is also shown for comparison. The lack of shear in the stellarator should be noted, in contrast to the tokamak case.

The authors would like to thank Dr. R. J. Bickerton for his advice and encouragement during this work.

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Interaction of Beams of Radiation of Finite Diameters within a Plasma

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Two beams of waves of finite diameters intersecting within a magnetized plasma emit a secondary beam of waves, provided a resonance condition is satisfied. The field of the secondary beam and the total power emitted are determined for arbitary directions and polarizations of the incident beams. This process could be used as a means of time- and space-resolved diagnostics since it yields two relations between the density, the temperatures, and the magnetic field. Explicit formulas are given for the case of two beams in the ordinary mode producing a lower hybrid wave.

When two beams of radiation, with ω_j and $\vec{k}_j, j = 1, 2$, intersect within a plasma, they generate in the interaction volume a second-order polarization which emits a new wave, provided that $\omega_0 = \omega_2 \pm \omega_1$ and $\vec{k}_0 = \vec{k}_2 \pm \vec{k}_1$ satisfy the dispersion relation $\Delta(\omega_0, \vec{k}_0) = 0$ of this wave. In this case, power is transferred from the primary beams to the new beam according to the Manley-Rowe[1] relations

$$\delta P_1 + \delta P_2 = P_0,$$

$$\pm \frac{\delta P_1}{\omega_1} = \frac{\delta P_2}{\omega_2} = \frac{P_0}{\omega_0}$$

(assuming $\omega_2 > \omega_1 > 0$). This process can in principle be used as the basis of a new method of plasma diagnostics. In order to implement this method one of the incident beams should be frequency-modulated over a range that covers the resonance $\Delta(\omega_0, \vec{k}_0) = 0$. The emerging beams will then be amplitude-modulated, allowing synchronous detection. Thus one determines ω_0 and $\vec{k}_0$ which satisfy $\Delta = 0$, which establishes a first relation between the density, the temperatures, and the magnetic field within the small volume defined by the intersection of the primary beams. The theory also gives explicitly the power P_0 in the form

$$P_0 = M(\mu) P_1 P_2 / P^*,$$

where $M(\mu)$ is a resonance factor, taking its maximum when the mismatch μ vanishes. Thus the measurement of the amplitude modulation (that is, of δP) of one of the emerging beams yields a second relation among the plasma parameters.

The characteristic power P^* gives a measure of the minimum power required to observe this process. Let $(\delta P_1/P_1)_{\min}$ be the smallest detectable modulation, then one must have

$$P_2 \geq \frac{\omega_0}{\omega_1} P^* \left(\frac{\delta P_1}{P_1}\right)_{\min}.$$

The power in the first beam can be small since δP_1 only has to be above the noise. While this establishes a lower limit for the power in one beam, there is no such limit for the electric fields, so that one can always avoid parametric effects by keeping the fields below threshold.

Nonlinear mixing of waves in plasmas has been considered in a number of papers.[2-9] Except for Ref. 9, all of them treat infinite plane-wave fields rather than beams of finite cross sections. Magnetized plasmas are considered in Refs. 4 and 5 but for special choices of the wave vectors and polarizations. References 6, 7, and 8 consider saturation in an unmagnetized plasma when the primary wave vectors are parallel.

In this Letter, we consider beams of finite diameters of arbitrary direction and polarization intersecting within a magnetized plasma. An explicit expression is obtained for the power transferred to the emitted beam.

The second-order polarization can be written in $\kappa=\{\omega, \vec{k}\}$ space as[10]

$$\mathcal{P}_\alpha^{(2)}(\kappa)=-i\sum(q/m)(\omega_p^2/\omega)(2\pi)^{-4}\int d\kappa'\,d\kappa''\delta(\kappa'+\kappa''-\kappa)\Lambda_{\alpha\beta\gamma}(\kappa',\kappa'')E_\beta(\kappa')E_\gamma(\kappa''), \tag{1}$$

with the summation extending over all species.[11] In the two-fluid theory, the matrix Λ is given by[9]

