Consistent Supergravity with Complex Spin-$\frac{3}{2}$ Gauge Fields

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We construct a new theory of supergravity which is invariant under complex local supersymmetry transformations. This theory is obtained by coupling the massless spin-$(\frac{3}{2},1)$ multiplet of ordinary global supersymmetry to the spin-$(2,\frac{3}{2})$ gauge multiplet of previous supergravity.

Ordinary supergravity is formulated[1-3] in terms of two real gauge fields, the spin-2 vierbein field and the Majorana spin-$\frac{3}{2}$ Rarita-Schwinger[4] field. The latter is the gauge field belonging to local supersymmetry transformations with anticommuting real (Majorana) spinorial parameters. In this Letter we formulate a new theory of supergravity in terms of the real spin-2 and spin-1 vierbein and photon fields, and a complex spin-$\frac{3}{2}$ Rarita-Schwinger field. This theory is obtained by coupling the real spin-$(2,\frac{3}{2})$ gauge multiplet of supergravity to the real spin-$(\frac{3}{2},1)$ matter multiplet of global flat-space supersymmetry. The two real (Majorana) spin-$\frac{3}{2}$ fields are combined into the single complex (Dirac) spin-$\frac{3}{2}$ field. The resulting action turns out to have a complex local supersymmetry whose gauge field is this complex spin-$\frac{3}{2}$ field. This theory unifies gravitation and electromagnetism, the photon now belonging to the gauge multiplet of complex local supersymmetry, and constitutes the first example of a consistent interacting theory of complex spin-$\frac{3}{2}$ fields.

Before discussing the derivation, we give the results. The complete action is the sum of the supergauge action $\mathcal{L}^G$ of ordinary supergravity, and the matter action $\mathcal{L}^M$. The former contains the vierbein field $e_{a\mu}$ (the graviton) and the Majorana Rarita-Schwinger field ψ_μ, while the latter describes a photon A_μ and a second Majorana spin-$\frac{3}{2}$ field φ_μ. The matter action couples, as expected, ψ_μ to the Noether current associated with the global supersymmetry invariance of the matter action, and contains, in addition, a four-fermion contact term $\mathcal{L}_4^M$.

$$\mathcal{L}=\mathcal{L}^G+\mathcal{L}^M,\quad \mathcal{L}^M=\hat{\mathcal{L}}^M+\mathcal{L}_4^M. \tag{1}$$

$$\hat{\mathcal{L}}^M=-\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\varphi_\mu\gamma_5\gamma_\nu D_\rho\varphi_\sigma-\tfrac{1}{4}eg^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}+(\kappa/\sqrt{2})\bar{\psi}_\mu[eF^{\mu\nu}+\tfrac{1}{2}\gamma_5\tilde{F}^{\mu\nu}]\varphi_\nu, \tag{2}$$

$$\begin{aligned}\mathcal{L}_4^M=&-\tfrac{1}{4}\kappa^2(\bar{\psi}_\mu\varphi_\nu)[e(\bar{\psi}^\mu\varphi^\nu-\bar{\psi}^\nu\varphi^\mu)+\epsilon^{\mu\nu\rho\sigma}(\bar{\psi}_\rho\gamma_5\varphi_\sigma)]\\&-(e\kappa^2/16)[(\bar{\psi}^b\gamma^a\psi^c)(\bar{\varphi}_b\gamma_a\varphi_c+2\bar{\varphi}_a\gamma_b\varphi_c)-4(\bar{\psi}^a\gamma\cdot\psi)(\bar{\varphi}_a\gamma\cdot\varphi)]\\&-(e\kappa^2/32)[(\bar{\varphi}^b\gamma^a\varphi^c)(\bar{\varphi}_b\gamma_a\varphi_c+2\bar{\varphi}_a\gamma_b\varphi_c)-4(\bar{\varphi}_a\gamma\cdot\varphi)^2],\end{aligned} \tag{3}$$

$$\begin{aligned}\mathcal{L}^G=&-(2\kappa^2)^{-1}eR(e)-\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma\\&-(e\kappa^2/32)[\bar{\psi}^b\gamma^a\psi^c)(\bar{\psi}_b\gamma_a\psi_c+2\bar{\psi}_a\gamma_b\psi_c)-4(\bar{\psi}_a\gamma\cdot\psi)^2].\end{aligned} \tag{4}$$

