$\rho - A_2$ exchange falls as $\alpha(0) = 0.5$. Within the context of these simple models, it is difficult to reconcile a value for α compatible with ρ-A_2 exchange with the presistence of the forward peak mentioned previously.

It is worth noting that the differential cross section at higher momentum transfers shows a shoulder developing at $|t| \cong 0.1$ (GeV/c)^2 as the incident momentum is increased, while beyond a $|t|$ of 0.25 the falloff appears to be exponential. As indicated by the decreasing values of $\alpha(t)$, there is some evidence for shrinkage in this exponential region.

With use of the $|t| = 0$ cross section derived from formula (1), bounds on the total cross-section difference may be estimated from the optical theorem and the assumption of charge symmetry[8]:

$$[4\sqrt{\pi}\hbar(A+B)^{1/2}] \geq |\sigma_{pp} - \sigma_{np}|. \quad (3)$$

These limits are presented in Table II, together with limits from the separate measurements of the total cross sections.[9,10] It is evident the limits from this experiment at the highest energies are comparable to those found from subtraction of the total cross sections.

Finally, formula (3) neglects both the real part of the amplitude and the double-flip zero net helicity-flip amplitude. Thus, any theoretical model which estimates these large contributions should be able to establish significantly stronger bounds on the cross-section difference than those presented in this paper.

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†Present address: Fermilab, Batavia, Ill. 69510.

‡Present address: California Institute of Technology, Pasadena, Calif. 91108.

§Present address: Johns Hopkins University, Baltimore, Md. 21218.

‖Present address: Department of Radiology, Stanford University, Stanford, Calif. 94305.

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One-Loop Renormalizability of Pure Supergravity and of Maxwell-Einstein Theory in Extended Supergravity

M. T. Grisaru*
Department of Physics, Brandeis University, Waltham, Massachusetts 02154

and

P. van Nieuwenhuizen† and J. A. M. Vermaseren†
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

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It is shown that the four-particle S matrix of pure supergravity and of the recently constructed theory of extended supergravity with O(2) internal symmetry is one-loop finite. An explicit calculation of photon-photon scattering confirms this latter result.

Whereas the one-loop quantum corrections to the S matrix of pure Einstein gravitation are known to be finite,[1] all endeavors to couple matter fields to gravitons (scalars,[1] photons,[2] fermions,[2] Yang-Mills bosons,[2] or quantum electrodynamics[3]) have led so far to divergent quantum field theories. Because of the dimensional character of the gravitational constant, proper renormalizability can never be obtained and instead one should look for theories where all divergences cancel in a miraculous way. The prime candidate for such a theory is supergravity[4-6] which connects intimately Fermi-Bose supersymmetry with gravitation by means of a local gauge theory. It is known that in globally supersymmetric field theories, cancellations do occur between the divergences stemming from boson and fermion loops.[7] In this Letter we prove that in quantum supergravity similar cancellations occur which render the four-particle S-matrix elements of the $(2, \frac{3}{2})$ gauge multiplet[4,5] and the $(2, \frac{3}{2}, \frac{3}{2}, 1)$ extended gauge multiplet[6] one-loop finite. These results can in principle be extended to many-particle S-matrix elements.

Our discussion consists of three parts. (i) We show that in any theory where matter is coupled covariantly to Einstein gravitation, the one-loop S-matrix elements with only external gravitons (but both internal gravitons and/or matter fields) are finite. (ii) If there is a global symmetry which relates the amplitudes with external matter fields to the amplitudes with only external gravitons, all one-loop S-matrix amplitudes will be finite. This is precisely the kind of symmetry provided by pure supergravity and extended supergravity where all matter fields are in the same multiplet as supergravitons. (iii) The proof of the pudding is an explicit calculation: The one-loop divergences of the S matrix for photon-photon scattering in the $(2, \frac{3}{2}, \frac{3}{2}, 1)$ multiplet do indeed cancel.