$$\Lambda_{\alpha\beta\gamma}(\kappa',\kappa'')E_\beta' E_\gamma''=(\omega'^2\omega'')^{-1}(\vec{k}'\cdot\overleftrightarrow{U}'\cdot\vec{E}')(\overleftrightarrow{U}''\cdot\vec{E}')_\alpha$$
$$+(\omega\omega'\omega'')^{-1}(\vec{k}''\cdot\overleftrightarrow{U}'\cdot\vec{E}')[(\overleftrightarrow{U}''-1)\cdot\overleftrightarrow{U}\cdot\vec{E}']_\alpha+(\omega\omega'\omega'')^{-1}(\vec{E}''\cdot\overleftrightarrow{U}'\cdot\vec{E}')(\overleftrightarrow{U}\cdot\vec{k}'')_\alpha .$$

The matrix U is defined by

$$\overleftrightarrow{U}\cdot\vec{a}=\frac{\omega^2}{\omega^2-\Omega^2}\left(\vec{a}-i\frac{\vec{\Omega}}{\omega}\times\vec{a}\right)-\frac{\vec{\Omega}(\vec{\Omega}\cdot\vec{a})}{\omega^2-\Omega^2}$$

with $\vec{\Omega}=q\vec{B}/m$. The incident beams are assumed to be of the form

$$\vec{E}_j=A_j\vec{e}_j\exp\{-(2\alpha)^{-1}[r^2-(\hat{g}_j\cdot\vec{r})^2]+i\vec{k}_j\cdot\vec{r}-i\omega_j t\}+\text{c.c.},$$

where $\vec{g}_j=(\partial\omega/\partial\vec{k})_j$ and $\hat{g}_j=\vec{g}_j/|\vec{g}_j|$; $\vec{e}_j$ is the polarization vector belonging to the mode $(\vec{k}_j,\omega_j)$; α is the area of the cross section of the beam which must be large compared to the wavelength: $\alpha k_j^2\gg 1$. The total power transmitted in such a beam is

$$P_j=\pi\alpha A_j^2\omega_j^{-1}|\partial\Delta_j/\partial\vec{k}|,$$

where

$$\Delta_j=\vec{e}_j^{\,*}\cdot[\omega^2(1+\overleftrightarrow{\chi})-k^2+\vec{k}\vec{k}]\cdot\vec{e}_j$$

and $\overleftrightarrow{\chi}$ is the linear susceptibility.

Substituting $\vec{E}=\vec{E}_1+\vec{E}_2$ into (1) and neglecting all but the terms oscillating at the difference frequency, one obtains

$$\mathcal{P}_\alpha^{(2)}(\vec{r},t)=A_1A_2U(\vec{r})\pi_\alpha\exp(i\vec{k}_0\cdot\vec{r}-i\omega_0 t)+\text{c.c.}, \tag{2}$$

where

$$U(\vec{r})=\exp[-(2\alpha)^{-1}(\vec{r}\cdot\overleftrightarrow{\Gamma}^{-1}\cdot\vec{r})]$$

with

$$\vec{r}\cdot\overleftrightarrow{\Gamma}^{-1}\cdot\vec{r}=2r^2-(\hat{g}_1\cdot\vec{r})^2-(\hat{g}_2\cdot\vec{r})^2$$

and

$$\pi_\alpha=-i\sum(q/m)(\omega_p^2/\omega)\{\Lambda_{\alpha\beta\gamma}(-\kappa_1,\kappa_2)e_{1\beta}^{\,*}e_{2\gamma}+\Lambda_{\alpha\beta\gamma}(\kappa_2,-\kappa_1)e_{2\beta}e_{1\gamma}^{\,*}\}.$$

The far field of the wave emitted by the polarization (2) can be obtained from the asymptotic form of the Green's function[12]

$$\overleftrightarrow{\mathcal{G}}(\omega_0,\vec{r})=\frac{\omega_0^2\vec{e}\,\vec{e}^{\,*}\exp(i\vec{k}\cdot\vec{r})}{2\pi K|\partial\Delta/\partial\vec{k}|r},$$

where the vector $\vec{k}=\vec{k}(\omega_0,\vec{r})$ must be so determined that $\Delta(\omega_0,\vec{k})=0$ and $\partial\omega/\partial\vec{k}=\lambda\vec{r}$ with $\lambda>0$ (that is,

the group velocity must point in the direction of $\vec{r}$). K is the Gaussian curvature of the surface $\Delta = 0$ at the point $\vec{k}$. Resonance means $\vec{k} = \vec{k}_0$.