The action is invariant under the following transformation laws

$$\delta A_\mu=\sqrt{2}(\bar{\epsilon}\varphi_\mu),\quad \delta e^a{}_\mu=\kappa\bar{\epsilon}\gamma^a\psi_\mu, \tag{5}$$

$$\delta\varphi_\mu=-(1/\sqrt{2})[F_{\mu\nu}\gamma^\nu+\tfrac{1}{2}e\tilde{F}_{\mu\nu}\gamma^\nu\gamma_5]\epsilon+\tfrac{1}{2}\kappa(\bar{\psi}_\mu\varphi_\nu-\bar{\psi}_\nu\varphi_\mu)\gamma^\nu\epsilon+\tfrac{1}{2}\kappa e\epsilon_{\mu\nu\alpha\beta}(\bar{\psi}^\alpha\varphi^\beta)\gamma^\nu\gamma_5\epsilon, \tag{6}$$

$$\delta\psi_\mu=\delta\psi_\mu{}^G+\tfrac{1}{4}\kappa\sigma^{ab}\epsilon[2\bar{\varphi}_\mu\gamma_a\varphi_b+\bar{\varphi}_a\gamma_\mu\varphi_b], \tag{7}$$

$$\delta\psi_\mu{}^G=2\kappa^{-1}D_\mu\epsilon+\tfrac{1}{4}\kappa\sigma^{ab}\epsilon[2\bar{\psi}_\mu\gamma_a\psi_b+\bar{\psi}_a\gamma_\mu\psi_b]. \tag{8}$$

In these equations D_μ is the gravitational covariant derivative (without torsion), $\tilde{F}_{\mu\nu}=\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, $\gamma\cdot\psi=\gamma^a\psi_a$, $e=\det(e^a{}_\mu)$, ϵ is a real space-time-dependent Majorana spinor, while Greek (Latin) indices consistently denote world (local Lorentz) tensors. We use the positive metric $g_{\mu\nu}=g^{\mu\nu}=\delta_\mu{}^\nu=(+,+,+,+)$ in flat space (the Pauli metric) and our gamma matrices are Hermitian with squares equal to one ($\gamma_5{}^2=\gamma_a{}^2=1$ with $a=1,4$).

As in previous matter-supergravity couplings,[5-8] the bilinear term in the transformation law of the matter fermion field φ_μ can be absorbed into the flat-space law by introducing supercovariant derivatives[5-7]

$$\hat{D}_\mu A_\nu = D_\mu A_\nu - (1/\sqrt{2})\kappa(\bar{\psi}_\mu \varphi_\nu). \tag{9}$$

The matter term in the transformation law of the gauge fermion field ψ_μ is exactly torsion. We introduce the gravitational covariant derivative with torsion $\bar{D}_\mu$ in terms of the connection $\omega_{\mu ab} = \omega_{\mu ab}{}^{(0)} + \kappa_{\mu ab}$,

$$\kappa_{\mu ab} = \tfrac{1}{4}\kappa^2[\bar{\psi}_\mu \gamma_a \psi_b - \bar{\psi}_\mu \gamma_b \psi_a + \bar{\psi}_a \gamma_\mu \psi_b] + \tfrac{1}{4}\kappa^2[\bar{\varphi}_\mu \gamma_a \varphi_b - \bar{\varphi}_\mu \gamma_b \varphi_a + \bar{\varphi}_a \gamma_\mu \varphi_b]. \tag{10}$$