In DeWitt's background-field method[1,8] the one-loop counter terms for any matter system which is covariantly coupled to Einstein gravitation are given by

$$\Delta I = \frac{1}{n-4} \int d^4x\, g^{1/2}[a(R_{\mu\nu})^2 + bR^2 + cR_{\mu\nu\rho\sigma}M^{\mu\nu\rho\sigma}(\varphi, e) + dN(\varphi, e)]. \tag{1}$$

Here M and N are polynomials in the background matter fields φ and their derivatives, dimensional regularization is used, and $e_{a\mu}$ is the background vierbein field.[9] No term proportional to $(R_{\mu\nu\rho\sigma})^2$ is present since it can be expressed in terms of $(R_{\mu\nu})^2$ and R^2 by means of the Gauss-Bonnet theorem.[1,8] For a given process the one-loop divergences of the S matrix of normal field theory are obtained[1,8] by inserting into ΔI the iterative solutions of the classical field equations (tree graphs), of the form

$$e_{a\mu}(x) = e_{a\mu}{}^{\rm in}(x) + k\int D(x-x')J_e(\varphi, e)dx', \tag{2}$$

$$\varphi(x) = \varphi^{\rm in}(x) + k\int D(x-x')J_\varphi(\varphi, e)dx', \tag{3}$$

and collecting all terms with the required number of in fields. Obviously the solutions of the classical field equations satisfy the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = -k^2T_{\mu\nu}(\varphi, e), \tag{4}$$

which relation can be viewed as an equality between two infinite sets of tree graphs, all starting at a given point x, and ending on in-field operators.

An equivalent expression for ΔI is therefore

$$\Delta I = \frac{1}{n-4}\int d^4x\, g^{1/2}[4ak^4(T_{\mu\nu})^2 + 4bk^4(T_\mu{}^\mu)^2 + cR_{\mu\nu\rho\sigma}M^{\mu\nu\rho\sigma} + dN]. \tag{5}$$

In this form ΔI contains no terms with only gravitational fields and since the tree graphs given by $\varphi(x)$ in Eq. (3) never have terms with only gravitational in fields, it is clear that ΔI will have vanishing matrix elements between any in states containing only gravitons.[10] Therefore, *all S-matrix amplitudes for processes with only external gravitons but any internal particles are one-loop finite.*

If the theory contains a photon field, the terms in the counter Lagrangian which depend on two photon fields but not on other matter fields are proportional to

$$eR(F_{\mu\nu})^2 + fR^{\mu\nu}F_{\mu\alpha}F_\nu{}^\alpha + hR^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + k(D_\alpha F^{\alpha\beta})^2. \tag{6}$$

After inserting the classical field equations, the first two terms become of the form FFT, while the last term vanishes. If the theory is invariant under the duality rotation $F_{\mu\nu} \rightarrow e\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ then $h = 0$.[11] Thus, *in any duality-invariant theory the S matrix for processes with two external photons and any number of external gravitons is one-loop finite.*

The one-loop divergences of the S matrix can be thought of as pseudo amplitudes given by tree graphs obtained by using the vertices defined by the original Lagrangian and by the counter Lagrangian but only to first order in the latter. Since both the Lagrangian and the counter Lagrangian are separately invariant under global supersymmetry transformations, there will be relations between the pseudo amplitudes for a given process. For a four-particle process these relations are derived as follows[12]: Let $\alpha(\vec{p})$, $\beta(\vec{p})$ be the in-annihilation operators for a pair of particles in a $(j, j+\frac{1}{2})$ multiplet of momentum $\vec{p}$ and the same helicity. Then

$$[\bar{\epsilon}Q, \alpha(\vec{p})] = \lambda(j)\Gamma^*(\vec{p}, \epsilon)\beta(\vec{p}), \tag{7}$$

$$[\bar{\epsilon}Q, \beta(\vec{p})] = \lambda(j)\Gamma(\vec{p}, \epsilon)\alpha(\vec{p}), \tag{8}$$

where λ is a numerical factor, Q the supersymmetry charge, and ϵ a Majorana spinor parametrized by two anticommuting complex c-numbers η_1 and η_2. For a massless particle with $\vec{p} = (p \sin\vartheta, 0, p\cos\vartheta)$,

$$\Gamma(p, \epsilon) = (2p)^{1/2}(\eta_1 \cos\tfrac{1}{2}\vartheta + \eta_2 \sin\tfrac{1}{2}\vartheta). \tag{9}$$

