Coulomb Corrections in Proton-Deuteron Scattering

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(Received 16 September 1976)

We present the first calculations of differential cross sections for elastic proton-deuteron scattering using a three-body formalism which *correctly* takes into account the Coulomb repulsion between the two protons.

The neutron-deuteron problem represents the most thoroughly investigated example in three-body scattering theory. Proton-deuteron collisions, on the other hand, have not yet received the attention that they deserve in view of the many precise experimental data available. The reason for this has been the lack of any *practical* theory which correctly deals with the difficulties arising from the long-range nature of the Coulomb repulsion between the two protons.[1]

Up to now, essentially two proposals exist for modifying three-body integral equations so as to allow for the incorporation of Coulomb forces in addition to short-range interactions. One is based on including the Coulomb potentials into the unperturbed part of the Hamiltonian, thereby formally separating the short-range and the long-range portions of the interaction.[2,3] In order to make this scheme manageable, approximations have been made by Adya[4] the reliability of which is difficult to assess. In the other attempt,[5] the Coulomb potentials are screened, and that part of the three-body kernel which becomes most singular in the limit of zero screening is inverted explicitly so that the resulting modified equations are well-behaved. However, they are useful only for energies below the break-up threshold.

Recently[6] we have proposed another approach for investigating elastic proton-deuteron scattering which neither has recourse to uncontrollable approximations nor has a restricted range of application. It is the purpose of this Letter to substantiate both assertions by discussing first numerical results for scattering cross sections.

Our method is based on the quasiparticle approach of Alt, Grassberger, and Sandhas.[7] There, by splitting the two-body transition operators T_γ occurring in the three-body kernel into a separable part $T_\gamma{}^s$ and a remainder $T_\gamma{}'$, equations could be derived for effective two-body amplitudes $\mathsf{T}_{\beta n,\,\alpha m}$ which directly describe the scattering of an elementary particle off a two-body bound state. The effective potentials $\mathsf{V}_{\beta n,\,\alpha m}$ occurring in these equations depend only on the remainder $T_\gamma{}'$, and can, under suitable circumstances, be evaluated perturbatively.

Application of this idea to the present problem is made particularly simple and transparent if the strong forces between the nucleons are chosen to be purely separable, so that $V_\gamma = \sum_r |\chi_{\gamma r}\rangle \times \lambda_{\gamma r}\langle\chi_{\gamma r}|$. (This restriction is not essential but only a matter of computational convenience.) Then the two-body transition operators are also separable except the one of the pp subsystem which, according to the two-potential formula, contains the pure Coulomb amplitude T_c as nonseparable part [i.e., $T_\gamma{}' = 0$ for $\gamma \neq (p,p)$ and $T_{pp}{}' = T_c$]. Under these circumstances $\mathsf{V}_{\beta n,\,\alpha m}$ reduces to the simple and *exact* form (see Ref. 7, Sec. 5)

$$\mathsf{V}_{\beta n,\,\alpha m} = \bar{\delta}_{\beta\alpha}\{\langle\chi_{\beta n}|G_0|\chi_{\alpha m}\rangle + \langle\chi_{\beta n}|G_0(T_\beta{}' + T_\alpha{}')G_0|\chi_{\alpha m}\rangle\} + \sum_\gamma \bar{\delta}_{\beta\gamma}\bar{\delta}_{\gamma\alpha}\langle\chi_{\beta n}|G_0 T_\gamma{}' G_0|\chi_{\alpha m}\rangle . \tag{1}$$

As usual, $\bar{\delta}_{\beta\alpha} = (1 - \delta_{\beta\alpha})$ and $G_0(z) = (z - H_0)^{-1}$. The first term in Eq. (1) describes the scattering by the

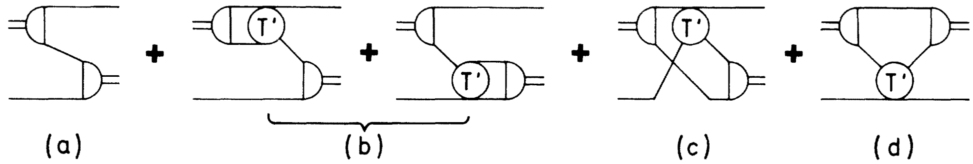

FIG. 1. Graphical representation of the effective potential for *pd* scattering, Eq. (1).

