Four-Body Model of Two-Nucleon Transfer Reactions*

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An exact four-body model of heavy-ion two-nucleon transfer reactions is described that includes all orders of interaction between the exchanged nucleons. The relative importance of the Q value of the reaction and of the nature of the nucleon-nucleon interaction in governing the magnitude of the transfer cross section is studied.

In the distorted-wave treatment of two-nucleon transfer reactions initiated by both light and heavy ions, there are persistent discrepancies (factor of ~ 5) in the predicted magnitude of the cross sections when compared with experimental results.[1] It has also been observed[2] that a large suppression (factor of ~ 50) of nn- and pp-exchange heavy-ion reactions exists relative to those involving np exchange. Some current attempts to understand these problems have centered on multistep contributions to the two-nucleon-exchange process,[3] and on damping due to Q-mismatch effects as well.[4] A possible alternative source of concern is that the conventional reaction theories do not include rescattering corrections among the nucleons being exchanged and, since the nucleon-nucleon interaction cannot be considered weak, the proper inclusion of the higher-order terms of that interaction could be a significant factor in a proper theoretical description of these processes. In this Letter, we study a simplified but exact four-body model of two-nucleon transfer reactions in order to assess the importance of higher-order terms of the nucleon-nucleon interaction.

Our four-body model is based on the extended Lee model of which a detailed description has been given elsewhere.[5] The spinless version to be used here involves a pair of nucleons (n) as well as two "heavy nuclei" a and b. Two-body n-n scattering proceeds as in the Amado model[6] by a coupling to a d quasiparticle so that the allowed process is $d \longleftrightarrow n+n$. The nature of this interaction[7] may be altered considerably by adjusting the wave-function renormalization constant Z_d of the d particle, subject to the constraint $0 \leq Z_d \leq 1$. If we take $Z_d = 1$, the d is an elementary particle with no coupling to $n+n$, while if we choose $Z_d = 0$, a separable-potential model with a single bound state may be obtained that is appropriate to the spin-triplet nucleon-nucleon interaction. We may also have n-n attraction with no bound state present, as would be required for the spin-singlet two-nucleon state. In the following we will make use of this freedom in the choice of the n-n interaction.

The three-body nuclei of the model, α and β, are generated by the additional couplings[8] $d+a \longleftrightarrow \alpha$ and $d+b \longleftrightarrow \beta$; and thus elastic da and db scattering are mediated by the spinless quasiparticles α and β. In the four-body sector of the model, the two-to-two reactions of interest are the elastic process $b\alpha \to b\alpha$ as well as the rearrangement collision $b\alpha \to a\beta$. Both of these processes are driven by successive d exchange; an odd number of exchanges are required for $b\alpha \to a\beta$, while $b\alpha \to b\alpha$ involves an even number. The dynamical integral equations that govern these processes are similar to those given previously.[5] It should be noted that not only have we neglected the Coulomb interaction but also that no n-a, n-b, or a-b interactions are present and thus a two-step exchange of the n's is not permitted. In addition to the reactions discussed above, the channels $b\alpha \to dab$ and $b\alpha \to nnab$ are open if the energy is sufficiently high. Although the model is an oversimplification of the nuclear physics of the two-nucleon-exchange reaction, it includes an exact treatment of the effect of the n-n interaction on the process. To our knowledge, this is the only model that includes all orders of interaction between the exchanged particles and whose numerical consequences have been worked out.

Most experience gained in recent years with the few-body problem has involved systems with equal-mass particles or with at most one heavy particle present. The spectrum of bound states and resonances obtained in such systems is rather limited, with only a small number of partial waves being necessary to describe low-energy scattering. In our present model involving two heavy objects that interact by exchanging the lighter d particle, we find a much richer spectrum of bound states and resonances.[9] We also find that angular distributions develop a pronounced diffraction structure not unlike that seen