Equating separately the η_1 and η_2 terms in expressions of the form

$$\langle 0|\alpha(\vec{p}_2)\alpha(\vec{q}_2)[\bar{\epsilon}Q, S]\,\alpha^\dagger(\vec{p}_1)\beta^\dagger(\vec{q}_1)|0\rangle = 0, \tag{10}$$

one obtains the following relations between the helicity amplitudes of the gauge multiplet:

$$\langle +\tfrac{3}{2}+\tfrac{3}{2}| +\tfrac{3}{2}+\tfrac{3}{2}\rangle = \langle +2+2| +2+2\rangle = \left(\frac{-u}{S}\right)^{1/2}\langle +\tfrac{3}{2}+2| +\tfrac{3}{2}+2\rangle, \tag{11}$$

$$\langle +\tfrac{3}{2}+\tfrac{3}{2}| \pm\tfrac{3}{2}-\tfrac{3}{2}\rangle = \langle +2+2| \pm 2-2\rangle = 0. \tag{12}$$

The $\pm$ signs denote the handedness of the particles. All other pseudo amplitudes can be related by crossing symmetry and parity and time-reversal invariance except $\langle +\frac{3}{2}+\frac{3}{2}| -2-2\rangle$. However, an argument based on the kinematical constraints[12] similar to those of Grisaru, van Nieuwenhuizen, and Wu[13] shows that this pseudo amplitude vanishes. Since the pseudo amplitudes for graviton-graviton scattering vanish, it follows that *all four-particle S-matrix elements in the (2, $\frac{3}{2}$) multiplet are finite.*

We consider next the recently constructed[6] gauge theory for the $(2, \frac{3}{2}, \frac{3}{2}, 1)$ multiplet of extended supergravity. In this case similar results hold. Again from global supersymmetry invariance the divergences for amplitudes involving particles only of the $(2, \frac{3}{2})$ or the $(\hat{\frac{3}{2}}, 1)$ multiplet are equal or zero (a carat distinguishes from now on the spin-$\frac{3}{2}$ field in the matter multiplet from that in the gauge multiplet). Since the theory is invariant under rotations in the $(\frac{3}{2}, \hat{\frac{3}{2}})$ plane, one has

$$\langle +\tfrac{3}{2}+\tfrac{3}{2}| +\tfrac{3}{2}+\tfrac{3}{2}\rangle = \langle +\hat{\tfrac{3}{2}}+\hat{\tfrac{3}{2}}| +\hat{\tfrac{3}{2}}+\hat{\tfrac{3}{2}}\rangle. \tag{13}$$

It follows that the S-matrix elements in the $(2, \frac{3}{2})$ and $(\hat{\frac{3}{2}}, 1)$ sectors are finite. Finally, each pseudo amplitude with two $(2, \frac{3}{2})$ particles and two $(\hat{\frac{3}{2}}, 1)$ particles is either zero, or, by global super symmetry, is equal to the pseudo amplitude $\langle +2+1| +2+1\rangle$. Although the theory itself is not duality invariant, the four-fermion terms which break duality invariance do not contribute to the one-loop divergences with two external photons and no other external matter lines. Therefore the counter term $R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$ in Eq. (6) is absent and the pseudo amplitude $\langle +2+1| +2+1\rangle$ vanishes. It follows that *all four-particle S-matrix elements of the (2, $\frac{3}{2}$, $\frac{3}{2}$, 1) multiplet are one-loop finite.*

For an explicit verification of the preceding general arguments we calculate, using normal field theory, the one-loop divergences of photon-photon scattering in the extended $(2, \frac{3}{2}, \frac{3}{2}, 1)$ multiplet of Ref. 6 to which we refer for details. This theory is invariant under two local supersymmetry transformations δ_ψ and δ_φ. For later use we mention the transformation law

$$\delta_\psi \varphi_\mu = \sigma^{\alpha\beta}\gamma_\mu \epsilon[(\sqrt{2})^{-1}F_{\alpha\beta} - k\bar{\psi}_\alpha\varphi_\beta]. \tag{14}$$

As gauge-fixing term we choose[1,2,14,16]

$$\mathcal{L}^B = -\tfrac{1}{2}(\partial_\mu h_{\mu\nu} - \tfrac{1}{2}\partial_\nu h_{\mu\mu})^2 - \alpha^{-1}(e_{a\mu} - e_{\mu a})^2 - \tfrac{1}{2}(\partial_\mu A_\mu)^2 - \tfrac{1}{2}(\bar{\psi}\cdot\gamma\not{\partial}\gamma\cdot\psi + \bar{\varphi}\cdot\gamma\not{\partial}\gamma\cdot\varphi), \tag{15}$$

