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Low-Temperature Limit of the Temperature-Dependent Part of the Resistivity of Potassium

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The electrical resistivity of potassium has been measured with high accuracy between 4.2 and 1.1 K. At temperatures above 2 K the dominant term is of the form $Te^{-\theta/T}$, with $\theta = 19.9 \pm 0.2$ K, which is due to electron-phonon scattering under phonon-drag conditions. At temperatures below 2 K a weak dependence of the form AT^s is dominant, where s is between 1 and 2. Tentatively one could ascribe this term to electron-electron scattering.

The theoretical understanding of transport properties in simple metals is presently in such a state that it is possible to make very detailed predictions of the low-temperature behavior. Unfortunately, because of the lack of high-precision measurements, calculations of the low-temperature electrical resistivity have not been submitted to a detailed experimental test. It is the purpose of this Letter to present such measurements on the simple metal potassium with a hitherto unobtained precision.

Recently it has been shown theoretically that in alkali metals the simple T^5 Bloch relation should be invalid because of the presence of a phonon-drag effect.[1-3] The contribution of the electron-phonon scattering $\rho_{\text{el-ph}}$ should decrease exponentially in the low-temperature phonon-drag limit if only umklapp processes can remove momentum from the electron system. The data of Gugan[4] and Ekin and Maxfield[5] are found to be consistent with an exponential behavior between 2 and 4 K. However, these data do not allow an unambiguous determination of the all-important parameter characterizing the umklapp processes, as a result, in part, of uncertainties in the extrapolation to $T = 0$ K. In addition, as has been realized long ago,[6] there should be another contribution to the electrical resistivity, $\rho_{\text{el-el}}$, due to electron-electron interaction which should be proportional to T^2. However, there is considerable uncertainty about the exact value of the coefficient of proportionality A.[7-9] Lawrence and Wilkins[7] predict that for potassium, $A = 0.17$ pΩ cm K^{-2}. Using this value as a guide one expects that below a temperature somewhere between 1 and 2 K, $\rho_{\text{el-el}}$ should become the dominant term in the temperature-dependent part of the resistivity $\rho(T)$ and should clearly be detectable by high-precision measurements.

In this Letter we present data which not only confirm unequivocally the presence of phonon drag in potassium but also allow the electron-phonon scattering under conditions of strong phonon-drag to be specified much more exactly. We also report the observation for the first time of a low-temperature resistivity effect which could be due

to electron-electron scattering; however, the strong sample dependence is in disagreement with the theoretical predictions.

To observe $\rho(T)$ a high-accuracy measurement of the total resistivity ρ is required to be able to separate $\rho(T)$ from the residual resistivity ρ_0. An experimental difficulty is the low resistance (10^{-5} Ω) of the samples due to the dimensions which are dictated by the experimental conditions. Moreover, even for the purest samples, $\rho(T)$ is only 0.1% of ρ_0 at 2 K and decreases rapidly at lower temperatures. A novel method had to be used to measure ρ with the necessary high relative accuracy (1 ppm).

The resistance of the potassium samples was measured by comparing the unknown resistance R_x with a known standard resistance R_s by means of a bridge circuit. The ratio of the two currents fed through the two resistances was controlled by a current comparator of the flux-gate type.[10] The unknown resistance, standard resistance, and bridge zero detector (a superconducting galvanometer also based on the flux-gate principle[11]) were all immersed in a liquid helium bath and were interconnected by superconducting wires. The voltage sensitivity of the galvanometer, with R_x and R_s of the order of 10^{-5} Ω, was 10^{-12} V. This enabled us to measure the ratio R_x/R_s with a resolution of 0.1 ppm and a long-term (24 h) reproducibility of 1 ppm. The details of the measuring circuit will be published elsewhere.[12]

The standard resistance R_s consisted of commercially available copper-beryllium. The slight temperature dependence of R_s was determined in a separate experiment with a relative accuracy of 2 ppm.

The geometrical factor for the samples was determined from a measurement at 77.6 K assuming a specific resistivity of 1.38 $\mu\Omega$ cm. Because of the limited sensitivity of the galvanometer at liquid nitrogen temperature, the accuracy of the determination was 1%. Thermal contraction between 77.6 K and liquid helium temperatures was neglected; this could produce a systematic error of at most 0.5%.[13]

In the presently described experiment, inaccuracies in the temperature determination were the most important sources of error. The temperature determinations were based on the He^4-58 temperature scale and had an accuracy of 1 mK below T_λ and 5 mK above T_λ. A measure of the overall accuracy is shown in Fig. 1. The temperature ranges over which the samples were measured are given in Table I.

