(-8\pi^2/g^2))$, where g is the gauge coupling constant; they are nonperturbative.

The fact that the classical field configuration which is responsible for the new results is in Euclidean four-dimensional space, i.e., imaginary time, leads one to suspect that the pseudoparticle is associated with quantum-mechanical tunneling by which field configurations in the ordinary three-dimensional space are joined in the course of the (real-time) evolution through the penetration of an energy barrier.[5] Also the exponentially small magnitude is indicative of tunneling. Here we wish to present a further explanation of this point, which we hope, will clarify the physical interpretation of the pseudoparticle solution and will supplement 't Hooft's more formal computations. Our considerations lead to a description of the quantum mechanical vacuum state of a Yang-Mills theory which is unexpectedly rich.

In the quantum field theory, a state of the system can be represented by a wave functional $\Psi[\vec{A}]$ of the field configuration. Having in mind a Yang-Mills theory, we have taken the potentials $\vec{A}(\vec{x})$ (anti-Hermitian matrices in the space of the infinitesimal group generators) as argument of the functional, excluding the time components $A^0(\vec{x})$, because they are dependent variables. In defining scalar products and matrix elements of observables one must avoid infinities associated with the volume of the gauge group. Without repeating details of the well-kown gauge-fixing procedure, let us only recall that it removes from the functional integral over $\vec{A}$ configurations of the fields which can be joined by a *continuous* gauge transformation to configurations already counted. In particular, one does not integrate over potentials of the form

$$\vec{A}(\vec{x})=g^{-1}(\vec{x})\nabla g(\vec{x}), \tag{1}$$

where g is the unitary matrix of a gauge transformation that can be joined to the identity through a one-parameter continuous family of transformations $g(\vec{x},\alpha)$:

$$g(\vec{x},1)=g(\vec{x});\quad g(\vec{x},0)=I. \tag{2}$$

The potentials of Eq. (1) are of course gauge equivalent to $\vec{A}=0$.

But it is important to realize that there are values of $\vec{A}$ that can be obtained from each other by gauge transformations which cannot be continuously joined with the identity transformation. For instance, we may consider

$$g_1(\vec{x}) = \frac{\vec{x}^2 - \lambda^2}{\vec{x}^2 + \lambda^2} - \frac{2i\lambda\vec{\sigma}\cdot\vec{x}}{\vec{x}^2 + \lambda^2} \tag{3}$$

which gives origin to

$$\vec{A}(\vec{x}) = g_1^{-1}(\vec{x})\nabla g_1(\vec{x}) = \frac{2i\lambda}{(\vec{x}^2+\lambda^2)^2}[\sigma(\lambda^2 - \vec{x}^2) + 2\vec{x}(\sigma\cdot\vec{x}) + 2\lambda\vec{x}\times\vec{\sigma}] \tag{4}$$

and of course to vanishing field strengths F_{ij}. Values of the potentials like those of Eq. (4), although gauge equivalent to $\vec{A}=0$, should *not* be removed from the integrations over the field configurations by the gauge fixing procedure, and indeed we shall argue that physical effects are associated with them.

Before proceeding, let us characterize the classes of gauge-equivalent, but not continuously gauge-equivalent, potentials. We study effects which are local in space and therefore, when we consider a gauge transformation g, we require

$$g(\vec{x}) \xrightarrow[|\vec{x}|\to\infty]{} I. \tag{5}$$

Thus g defines a mapping of the three-dimensional space, *with all the directions at* ∞ *identified,* into the group space. From the topological point of view, the Euclidean space E^3 with points at ∞ identified is equivalent (homeomorphic) to a three-dimensional sphere S^3; but the manifold of SU(2) is also homeomorphic to S^3, so that g defines a mapping

$$S^3 \xrightarrow[g]{} S^3. \tag{6}$$

It is known that these mappings fall into homotopy classes (mappings belonging to different classes cannot be continuously distorted into each other) classified by an integer n,

$$g_n(\vec{x}) = [g_1(\vec{x})]^n \tag{7}$$

with g_1 given in Eq. (3) being a representative of the nth class.

We can make contact now with the pseudoparticle solution.[1] Observe that the field configuration of Eq. (4) has zero potential energy, and that there is no energy-conserving evolution of the system which adiabatically connects that configuration with $\vec{A}=0$. Such an evolution should be a continuous gauge transformation; but this is impossible because g_1 and the identity belong to different homotopy classes. All paths joining the two field configurations in real time must go over an energy barrier. To exemplify this, let us multiply the potentials of Eq. (4) by $\frac{1}{2}-\alpha$ and increase α adiabatically from $-\frac{1}{2}$ to $+\frac{1}{2}$. Now the field strength is nonvanishing, but proportional to $\frac{1}{4} - \alpha^2$. The energy, $-\frac{1}{8}\int d^3x\, \mathrm{Tr} F_{ij}F^{ij} \geq 0$, becomes proportional to $(\alpha^2 - \frac{1}{4})^2$ and exhibits a barrier shape as α varies from $-\frac{1}{2}$ to $\frac{1}{2}$.