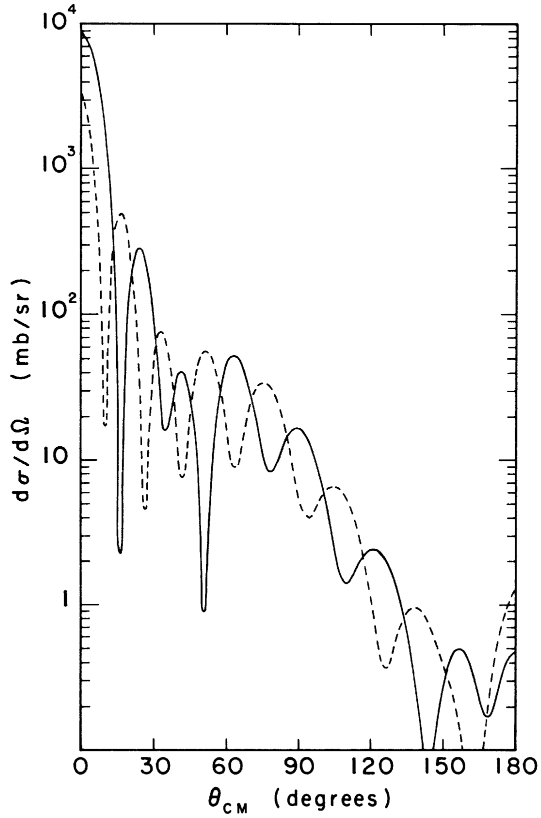

FIG. 1. Angular distributions for $b\alpha \rightarrow b\alpha$ (solid line) and $b\alpha \rightarrow a\beta$ (dashed line) at $E_{c.m.}=30$ MeV. The parameters of the n-n interaction are appropiate to the spin-triplet nucleon-nucleon interaction. Partial waves up to $l=16$ were included in the calculation of the cross sections.

in actual heavy-ion data. In Fig. 1 we show typical angular distributions for $b\alpha \rightarrow b\alpha$ and for $b\alpha \rightarrow a\beta$ at a c.m. energy of 30 MeV. We have chosen $m_a = m_b = 12m_n$ and $\epsilon_\alpha = \epsilon_\beta = 21$ MeV. At this energy all two-, three-, and four-body channels are open, and we note the very oscillatory structure of the cross section as well as the shift in phase between the elastic and rearrangement angular distributions. We are somewhat surprised that so much structure can be obtained with the simple interactions that have been used. The dependence of this structure on the mass ratios of the system and on the amount of three- and four-body inelasticity present in the model is currently being studied.

The major question that we wish to study here is the dependence of the transfer cross section on the two-nucleon interaction between the exchanged particles. It has recently been pointed out that the suppression of nn- and pp-transfer

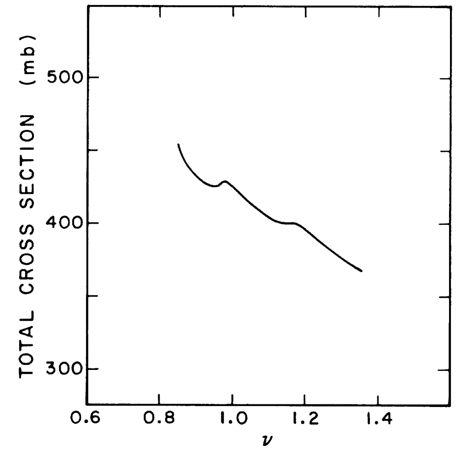

FIG. 2. Total cross section for $b\alpha \rightarrow a\beta$ as a function of ν. $\epsilon_\alpha = \epsilon_\beta = 21$ MeV and $E_{c.m.}=30$ MeV.

reactions relative to np transfer is more likely due to Q effects than to the nature of the two-nucleon interaction. To test this assertion we have performed a model calculation for the reactions $b\alpha \rightarrow b\alpha$ and $b\alpha \rightarrow a\beta$, varying the nature of the n-n interaction as well as the Q of the reaction (with $Q = \epsilon_\beta - \epsilon_\alpha$). In the context of our model, we start with a system that has an n-n interaction corresponding to the physical deuteron and study the two-nucleon-transfer cross section as we weaken the n-n interaction until it corresponds to the spin-singlet virtual state of two nucleons.[10] We find it convenient to define a coupling strength $\nu = (\gamma/\gamma_0)^2$, where γ is the n-n coupling constant and γ_0 corresponds to the strength necessary for a zero-energy bound state. In this manner, $\nu = 1.356$ corresponds to the deuteron, whereas $\nu = 0.944$ is appropriate to the singlet virtual state. We show in Fig. 2 the total cross section for $b\alpha \rightarrow a\beta$ as a function of ν. It is seen that the total cross section[11] increases as the d is continued from the deuteron to the singlet state; and thus in the model, any attempt to explain the nn and pp suppression as due to nucleon-nucleon correlations is ruled out.