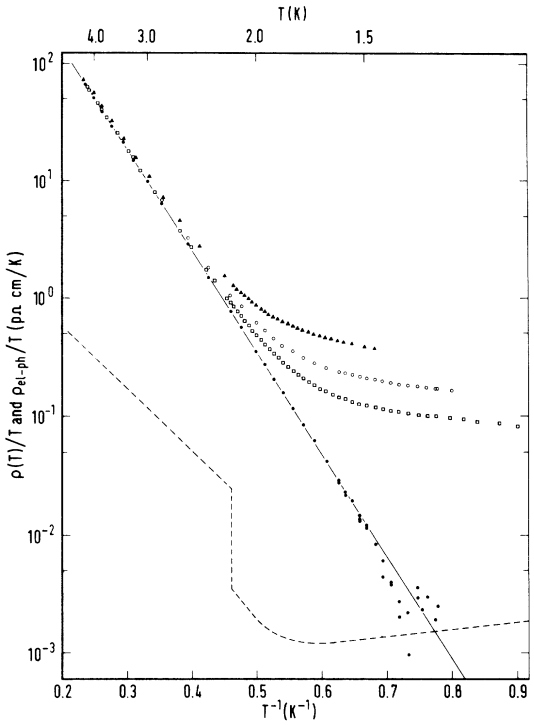

FIG. 1. $\rho(T)/T$ and ρ_{el-ph}/T versus $1/T$. The difference between $\rho(T)$ and ρ_{el-ph} is the AT^2 term. For clarity only ρ_{el-ph}/T of sample 2b is given; the other samples practically coincide with sample 2b. Closed triangles, $\rho(T)/T$ of sample 1; open squares, $\rho(T)/T$ of sample 2c; open circles, $\rho(T)/T$ of sample 2b; closed circles, ρ_{el-ph}/T of sample 2b. The solid line gives $\rho_{el-ph}/T = 7.34\, e^{-19.9/T}$. The dashed line indicates the estimated measuring error in $\rho(T)/T$, taking also into account errors in $R_s(T)$ and T, but not the 1% error in the form factor.

The samples were prepared by forcing molten potassium[14] (purity 99.97%) under helium pressure into a 1-m-long polyethylene tube with an inner diameter of 0.9 mm which was equipped with copper contacts. We report on measurements on two samples. Sample 1 had a residual resistance ratio (RRR) of 3100. (Here the RRR is defined as the ratio of the 300-K resistance to the 0-K resistance.) Sample 2a was cooled down immediately after preparation and had a RRR of 3000. After annealing this sample for two days at room temperature in a helium atmosphere, the sample (now called 2b) had a RRR of 6300. After storing this sample for 80 days at room temperature in vacuum the RRR became 8100 (sample 2c). Sample 1 was measured with an accuracy and resolution a factor of 10 worse than sample 2. Preliminary results on sample 1 were reported by van Kempen.[15]

TABLE I. Summary of the results of the resistivity measurements of potassium. The measured resistivity is described by $\rho = \rho_0 + \rho(T)$ with $\rho(T) = BTe^{-\theta/T} + AT^2$. The error in ρ (4.0 K) includes the uncertainty in the sample form factor.

Sample, [RRR]	Temperature range (K)	θ (K)	B (10^3 pΩ cm K^{-1})	A (pΩ cm K^{-2})	ρ_0 (pΩ cm)	ρ (4.0 K) (pΩ cm)
1 [3100]	1.47–4.2	20.0 ± 0.2	8.05	0.27 ± 0.02	2335	222 ± 1.5%
2a [3000]	1.22–4.2	19.8 ± 0.2	7.81	0.16 ± 0.02	2394	223 ± 1.5%
2b [6300]	1.25–4.2	19.9 ± 0.1	7.34	0.135 ± 0.01	1134	206 ± 1.5%
2c [8100]	1.10–4.2	20.0 ± 0.1	7.63	0.075 ± 0.01	878	205 ± 1.5%

The measurements were analyzed under the assumption that the temperature-dependent part of the resistivity $\rho(T)$ can be written as

$$\rho(T) = \rho_{el\text{-}el} + \rho_{el\text{-}ph} = AT^2 + BT^n e^{-\theta/T}. \quad (1)$$

A meaningful analysis was possible because the measurements extend to temperatures at which one or the other term dominates (see Fig. 1). A least-mean-square fit of the expression (1) to the experimental points for each sample was achieved within experimental accuracy. For simplicity we present the results of the analysis for $n = 1$ only, although satisfactory results were obtained for n between 0.7 and 1.1. The results are summarized in Table I, together with the value of the residual resistance ρ_0 which was obtained by extrapolation by means of formula (1).