In the quantum theory, tunneling will occur across this barrier. It is well known that a semiclassical description of tunneling can be given by solving the classical equations of motion with *imaginary* time, thus achieving an evolution which would be classically forbidden for real time.[5] The pseudoparticle solution[1] serves precisely this purpose: It carries zero energy (the Euclidean stress tensor vanishes); it can be arranged to connect $g=g_1$ at $x_4 = -it = -\infty$ with $g=I$ at $x_4=\infty$. The physical implication of the pseudoparticle solution is that the quantal description of the vacuum state cannot be limited to fluctuations around any definite classical configuration of zero energy.

Let us now describe in greater detail the nature of the vacuum wave functional. Consider any of the field configurations

$$\vec{A}_n(\vec{x}) = g_n^{-1}(\vec{x})\nabla g_n(\vec{x}) \tag{8}$$

with vanishing F^{ij}. Neglecting tunneling effects we might expect the vacuum to be of the form

$$\psi_n[\vec{A}] = \varphi[\vec{A} - \vec{A}_n], \tag{9}$$

where the wave functional φ is peaked about zero and has a spread due to quantum fluctuations and any $\vec{A}_n$ can be chosen as representative of the classical vacuum, i.e., the classical zero-energy configuration.

But the pseudoparticle solution connects $\vec{A}_n$ with $\vec{A}_{n+1}$, giving origin to tunneling between the different ψ_n. The true quantal vacuum state will therefore be a superposition of the form

$$\Psi[\vec{A}] = \sum_n c_n \psi_n[\vec{A}] + O(\exp(-8\pi^2/g^2)). \tag{10}$$

To determine the coefficients c_n in this equation let us observe that the finite gauge transformation g_1 changes ψ_n into ψ_{n+1}. Requiring the vacuum state to be stable against gauge transformations determines the coefficients to be

$$c_n = e^{in\theta}. \tag{11}$$

Thus we find a family of vacua, parametrized by an angle θ, where under the gauge transformation g_1

$$\Psi_\theta[\vec{A}] \underset{g_1}{\rightarrow} e^{-i\theta}\Psi_\theta[\vec{A}]. \tag{12}$$

The occurrence of multiple vacua is intriguing and is reminiscent of the situation encountered in the Schwinger model.[6]

The significance of the phase θ in Eqs. (10) and (11) becomes apparent when massless fermions are coupled to the Yang-Mills fields. One may then introduce the U(1) axial-vector current

$$J_5^{\mu} = i\bar{\psi}\gamma^{\mu}\gamma^5\psi$$

which however is not conserved because of the anomaly.[3] A conserved, but gauge-variant, current is given by

$$\tilde{J}_5^{\mu} = J_5^{\mu} - 4\pi^{-2}\epsilon^{\mu\nu\alpha\beta} \times \mathrm{Tr}(A_\nu \partial_\alpha A_\beta + \tfrac{2}{3}A_\nu A_\alpha A_\beta) \tag{13a}$$

and the conserved axial charge is

$$\tilde{Q}_5 = \int d^3x\, \tilde{J}_5^{0}. \tag{13b}$$

To exhibit the gauge dependence of $\tilde{Q}_5$, we perform a finite gauge transformation g, with $g(\vec{x}) \underset{|\vec{x}|\to\infty}{\longrightarrow} I$, and find

$$\Delta\tilde{Q}_5 = \frac{1}{12\pi^2}\int d^3x\, \mathrm{Tr}\,\epsilon_{ijk}(g^{-1}\partial_i g)(g^{-1}\partial_j g)(g^{-1}\partial_k g) = \frac{1}{12\pi^2}\int d\mu(g) = 2n, \tag{14}$$

where $d\mu(g)$ is the invariant measure of the group and n is the integer which characterizes the homotopy class of g. (g belongs to the nth homotopy class when it is continuously deformable to g_n.) The fact that $\tilde{Q}_5$ commutes with the Hamiltonian, and that it changes by two units under the gauge transformation g_1, together with Eq. (11), implies that

$$\exp(-\tfrac{1}{2}i\theta'\tilde{Q}_5)\Psi_\theta[\vec{A}] = \Psi_{\theta+\theta'}[\vec{A}] \tag{15}$$

which in turn shows that all the vacua are degenerate in energy and define the same theory. Equation (15) also demonstrates the possibility of symmetry breaking without Goldstone bosons: This may provide a solution to the U(1) problem.[2,4]

An explanation of the nonconservation of the gauge-invariant fermionic axial charge

$$Q_5(t) = \int d^3x\, J_5^{0}(t,\vec{x}) \tag{16}$$

may be given. The tunneling process between adjacent vacuum components $\psi_n[\vec{A}]$ and $\psi_{n+1}[\vec{A}]$ is equivalent to a gauge transformation g_1 which changes $\tilde{Q}_5 - Q_5$ by two units; see Eq. (14). But $\tilde{Q}_5$ is conserved, and therefore $\Delta Q_5 = -2$.