strong potentials alone, relevant for *nd* reactions. The other terms represent the *full* contribution of the Coulomb repulsion between the two protons. Equation (1) is displayed graphically in Fig. 1. After symmetrization, the Lippmann-Schwinger–type equations for the effective two-body transition operators assume the form

$$\mathrm{T}_{nm} = \mathrm{V}_{nm} + \sum_r \mathrm{V}_{nr}\,\mathrm{G}_{0,r}\,\mathrm{T}_{rm}\,. \tag{2}$$

Here V_{nm} is the symmetrized version of expression (1), and $\mathrm{G}_{0,r}$ is the effective free Green's function. The indices n, m, and r characterize the various possible two-particle channels [d for np ($^3S_1$), s for np ($^1S_0$), and $\bar{s}$ for pp ($^1S_0$)]. However, Eq. (2) is not yet suited for the investigation of pd scattering since its kernel is not of the Fredholm type. Indeed, no difficulties arise from diagrams (a)–(c) of Fig. 1, whereas diagram d contains a part the singularity structure of which is the same as in the genuine two-particle Coulomb case. Realization of this fact enables us to apply the methods developed for the latter problem[8] to our effective two-body equation (2).

Let us briefly outline this approach. We start by assuming the Coulomb potential to be exponentially screened, so that $V_c(r) \to V_{(\mu)}(r) = e^2 \exp(-\mu r)/r$. Consequently the corresponding amplitude T_γ', and hence the effective two-body quantities T, V, and G_0, also depend on μ, and will therefore be labeled by μ. The difficulties with Eq. (2) mentioned previously can then be paraphrased by saying that its solution $\mathrm{T}^{(\mu)}$ has no limit as $\mu \to 0$. According to the above discussion the kernel of Eq. (2) is decomposed into a "dangerous" part originating from graph d, and another part $\tilde{\mathrm{V}}^{(\mu)}\mathrm{G}_0^{(\mu)}$ which poses no problems, in the limit of zero screening,[6]

$$(\mathrm{V}^{(\mu)}\mathrm{G}_0^{(\mu)})_{nm}(\vec{q}\,',\vec{q}) = \delta_{nm}\delta_{nd}K_{(\mu)}(\vec{q}\,',\vec{q}) + (\tilde{\mathrm{V}}^{(\mu)}\mathrm{G}_0^{(\mu)})_{nm}(\vec{q}\,',\vec{q}), \tag{3}$$

with

$$K_{(\mu)}(\vec{q}\,',\vec{q}) = (e^2/2\pi^2)\{[(\vec{q}\,'-\vec{q})^2+\mu^2][(3/4M)(q_e^{\,2}+i\epsilon-q^2)]\}^{-1}. \tag{4}$$

Here, M denotes the nucleon mass, and q_e the on-shell momentum in the pd center-of-mass system. After introducing the splitting (3) in Eq. (2), the "dangerous" portion of $\mathrm{V}\mathrm{G}_0\mathrm{T}$ is shifted to the left and the resulting factor multiplying T is explicitly inverted; thereby, we end up with an equation of familiar structure for $\mathrm{T}^{(\mu)}$,

$$\mathrm{T}_{nm}^{(\mu)}(\vec{q}\,',\vec{q}) = \delta_{nm}\delta_{nd}\,T_{(\mu)}(\vec{q}\,',\vec{q}) + \mathrm{T}_{sc,\,nm}^{(\mu)}(\vec{q}\,',\vec{q}). \tag{5}$$

The quantity $T_{(\mu)}$ is defined by

$$T_{(\mu)} = V_{(\mu)} + K_{(\mu)}T_{(\mu)} = V_{(\mu)} + T_{(\mu)}K_{(\mu)} \tag{6}$$

which is a *genuine two-body* equation describing the scattering of a particle of mass M off another particle of mass $2M$ via a screened Coulomb potential $V_{(\mu)}$. The Coulomb-modified strong amplitude $\mathrm{T}_{sc}^{(\mu)}$ fulfills the equation

$$\mathrm{T}_{sc,\,nm}^{(\mu)} = \Omega_n^{(\mu)T}\tilde{\mathrm{V}}_{nm}^{(\mu)}\Omega_m^{(\mu)} + \sum_r \Omega_n^{(\mu)T}\tilde{\mathrm{V}}_{nr}^{(\mu)}\mathrm{G}_{0,r}^{(\mu)}\mathrm{T}_{sc,\,rm}^{(\mu)}. \tag{7}$$