which contains only terms quadratic in the quantum fields, and let α tend to infinity in which gauge the vierbein field is symmetric. The corresponding ghost Lagrangian is found to be

$$\mathcal{L}^G = -\bar{C}_\psi\gamma^a\delta_a{}^\mu D_\mu C_\psi - \bar{C}_\varphi\gamma^a\delta_a{}^\mu D_\mu C_\varphi + \mathcal{L}^G(BB) + \mathcal{L}^G(BF), \tag{16}$$

where the ghost terms in the gravitational-electromagnetic sector $\mathcal{L}^G(BB)$ are known from the Dirac-Einstein system.[2] The ghost terms in the supersymmetry sector are diagonal in the commuting supersymmetry transformations δ_ψ and δ_φ because

$$\delta_\psi(\gamma\cdot\varphi) = \delta_\varphi(\gamma\cdot\psi) = 0, \tag{17}$$

as the reader can easily verify from Eq. (14). The off-diagonal terms $\mathcal{L}^G(BF)$, containing one commuting bosonic ghost field, also contain either a ψ or a φ field and will not contribute to the process considered.

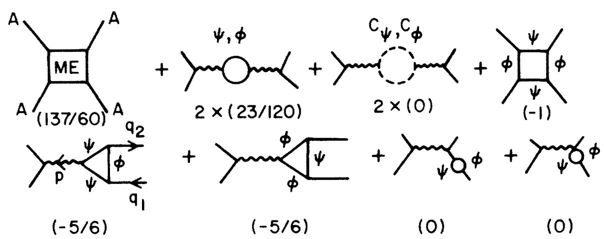

FIG. 1. The divergences of photon-photon scattering. The numbers below the graphs denote the coefficients of $(n-4)^{-1}k^4[T_{\mu\nu}(\gamma)]^2$, where $T_{\mu\nu}(\gamma)$ is the photon energy tensor, while ME stands for all graphs present in ordinary Maxwell-Einstein theory.

The calculation is straightforward and for details we refer to Ref. 15. Dimensional regularization was used and tedious γ-matrix algebra was avoided by using Veltman's algebraic manipulation program Schoonschip.[16] Our results are not expected to depend on the choice of regularization scheme nor to contain axial anomalies. The graviton pole in the diagrams with vertex corrections cancels if one uses the identity

$$2(q_\alpha{}^1 T_{\alpha\beta} F_{\beta\gamma}{}^2 F_{\gamma\delta}{}^1 q_\delta{}^2) = (q_\alpha{}^2 T_{\alpha\beta} a_\beta{}^2)(F_{\alpha\beta}{}^1 F_{\alpha\beta}{}^2) \qquad (18)$$

so that the equivalence principle still holds at this quantum level.[3] All contributions are proportional to $(T_{\mu\nu})^2$ as expected from electromagnetic duality.[16] When we add all results as indicated in Fig. 1, *the sum of all divergences vanishes.*

This is the first time after many negative results that the one-loop corrections to the S matrix of a matter-gravity system turn out to be finite. Green's functions such as the vacuum polarization[2] are of course still divergent. Our results can in principle be extended to many-particle S-matrix elements and to more matter fields by putting these in larger irreducible multiplets with larger internal symmetry groups. It is conceivable that these multiplets arise as a limiting case[17] of the theory of local supersymmetry of Nath.[18] One cannot expect finiteness if matter is not in the same irreducible multiplet as the supergravitons.[16]

As to higher loops, the crucial question is here whether the two-loop S matrix without external matter lines is finite. In the background-field method such two-loop divergences are proportional to $(R_{\mu\nu\alpha\beta})^3$ and since this invariant does not vanish on shell nor satisfy a Gauss-Bonnet-type relation,[19] its coefficient would have to vanish. The computation of this coefficient seems tedious, but its result is worth it.

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Characteristics of the S-Wave $\bar{K}K$ Enhancement at 1300 MeV*

A. J. Pawlicki, D. S. Ayres, D. Cohen, R. Diebold, S. L. Kramer, and A. B. Wicklund
Argonne National Laboratory, Argonne, Illinois 60439
(Received 7 September 1976)

Our high-statistics experiment on $\pi^-p \to K^-K^+n$ and $\pi^+n \to K^-K^+p$ at 6 GeV/c confirms the $\bar{K}K$ S-wave enhancement near 1300 MeV recently observed by Cason *et al.* in $\pi^-p \to K_S{}^0K_S{}^0n$. Using the F-wave amplitude and the isospin dependence of interfering $\bar{K}K$ states, we resolve partial-wave ambiguities and find the S wave to be dominantly isospin $I = 0$, with a slow variation of phase. This excludes the interpretation of Cason *et al.* that the S wave enhancement is a narrow $I = 1$ state.