Finally, we remark that, whereas in the massless case conservation of $\tilde{Q}_5$ renders all vacua degenerate, we expect that if the fermions are massive, so that $\tilde{Q}_5$ is no longer conserved, different values of θ define nonequivalent theories as in the Schwinger model.[6] A nonzero value of θ could describe CP nonconservation,[2] but in the theory as developed thus far there is no indication how to compute θ.

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Added note.—After completion of this manuscript, we received a paper by C. Callan, R. Dashen, and D. Gross (to be published) who arrive at conclusions similar to ours.

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[5]Use of imaginary time in discussions of tunneling is well known; see, e.g., K. Freed, J. Chem. Phys. 56, 692 (1972); D. McLaughlin, J. Math. Phys. (N.Y.) 13, 1099 (1972).

[6]J. Schwinger, Phys. Rev. 128, 2425 (1962); J. Lowenstein and A. Swieca, Ann. Phys. (N.Y.) 68, 172 (1971); S. Coleman, R. Jackiw, and L. Susskind, Ann. Phys. (N.Y.) 93, 267 (1975). (This analogy was developed in conversations with Coleman and Susskind.) There is no pseudoparticle solution for *spinor* electrodynamics in two space-time dimensions. However, by adding to the model charged spinless fields, with a Higgs potential and minimal electromagnetic coupling, a pseudoparticle solution exists in Euclidean two-dimension space—it is just the Nielsen-Olesen string

[Nucl. Phys. B61, 45 (1973)]. The topologically conserved object is $\int d^2x \epsilon_{\mu\nu} F^{\mu\nu}$, and $\epsilon_{\mu\nu} F^{\mu\nu}$ is also proportional to the anomalous divergence of the axial vector current; K. Johnson, Phys. Lett. 5, 253 (1963); R. Jackiw, in *Laws of Hadronic Matter*, edited by A. Zichichi (Academic, New York, 1975).

Direct Evidence for the Bose-Einstein Effect in Inclusive Two-Particle Reaction Correlations*

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Two-particle correlations are studied as a function of t_{12}, the square of the difference of the four-momentum of the particles. We observe that the $\pi^-\pi^-$ correlation differs from the $\pi^-\pi^+$ correlation only at low values of t_{12}. This difference can quantitatively be understood as a consequence of Bose-Einstein statistics.

Short-range correlations for particles produced in high-energy inclusive reactions are known to exist. Various models for multiparticle-production processes, e.g., clustering,[1] have been considered to account for the observed correlations. These models yield correlations arising from dynamical production processes which have a range of 1–2 units of rapidity and are essentially independent of the azimuthal angle between the transverse momenta. Both of these features are observed for correlations involving unlike pions; when the pion pairs have like charges, however, a striking narrow correlation (width ~ 0.4 units of rapidity) for small azimuthal-angle separation is observed.[2,3] It has been suggested that this narrow correlation may be a Bose-Einstein (BE) symmetry effect,[4] rather than a consequence of charge-structure dynamics in particle production. Experimental evidence supporting this explanation has so far been lacking. We present here an analysis which shows evidence for the presence of both dynamical and BE symmetry effects for like charges.

The data presented here are obtained from π^-p interactions at 200 GeV/c using the Fermilab 30-in. bubble-chamber, wide-gap spark-chamber hybrid system. Experimental details have been reported elsewhere.[2] The data sample consists of ~ 17 000 events of all topologies.

The normalized two-particle correlation function is given by

$$R(y_1, y_2 \varphi, Q_1, Q_2) = \frac{\rho_2(y_1, y_2, \varphi, Q_1, Q_2)}{\rho_1(y_1, Q_1)\rho_1(y_2, Q_2)} - 1. \quad (1)$$

Here ρ_2 and ρ_1 are the two-particle and single-particle densities, respectively, y is the center-of-mass–system rapidity, Q is the magnitude of the transverse momentum, and the azimuthal angle φ is defined by $\cos\varphi = \vec{Q}_1 \cdot \vec{Q}_2 / Q_1 Q_2$. We integrate Eq. (1) over y_1 and consider the resulting $R(\Delta y, \varphi, Q_1, Q_2)$, where $\Delta y = y_1 - y_2$. We denote the correlations for $\pi^-\pi^-$ (like charges) and for $\pi^-\pi^+$ (unlike charges) as $R(--)$ and $R(-+)$, respectively.

In Fig. 1, we present $R(--)$ and $R(-+)$ as a function of Δy for various choices of φ and Q. The values of R, summed over all Q_1 and Q_2, are shown for three regions of φ (0°–45°, 45°–135°, and 135°–180°) in Figs. 1(a) (like charges) and 1(b) (unlike charges). As noted in Ref. 2, $R(--)$ and $R(-+)$ manifest different characteristics in the φ dependence of the rapidity correla-