The total power radiated can be obtained by integrating the power flow vector

$$\vec{S} = -|\vec{E}|^2 \omega_0^{-1} \partial\Delta/\partial\vec{k},$$

over the surface of a large sphere, using only the far field. Thus one obtains

$$P_0 = P_1 P_2 M(\mu) \frac{e^2}{m_e^2} \frac{\omega_{pe}^4 \omega_0 \omega_1 \omega_2}{|\partial\Delta_1/\partial\vec{k}_1||\partial\Delta_2/\partial\vec{k}_2||\partial\Delta/\partial\vec{k}|} |\Lambda_{\alpha\beta\gamma}(-\kappa_1,\kappa_2) e_\alpha^* e_{1\beta}^* e_{2\gamma} + \Lambda_{\alpha\beta\gamma}(\kappa_2,-\kappa_1) e_\alpha^* e_{2\beta} e_{1\gamma}^*|^2. \quad (3)$$

The function $M(\mu)$ is very sharply peaked at resonance and

$$M(\mu) = (\sin\gamma)^{-2}[(\Gamma_{11} - \mu K_1)(\Gamma_{22} - \mu K_2) - \Gamma_{12}^2]^{1/2} \exp(-\alpha\mu^2 \vec{t}_0 \cdot \vec{\vec{\Gamma}}^{-1} \cdot \vec{t}_0).$$

In this last formula γ is the angle between the group velocities $\vec{g}_1$ and $\vec{g}_2$; K_1 and K_2 are the principal curvatures of the surface $\Delta(\omega_0, \vec{k}) = 0$; $\vec{t}_1$ and $\vec{t}_2$ are the principal tangents of this surface and $\vec{t}_0 = \vec{t}_1 \times \vec{t}_2$, finally $\Gamma_{jk} = \vec{t}_j \cdot \vec{\vec{\Gamma}} \cdot \vec{t}_k$. The mismatch μ is defined by

$$\mu \vec{\vec{\Gamma}}^{-1} \cdot \vec{t}_0 = \vec{k} - \vec{k}_0.$$

At resonance, $\mu = 0$, $M(\mu)$ is very sharply peaked due to the large value of α.

As an example I consider two beams polarized in the ordinary mode with $\vec{k}_1 \cdot \vec{B} = \vec{k}_2 \cdot \vec{B} = \vec{k}_1 \cdot \vec{k}_2 = 0$. The emitted beam will be in the lower hybrid mode, provided that $\omega_2 = \omega_1 + \omega_0$ with

$$\omega_0 = \omega_{LH} + \frac{k_B}{c^2 m_i}\left[\frac{3\beta^2}{(1+\beta)^2} T_i + T_e\right] \frac{\omega_1^2 - \omega_{pe}^2}{\omega_{LH}}$$

(in mks units), where $k_B = 1.38 \times 10^{-23}$ J/°K, $c = 3 \times 10^8$ m/s, $\beta = \omega_{pe}^2/\omega_{ce}^2$, $\epsilon = m_e/m_i$, $\omega_{LH}^2 = \omega_{ce}^2 \epsilon\beta/(1+\beta)$. P^* becomes

$$P^* = 2^{5/2} \frac{c k_B}{\mu_0} \frac{(m_e m_i)^{1/2}}{e^2} [T_e + 3(1+\beta^{-1})^2 T_i](1+\beta)^{1/2} \nu^2 (\nu^2 - 1)^{1/2}$$

(in mks units), where $\mu_0 = 1.256 \times 10^{-6}$ Vs/Am, $\nu = \omega_1/\omega_{pe}$. The minimum power required in the second beam is

$$P_{2\min} = 2^{5/2} \frac{c k_B}{\mu_0} \frac{m_e}{e^2} [T_e + 3(1+\beta^{-1})^2 T_i] \nu (\nu^2 - 1)^{1/2} \left(\frac{\delta P_1}{P_1}\right)_{\min}$$

(in mks units). Although one would like to reduce $P_{2\min}$ as much as possible one cannot exploit the factor $(\nu^2 - 1)^{1/2}$. The inverse of this factor is simply the WKB approximation of the swelling of the beams as they approach cutoff. This swelling, of course, is finite.