The previous action can now be rewritten as follows:

$$\mathcal{L} = -(2\kappa^2)^{-1} eR(e,\omega) - \tfrac{1}{2}\bar{\psi}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho \psi_\sigma - \tfrac{1}{4} e g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \tfrac{1}{2}\bar{\varphi}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu \hat{D}_\rho \varphi_\sigma + (\kappa/2\sqrt{2})\bar{\psi}_\mu[eF^{\mu\nu} + \tfrac{1}{2}\gamma_5 \tilde{F}^{\mu\nu}]\varphi_\nu \tag{11}$$

or, equivalently,

$$\mathcal{L} = -(2\kappa^2)^{-1} eR(e,\omega) - \tfrac{1}{2}\bar{\psi}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu \hat{D}_\rho \psi_\sigma - \tfrac{1}{4} e g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \tfrac{1}{2}\bar{\varphi}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu \hat{D}_\rho \varphi_\sigma + \tfrac{1}{4}\kappa^2(\bar{\psi}_\mu \varphi_\nu)[e(\bar{\psi}^\mu \varphi^\nu - \bar{\psi}^\nu \varphi^\mu) + \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\rho \gamma_5 \varphi_\sigma], \tag{12}$$

where the supercovariant derivative of φ_μ (and *idem* ψ_μ) is given by

$$\hat{D}_\mu \varphi_\nu = D_\mu \varphi_\nu + (\kappa/2\sqrt{2})(\hat{F}_{\nu\lambda}\gamma^\lambda + \tfrac{1}{2} e\hat{\tilde{F}}_{\nu\lambda}\gamma^\lambda \gamma^5)\psi_\mu. \tag{13}$$

The transformation laws can be rewritten as

$$\delta A_\mu = \sqrt{2}(\bar{\epsilon}\varphi_\mu), \quad \delta e^a{}_\mu = \kappa\bar{\epsilon}\gamma^a \psi_\mu, \tag{14}$$

$$\delta\varphi_\mu = -(1/\sqrt{2})[\hat{F}_{\mu\lambda}\gamma^\lambda + \tfrac{1}{2} e\hat{\tilde{F}}_{\mu\lambda}\gamma^\lambda\gamma^5]\epsilon, \tag{15}$$

$$\delta\psi_\mu = (2/\kappa)\bar{D}_\mu \epsilon \equiv (2/\kappa) D_\mu \epsilon + \kappa\sigma^{ab}\epsilon\kappa_{\mu ab}. \tag{16}$$

We now observe that the action has a global U(1) invariance, corresponding to rotations in the ψ-φ plane.[9] As a consequence, the action is invariant under a second real local supersymmetry transformation, obtained by interchanging the roles of ψ and φ. The action is therefore invariant under the following complex local supersymmetry transformations

$$\delta A_\mu = -i\sqrt{2}(\bar{\epsilon}_c \chi_\mu - \bar{\chi}_{\mu c}{}^{\epsilon}), \quad \delta e^a{}_\mu = \kappa(\bar{\epsilon}_c\gamma^a\chi_\mu - \bar{\chi}_\mu\gamma^a\epsilon_c), \tag{17}$$

$$\delta\chi_\mu = (2/\kappa) D_\mu{}^c \epsilon_c, \quad D_\mu{}^c = \bar{D}_\mu - (i\kappa/2\sqrt{2})(\hat{F}_{\mu\lambda}\gamma^\lambda + \tfrac{1}{2} e\hat{\tilde{F}}_{\mu\lambda}\gamma^\lambda\gamma_5), \tag{18}$$

where the complex fermionic gauge field and complex gauge parameters are given by

$$\epsilon_c = (\epsilon_1 + i\epsilon_2)/\sqrt{2}, \quad \chi_\mu = (\psi_\mu + i\varphi_\mu)/\sqrt{2}. \tag{19}$$

We now discuss the derivation of this theory. The method is the same[5,6,8] as used to couple supergravity to previously known matter multiplets. Since the transformation laws as well as the Noether current of the flat global supersymmetric spin-($\frac{3}{2}$, 1) multiplet[10] were unknown, we start by discussing them. The flat-space Lagrangian

$$\mathcal{L}^0 = -\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\bar{\varphi}_\mu\gamma_5\gamma_\nu\partial_\rho\varphi_\sigma) - \tfrac{1}{4}(F_{\mu\nu})^2 \tag{20}$$

is invariant under the transformations

$$\delta A_\mu = \sqrt{2}(\bar{\epsilon}\varphi_\mu), \quad \delta\varphi_\mu = -(1/\sqrt{2})(F_{\mu\lambda}\gamma^\lambda + \tfrac{1}{2}\tilde{F}_{\mu\lambda}\gamma^\lambda\gamma_5)\epsilon. \tag{21}$$