A new $\bar{K}K$ S-wave state with mass 1255 ± 5 MeV and width 79 ± 10 MeV has been reported by Cason *et al.*[1] in an experiment which studied the reaction

$$\pi^-p \to K_S{}^0K_S{}^0n, \qquad (1)$$

at 6 and 7 GeV/c. The mass and width were determined from one of two phase-shift solutions by using the phase variation with mass given by the S-D interference in the $Y_2{}^0$ moment of the $K_S{}^0K_S{}^0$-decay angular distribution for $-t < 0.2$ GeV^2. Measurement of Reaction (1) alone does not determine the isospin I of the S wave. While Cason *et al.* suggested that the effect has $I = 1$, our results show that the isospin is zero. We also find that the more slowly varying phase solution is preferred, and not the rapidly varying solution which yielded the above values for the mass and width.

We have performed a high-statistics comparison of

$$\pi^-p \to K^-K^+n \quad (110\,000 \text{ events}), \qquad (2)$$

and

$$\pi^+n \to K^-K^+p \quad (50\,000 \text{ events}), \qquad (3)$$

at 6 GeV/c using the Argonne effective-mass spectrometer. We studied the region of K^-K^+ mass $M < 1750$ MeV and momentum transfer $-t < 0.40$ GeV^2; details are given elsewhere.[2,3] Comparison of the K^-K^+-decay moments from Reactions (2) and (3) directly isolates contributions from interferences between K^-K^+ states of differing isospin. It has been shown[2-4] that if A_0 and A_1 are the amplitudes for production of $I = 0$ and $I = 1$ K^-K^+ states in Reaction (2), then the amplitude for Reaction (2) is $A_0 + A_1$, while for Reaction (3) the amplitude is $A_0 - A_1$. Symbolically, we can write the cross sections for Reactions (2) and (3) as

$$\sigma^\mp \propto |A_0 \pm A_1|^2 = |A_0|^2 + |A_1|^2 \pm 2\,\mathrm{Re}(A_0A_1{}^*),$$

where $\sigma \equiv (4\pi)^{1/2}d^2\sigma/dtdM$ and the superscripts $-$ and $+$ refer to Reactions (2) and (3), respectively. Summing the two cross sections eliminates the $A_0A_1{}^*$ interference term; taking the difference isolates that term:

$$\sigma_{\text{sum}} \propto |A_0|^2 + |A_1|^2,$$

$$\sigma_{\text{dif}} \propto \mathrm{Re}(A_0A_1{}^*),$$

where $\sigma_{\text{sum}} \equiv \sigma^- + \sigma^+$, $\sigma_{\text{dif}} \equiv \sigma^- - \sigma^+$. Similar relations hold for the various K^-K^+-decay moments $\sigma\langle Y_l{}^m\rangle$.

By charge independence, the amplitude for $\pi^-p \to \bar{K}^0K^0n$ is the same as that for Reaction (3), and the even partial-wave amplitudes for Reactions (1) and (3) are the same except for a factor of $\sqrt{2}$ (since $K_S{}^0K_S{}^0$ and $K_L{}^0K_L{}^0$ each take one half of the even partial-wave cross section for $\pi^-p \to \bar{K}^0K^0n$).

We can determine the S-wave contribution to the cross section at small t. In Ref. 3 we have identified the non-S-wave contributions to the sum cross section $\sigma_{\text{sum}}\langle Y_0{}^0\rangle$ and these can be subtracted off to give $\sigma_{\text{sum}}{}^S\langle Y_0{}^0\rangle = 2(|I=0\ S\text{ wave}|^2 + |I=1\ S\text{ wave}|^2)$, which is shown in Fig. 1. The S^* peak below 1100 MeV is clearly exhibited, as well as a second peak of mass ~ 1300 MeV and width ~ 150 MeV. The S wave accounts for nearly half of the K^-K^+ cross section at 1300 MeV, and the systematic uncertainty in $\sigma_{\text{sum}}{}^S\langle Y_0{}^0\rangle$ is about $\pm 10\%$ in this region.