Choosing $\beta = 1$, $\nu = \sqrt{2}$, $T_e = T_i = 10^8$ °K, and $\delta P_1/P_1 = 0.01$, one obtains $P_{2\min} = 12$ MW which, for a laser, is not exorbitant. Equation (3) applies also for a secondary beam oscillating at the sum of the incident frequencies except that the term involving Λ must be replaced by

$$|\Lambda_{\alpha\beta\gamma}(\kappa_1,\kappa_2) e_\alpha^* e_{1\beta} e_{2\gamma} + \Lambda_{\alpha\beta\gamma}(\kappa_2,\kappa_1) e_\alpha^* e_{2\beta} e_{1\gamma}|^2.$$

In this case both primary beams give up energy to the secondary beam so that such a process could be useful as a heating mechanism which does not have the defects of lower hybrid heating. Indeed it has been shown[13] that lower hybrid heating suffers from parametric decay of the incoming beam which heats the plasma at the surface while depleting the beam before it reaches the center. When mixing two beams one can keep their fields low enough so as to avoid parametric decay along their separate trajectories and yet inject enough power so as to convert a large fraction into a secondary wave which will be absorbed.

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Low-Energy Electron Diffraction Determination of the Atomic Arrangement on Impurity-Stabilized Unreconstructed Si{111} Surfaces

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An impurity-stabilized Si{111} 1×1 structure has been obtained by depositing minute amounts of Te on a clean Si{111} 7×7 surface. A low-energy electron diffraction structure analysis of this 1×1 structure reveals that the atomic arrangement is essentially bulklike, but involves a slight contraction of the first interlayer spacing by about 15% with respect to the bulk value.

It is a matter of considerable interest that, despite the great deal of attention that has been devoted to Si surfaces in recent years, no direct determination has been made of the atomic arrangements on such surfaces. The problem of acquiring such knowledge is complicated by the fact that at least the three lowest index surfaces of Si (i.e., the {001}, {110}, and {111} surfaces) in their clean state have structures different from those of the equivalent bulk planes or, in the language of surface crystallography, are "reconstructed." The Si{001} surface, for example, exhibits a so-called 2×1 structure, the periodicity in one of the ⟨110⟩ directions on the surface being twice as large as the one on bulk {001} planes. The vacuum-cleaved Si{111} surface also exhibits a 2×1 structure, but the annealed Si{111} surface exhibits a 7×7 structure, which has a unit mesh 49 times larger than a bulk {111} plane. The electronic structures of both Si{001} and Si{111} surfaces have been objects of extensive experimental[1-8] and theoretical[9-14] investigations, while the corresponding atomic structures have only been objects of sophisticated speculations.[15-21]

In recent years, a great deal of effort has been devoted to the study of the Si{001} 2×1 structure by several of the groups active in LEED (low-energy electron diffraction) crystallography,[22-26] but no solution of the problem has yet been reported. One of the most puzzling aspects of this failure has been the unanswered question about the basic capability of either the multiple-scattering or the data-averaging methods used in LEED crystallography[27] to treat the diamond lattice properly and accurately. Legitimate questions have been raised, in particular, about the correctness of the muffin-tin model for the description of the Si lattice.

Somewhat less attention has been devoted to the atomic structure of the Si{111} surface, probably because the 2×1 superstructure requires *in situ* cleaving of a Si single crystal (not a universally available facility) and the 7×7 superstructure is just too complicated to handle at the present time. However, Florio and Robertson[28] established a few years ago that the Si{111} 7×7 structure reversibly transforms into a 1×1 structure at 900°C and that the 1×1 structure can be "impurity-stabilized" by minute amounts of Cl on the surface at room temperature. Hagstrum and Becker[29] reported that the 1×1 structure can be quenched on the clean Si{111} surface by rapid cooling from