We study next the dependence of the total cross sections for $b\alpha \rightarrow b\alpha$ and $b\alpha \rightarrow a\beta$ on the Q of the rearrangement reaction. We vary Q by fixing the binding energy of the α at $\epsilon_\alpha = 21$ MeV and allowing ϵ_β to vary from 3 to 39 MeV. The results of the calculation are shown in Fig. 3, both for elementary and for fully interacting d exchange (that is, we have chosen $Z_d = 1$ corresponding to a structureless elementary d, as well as $Z_d = 0$ corresponding to a composite d with the deuteron parameters). We note that for both reactions the

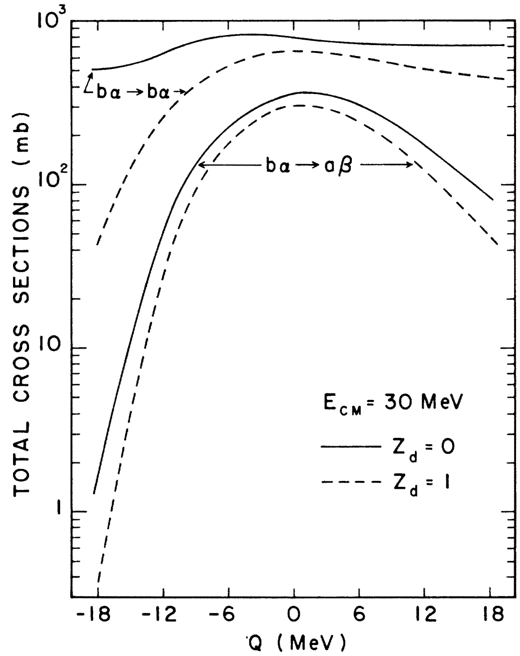

FIG. 3. Total cross sections for $b\alpha \to b\alpha$ for $b\alpha \to a\beta$ as a function of the Q of the rearrangement reaction.

fully interacting d produces a larger total cross section. In the rearrangement collision, the neglect of n-n correlations leads to a cross section that is 20% smaller than the exact result for Q values near zero, but this discrepancy increases to as much as 500% for larger values of Q. These percentage values are somewhat dependent on the energy of the reaction and on the binding energies of the α and β nuclei. If $|Q|$ is large, a strong suppression of the rearrangement collision[12] is clearly evident in Fig. 3 and this is usually ascribed to damping due to mismatching of channels. An alternative point of view is that the magnitude of the two-nucleon-exchange Born term that drives the integral equation for the rearrangement amplitude is governed by the nearness of the exchange pole in the $\cos\theta$ plane. In heavy-ion reactions, the position of this pole is much more sensitive to the Q of the reaction than in light-ion processes.

The results of our model calculations indicate that variations in the n-n interaction or the complete neglect of the higher-order terms of the interaction lead to cross section variations of a factor of 5 or less. It therefore seems quite likely that the inclusion of nucleon-nucleon correlations into the conventional reaction theories may be required in order to bring their predictions closer to experiment, particularly for reactions involving a large Q mismatch. Our model calculations are in accord with conclusions of Nair *et al.*[4] that the *nn* and *pp* suppression is due to Q effects rather than to nucleon-nucleon correlations.

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[1]See, for example, J. F. Petersen, D. A. Lewis, D. Dehnhard, H. P. Morsch, and B. F. Bayman, Phys. Rev. Lett. 36, 307 (1976).

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[5]A. C. Fonseca and P. E. Shanley, Phys. Rev. D 13, 2255 (1976).

[6]R. D. Amado, Phys. Rev. 132, 485 (1963).

[7]For the n-n interaction we have chosen the vertex function $f_d(n) = (n^2 + \beta_d^2)^{-1}$ where the range parameter is always fixed at $\beta_d = 3.04$ no matter what the nature of the n-n interaction is. The units are such that $\hbar^2 = 2m_n = 1$ and the binding energy of the deuteron is $\epsilon_d = 0.5$.

[8]For d-a and d-b interactions we have chosen vertex functions of identical shape so that $f_\alpha(n) = (n^2 + \beta_\alpha^2)^{-3}$ and $\beta_\alpha = \beta_\beta = 4.0$. In all calculations $Z_\alpha = Z_\beta = 0$. It should be noted that the nuclei α and β are smaller than real nuclei since our model does not generate the usual connection between the nuclear radius and mass number.

[9]This effect has been noted in one-dimensional three-body models by L. R. Dodd, Aust. J. Phys. 25, 507 (1972).

[10]A somewhat unrealistic feautre of our model is that as the n-n interaction is varied, the binding energies of the α and β particles remain fixed.

[11]The oscillations in the total cross section are due to changes in the spectrum of bound states and resonances as ν varies.

[12]We see an analogous strong damping of the rearrangement cross section if the n-n interaction is chosen to be appropiate to the singlet state.