To establish this result, one uses the cyclic identity $\partial_\mu \tilde{F}_{\mu\nu} = 0$. We now discuss local invariance. At the order κ^0 level, the requirement of invariance fixes uniquely the coupling of ψ_μ to the Noether current

$$\mathcal{J}_\mu = (F_{\mu\nu} + \tfrac{1}{2}\gamma_5\tilde{F}_{\mu\nu})\varphi_\nu \tag{22}$$

which is conserved because of the following identity:

$$\partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu + \gamma_5\epsilon_{\mu\nu\rho\sigma}\partial_\rho\varphi_\sigma = -\gamma_\alpha\sigma_{\mu\nu}(\epsilon^{\alpha\beta\rho\sigma}\gamma_5\gamma_\beta\partial_\rho\varphi_\sigma). \tag{23}$$

At the order κ level, the usual[5-8] minor miracle happens as the $F^2\psi$ terms in $\delta\mathcal{L}$ cancel. At the same order, the $\partial\varphi\varphi\psi$ terms lead to the $\kappa^2\psi^2\varphi^2$ contact terms in the action and all κ-dependent terms in the transformation laws. All κ^2 terms in the varied action cancel identically, if one adds the $\kappa^2\varphi^4$ terms to the action. Since $\mathcal{L}^0$ and the Noether current are U(1) invariant, it comes as no surprise that the φ^4 term is only torsion. For the details of the method and manipulations, we refer to Ref. 6. We here only quote the following identity, which was used to cast part of the terms in $\mathcal{L}_4{}^{\mathrm{M}}$ in the covariant-derivative form $(\bar{\psi}\varphi)^2$

$$(\overline{\varphi}_\rho\sigma_{\alpha\beta}\varphi_\sigma)(\overline{\psi}_\mu\sigma^{\alpha\beta}\gamma_5\psi_\nu)\epsilon^{\mu\nu\rho\sigma}=2(\overline{\varphi}_\rho\psi_\nu)(\overline{\psi}_\mu\gamma_5\varphi_\sigma)\epsilon^{\mu\nu\rho\sigma}. \tag{24}$$

In agreement with a very general power counting argument[6] and electromagnetic gauge invariance, $\mathcal{L}_4$ only contains fermion fields.

We now comment on our results. The theory is gravitational (general coordinate and local Lorentz) invariant, electromagnetic invariant, and invariant under local complex supersymmetry, as well as globally invariant under the infinitesimal chiral transformation

$$\delta\psi_\mu=i\gamma_5\psi_\mu,\quad \delta\varphi_\mu=-i\gamma_5\varphi_\mu. \tag{25}$$

As far as matter coupling is concerned, it is clear that consistency requires that complex supergravity can only be coupled to matter systems with a complex gobal supersymmetry.[11] Such systems[12] already exist. For example, the complex globally supersymmetric Yang-Mills multiplet contains the Yang-Mills field $A_\mu{}^a$, a Dirac spin-$\frac{1}{2}$ field λ^a, and a real scalar and pseudoscalar field A^a and B^a, all massless and in the adjoint representation of the internal symmetry group. As far as the superfield approach is concerned, the present theory should be related to a superspace with complex (Dirac) anticommuting coordinates.[13]

In this highly gauge-invariant theory, the photon joins the graviton as part of the gauge multiplet of complex local supersymmetry. Real locally supersymmetric Maxwell-Einstein theory[5] is one-loop–nonrenormalizable.[14] It has not gone unnoticed that this theory could be, at least at the one-loop level, a renormalizable theory unifying gravitation and electromagnetism.

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Note added.—Recently, the one-loop corrections in this extended supergravity have been calculated by M. T. Grisaru, P. van Nieuwenhuizen, and J. A. M. Vermaseren [this issue, Phys. Rev. Lett. 37, 1662 (1976)] and they are indeed finite.

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