Here we have introduced a quantity $\Omega_m^{(\mu)}$ with components $\Omega_d^{(\mu)} = \Omega^{(\mu)}$, and $\Omega_m^{(\mu)} = 1$ for $m \neq d$. The operator $\Omega^{(\mu)}(q_e^{\,2}+i\epsilon)$, and its transpose $\Omega^{(\mu)T}(q_e^{\,2}+i\epsilon) = \Omega^{(\mu)\dagger}(q_e^{\,2}-i\epsilon)$ are the two-body Møller operators for the screened potential $V_{(\mu)}$. In fact, $\Omega^{(\mu)}(q_e^{\,2}+i\epsilon)$ maps the plane wave state $|\vec{q}\rangle$ into the scattering state $|\vec{q}_{(\mu)}^{\,(+)}\rangle = \Omega^{(\mu)}|\vec{q}\rangle$, for $|\vec{q}| = q_e$.

Thus Eq. (5) represents a decomposition of the full amplitude $\mathrm{T}^{(\mu)}$ into a *genuine two-particle* Coulomb amplitude $T_{(\mu)}$, and an effective two-body amplitude $\mathrm{T}_{sc}^{(\mu)}$ in which the long-range distortion shows up through the *two-particle* Møller operators $\Omega^{(\mu)}$. This allows us to take over directly the result of Ref. 8, which implies that for on-shell values of the momenta ($|\vec{q}'|=|\vec{q}|=q_e$), the renormalized amplitudes $Z_\mu^{-1}T_{(\mu)}(\vec{q}',\vec{q})$ and $Z_\mu^{-1}\mathrm{T}_{sc,dd}^{(\mu)}(\vec{q}',\vec{q})$, and consequently also $Z_\mu^{-1}\mathrm{T}_{dd}^{(\mu)}(\vec{q}',\vec{q})$, remain finite when μ goes to zero. After performing this limit the last expression, therefore, represents the correct amplitude describing pd scattering with unscreened Coulomb potentials. Note that Z_μ is the well-known renormalization factor.

$$Z_\mu = \exp\{-2i\eta[\ln(2q_e/\mu) - C]\},$$

with $\eta = 1/(Rq_e)$, R being the Bohr radius of the pd system, and $C = 0.5772\ldots$, the Euler number.

In order to demonstrate that this approach is also practical, we present the first results of a numerical investigation of elastic pd scattering. To simplify the numerical work we have approximated in $\tilde{\mathrm{V}}^{(\mu)}$ and in $\mathrm{G}_0^{(\mu)}$ the screened two-body Coulomb amplitudes $T_{(\mu)\gamma}'$ by their Born terms $V_{(\mu)\gamma}$. This should be reasonable for the case of like-charged particles.

Allowing for charge dependence, our separable potentials have been chosen to be of the Yamaguchi type, with parameters fitted to the low-energy data for ${}^1S_0$ proton-proton scattering (together with V_c), and for neutron-proton scattering in the ${}^1S_0$ and the ${}^3S_1$ channels. Under the assumption of charge symmetry of the nuclear forces and by switching off the Coulomb interaction, these potentials provide at the same time a charge-dependent description of neutron-deuteron scattering.

For the numerical calculations we use the screened version of Eq. (2) for $\mathrm{T}^{(\mu)}$. After partial-wave decomposition we obtain from it the total (i.e., screened Coulomb plus Coulomb-modified nuclear) pd phase shifts $\delta_l^{(\mu)}$. Simultaneously we calculate from Eq. (6) for $T_{(\mu)}$ the screened Coulomb phase shifts $\sigma_l^{(\mu)}$. Both $\delta_l^{(\mu)}$ and $\sigma_l^{(\mu)}$ depend on the cut-off parameter μ and diverge when $\mu \to 0$. However, for the difference between the two which is just the desired Coulomb-modified nuclear phase shift, the zero-screening limit can be proved to exist by means of arguments analogous to those of Ref. 8. Our calculated values showed that within our numerical accuracy this limit is reached already for $\mu^{-1} \gtrsim 30$ fm at the energies considered up to now. Thus, the construction of a reliable Coulomb-modified strong amplitude for *unscreened* Coulomb potentials is made possible. Adding to it the analytically known pure Coulomb contribution yields the final pd scattering amplitude. For further details we refer to Ref. 6.