The basic nature of the parameter θ is demonstrated in Table I, where it is seen to be sample independent with a value of $\theta = 19.9 \pm 0.2$ K. The temperature dependence of $\rho_{el\text{-}ph}$ implies[16] an average energy of phonons weighted according to their contribution to $\rho_{el\text{-}ph}$ equal to $k\theta + 2kT$. It has been suggested[16] that predominantly those phonons that can participate in two separate nonequivalent umklapp processes contribute to $\rho_{el\text{-}ph}$, at temperatures for which kT is much smaller than the threshold energy of such processes (approximately 19 K in potassium). The differences between the values of B for the various samples are reduced when precisely the same value of θ is used for all samples. The remaining differences reflect, as does the value of $\rho(T)$ at 4.0 K, the small deviations from Matthiessen's rule similar to observations by other authors.[4,5]

As the results for samples 2a–2c show, A depends strongly on the history of the sample, that is, on the degree of annealing. Similarly, Gugan[4] observed that deviations from Matthiessen's rule due to dislocations were most pronounced at the lowest temperatures. The order of magnitude of $\rho_{el\text{-}el}$ corresponds fairly well with the predicted[7] value of $A = 0.17$ pΩ cm K^{-2}. However, no strong sample dependence of the electron-electron scattering has been predicted. The strong annealing dependence observed therefore poses serious questions. On the one hand, if this weakly temperature-dependent term is indeed due to electron-electron scattering, better theoretical understanding of its magnitude will be necessary, especially in regard to the dependence on crystal defects. On the other hand, if this term is not due to electron-electron scattering, then the present theoretical predictions for A are at least an order of magnitude too high. In such a case the origin of the observed term must still be found. Surely, the strongly coupled electron and phonon distribution at temperatures at which practically all umklapp processes are frozen out represent a system for which novel resistance mechanisms should be considered.

It should be pointed out that instead of a term AT^2 one can describe the data equally well with a term proportional to T^s with s between 1 and 2. A term T^5 can be ruled out.

In summary, the resistivity of potassium has been measured with a resolution much better than hitherto obtained. The data expressed by the empirical formula (1) with an activation energy $\theta = 19.9$ K support the picture of an electron-phonon scattering under strong phonon-drag conditions, for which the resistivity is controlled by specific umklapp processes. In addition, a weakly temperature-dependent term roughly proportional to T^2 has unambiguously been found and might represent the first observation of electron-electron scattering in the resistivity of the alkali metals; however, the constant of proportionality A is very much sample dependent, contrary to the theoretical expectations.[7]

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Phase Transitions in Long-Range Ferromagnetic Chains

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The critical behavior of one-dimensional ferromagnetic chains with isotropic long-range interactions between n-component spins is investigated near the critical range of r^{-2}. Different formulations of the renormalization group are used for $n>1$ and $n=1$. It is shown that the behavior is quite different in the two cases, with very close analogies to the short-range problem near two dimensions. Critical exponents are calculated to first order.

There has been renewed interest in critical phenomena in low-dimensional systems since the work by Migdal[1] and Polyakov,[2] who extended the ideas of Berezinskii and Blank[3] to expansions about two dimensions for isotropic n-component spin systems with short-ranged interactions. The special case $n=d=2$ has been investigated by Thouless and the author.[4,5] The critical temperature of the latter system is finite while for $n>2$ it is $O(\epsilon)$ in $2+\epsilon$ dimensions,[1,2,6] and the critical behavior is quite different in the two cases. This is in accord with the idea that the topology of the order parameter is important in such systems.[4,7] For a vector order parameter with $n=d$, there are stable-point singularities in the spin configurations[7] which may contribute significantly to the partition function, depending on the interaction range. An explicit realization of this has been performed for $n=d=2$[4,5] and $n=d=1$.[8,9]

In this Letter I investigate systems with long-ranged interactions in one dimension for various values of n. Dyson[10] has shown that the Ising model with interactions falling off as $r^{-1-\sigma}$ with $0<\sigma<1$ has a transition to an ordered state, while for $\sigma>1$ and $\sigma<0$ there is no transition. The limiting case $\sigma=1$ has an unusual transition at finite temperature in which the spontaneous magnetization has a discontinuity, but the free energy has an essential singularity such that all thermal properties are smooth at T_c.[8,9] The nature of this transition is very similar to the $n=d=2$ case[5] except that there the magnetization is zero for all temperatures.[11] This implies that $\sigma=1$ is a critical range of interaction (analogous to the lower critical dimension $d=2$ for short-ranged interactions). Here I treat in more detail the situation near $\sigma=1$ by renormalization-group techniques. For $n>1$ a modification of Polyakov's[2] method is used and of Anderson's[8,9] for $n=1$. The necessity of using different methods for the two cases arises because Polyakov's technique can be used only when long-wavelength fluctuations drive the transition but local defects drive the Ising transition.