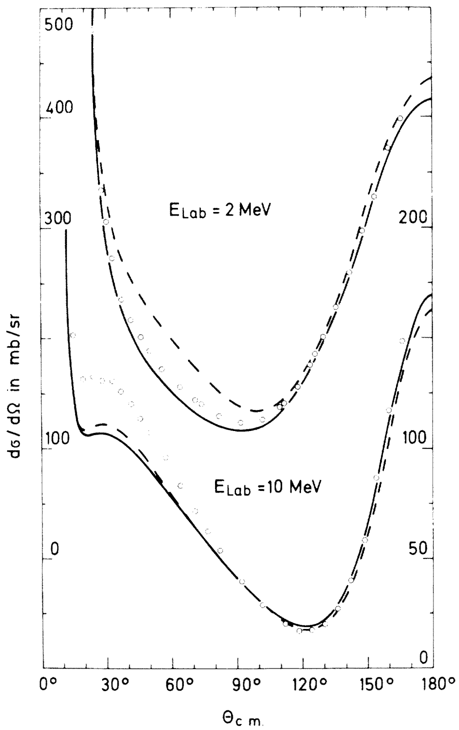

FIG. 2. Differential cross section for pd scattering at 2-MeV (left-hand scale) and 10-MeV (right-hand scale) incident proton energy. Solid line: full theory; dashed line: neglect of Coulomb modifications of the strong amplitude; circles: experimental results of Ref. 9.

In Fig. 2 we compare calculated pd cross sections at 2- and 10-MeV laboratory energy of the incident proton with experimental data from Kocher and Clegg.[9] The agreement is generally good, except in the Coulomb interference region and in the backward direction, where the theoretical curves are lower than the experimental ones, with the discrepancy increasing with increasing energy. This is not surprising since we already know from elastic nd scattering that the cross sections become too low in the forward and backward directions if the nucleon-nucleon potentials employed are too attractive. But there, by using less attractive (i.e., more realistic) potentials, good agreement with experiment can be achieved (see e.g., Alt and Bakker[10] and Ziegelmann[10]). The same remedy is also expected to work in the case of pd scattering.

In the framework of integral equations, the only theoretical method to describe pd cross sections has been, up to now, calculation of the corresponding *nd* amplitudes and coherent addition of the pure Coulomb amplitude.[11] This amounts to neglecting the Coulomb modifications of the strong *nd* amplitudes which, in the present formalism, means neglect of all shorter-range Coulomb effects in $\tilde{V}'^{(\mu)}$ and $G_0^{(\mu)}$. However, we are now in the position to check the reliability of such a procedure. For this purpose we have included in Fig. 2 the cross sections obtained in such a way. It appears that at low energies this approximate treatment is rather unsatisfactory, whereas it may become more reliable at higher energies.

In conclusion we emphasize once more that our approach to the Coulomb corrections in *pd* scattering is not only mathematically correct but also well suited for practical applications with no need for drastic approximations. (This is in contrast to the use of the formalism of Ref. 2 made in Ref. 4. Indeed, the results obtained there bear only a faint resemblance to ours or to the experimental data.) Apart from employing separable nuclear potentials, *only one* approximation has been made in the present calculations. Namely, the effective potentials and Green's functions which determine, via Eq. (7), the Coulomb-modified strong amplitude $T_{sc}^{(\mu)}$, are evaluated to lowest order in e^2 only. However, the neglect of higher-order terms can and will be checked. Furthermore, the difficult question which is bound to plague most other approaches of how many partial waves in the *pp* subsystem should be taken into account never does arise in our method where, characteristically, the full three-dimensional Coulomb potential is built in.

One of us (H. Ziegelmann) acknowledges an interesting conversation concerning this subject with Professor E. W. Schmid.

*Supported in part by Fonds zur Förderung der Wissenschaftlichen Forschung in Österreich, Project 2493.

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Generation of Coherent Radiation at 53.2 nm by Fifth-Harmonic Conversion*

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(Received 14 September 1976)

The generation of coherent radiation of 53.2 nm by fifth-harmonic conversion of laser pulses at 266.1 nm in both Ne and He is reported.

Frequency upconversion using third-harmonic generation and four-wave mixing has received attention in recent years for the generation of coherent radiation in the vacuum ultraviolet (VUV) region of the spectrum.[1-3] Such processes have been used to produce coherent radiation at wavelengths as short as 57.0 nm.[4] The generation of coherent light in the extreme ultraviolet region by third-order processes becomes increasingly difficult because of the scarcity of intense coherent sources at the required pumping wavelengths. The development of frequency conversion techniques utilizing higher-order nonlinearities offers an attractive alternative to this approach, since it would allow larger steps along the frequency scale to be made in a single conversion process.

Several of these processes have been suggested in the literature.[5,6] Although reasonable conversion efficiencies have been predicted for some of these interactions, the only published experimental evidence of such processes has been the fifth-