Charged-Pion Electroproduction from Protons up to $Q^2 = 9.5\ \mathrm{GeV}^2$*

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We report new measurements of inclusive pion electroproduction. Data were taken with a hydrogen target at the (W,Q^2) points (2.2 GeV, 1.2 GeV^2), (2.7, 2.0), (2.7, 3.3), (2.7, 6.2), (2.7, 9.5), and (3.1, 4.5). The invariant structure function is studied as a function of W, Q^2, ω, and x'.

Two of the predictions of the parton-quark model of nucleon structure are that the inclusive cross section for particle production will display Bjorken scaling[1] and that for production along the direction of the virtual photon the π^+/π^- ratio will increase as $1/\omega$ increases.[1,2] The present evidence for these two predictions is partially obscured by the fact that one must compare measurements made at different virtual-photon–target-nucleon total center-of-mass energies, W; and there is, as a result, some confusion between the W dependence and the ω dependence.[3,4] This Letter reports new data which cover a wide range in Q^2 at fixed W.

Measurements of the inclusive pion electroproduction reaction

$$e + p \to e + \pi^{\pm} + \text{anything} \tag{1}$$

were carried out at the Wilson Synchrotron Laboratory at Cornell University. Reaction (1) is analyzed in terms of the virtual photoproduction reaction

$$\gamma_v + p \to \pi^{\pm} + \text{anything}, \tag{2}$$

where the square of the virtual-photon mass $-Q^2$, energy ν, direction, and polarization parameter ϵ are tagged by the scattered electron. The cross section for Reaction (1) is written[5]

$$\frac{d^5\sigma}{d\Omega_e dE' dp^3} = \Gamma \frac{d^3\sigma}{dp^3}, \tag{3}$$

where Γ is the "flux" of virtual photons and $d^3\sigma/dp^3$ is the cross section for Reaction (2). The virtual photoproduction cross section is a function of W, Q^2, ϵ, p_T, the component of the pion momentum normal to the direction of the virtual photon, $x' = p_{\parallel}^*/(p_{\max}^{*2} - p_T^2)^{1/2}$, the component of the momentum of the pion parallel to the direction of the virtual photon normalized to the maximum longitudinal momentum consistent with the value of p_T, and φ, the angle between the electron–virtual-photon plane and the virtual-photon–pion plane. The asterisk denotes the virtual-photon–target-nucleon center of mass and $p_{\max}^*$ is the maximum pion momentum kinematically allowed with use of the $\pi^+ n$ final state for both the π^+ and π^- data. The virtual photoproduction cross section is a function of φ of the form

$$\frac{d^3\sigma}{dp^3} = A + \epsilon C + \epsilon B \cos 2\varphi + \left[\frac{\epsilon(\epsilon+1)}{2}\right] D \cos\varphi. \tag{4}$$

The results presented here have been averaged over φ so as to eliminate the B and D terms. Other experiments have shown that the B and D terms are small in the kinematic range of the data reported here.[6]

The data are presented in terms of the invariant structure function $F(x', Q^2, W)$ obtained by averaging

$$\frac{E}{\sigma_T}\frac{d^3\sigma}{dp^3} = \frac{E^*}{\sigma_T}\frac{2}{(p_{\max}^{*2} - p_T^2)^{1/2}}\frac{d^3\sigma}{dx' dp_T^2 d\varphi} \tag{5}$$

over the range $p_T^2 < 0.02\ \mathrm{GeV}^2$ and $0 < \varphi < 2\pi$. Here σ_T is the total virtual-photon–proton cross section. The quark-parton model predicts that F will be a function of x' and ω. The data reported here extend measurements reported earlier of the invariant structure function to higher Q^2 at $W = 2.7$ GeV.[3,6,7]

The two-arm spectrometer system described previously[8] was used to obtain data at the (W,Q^2) points (2.2 GeV, 1.2 GeV^2), (2.7, 2.0), (2.7, 3.3), (2.7, 6.2), (2.7, 9.5), and (3.1, 4.5). A lead-Lucite shower counter was used to identify the scattered electrons. Pions were identified by their time of flight for momenta less than 1.8 GeV and by a threshold Freon Cherenkov counter for momenta greater than 1.8 GeV. The data have been corrected for random coincidences (~2%), electronics deadtime (~5%), absorption in the counters (~5%), electron contamination of π^- data (~6%), and target wall events (~2%). The correction for pion background on the electron arm ranged from 5 to 30% with an estimated error equal to 10% of

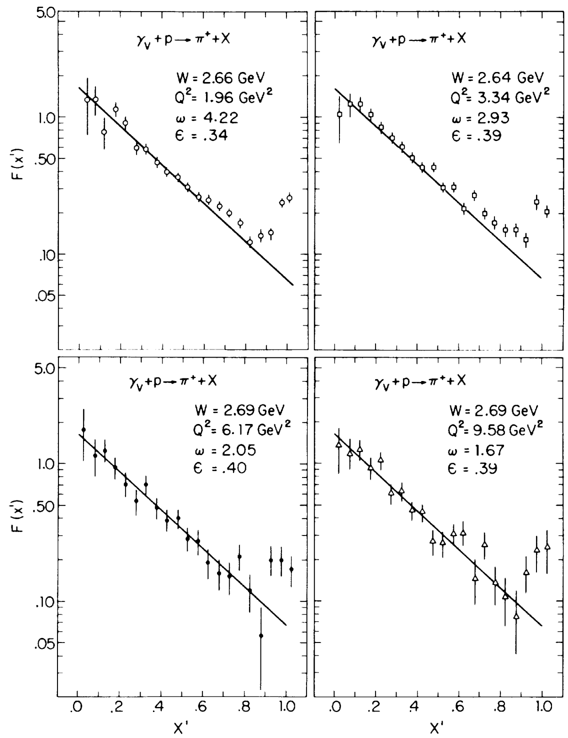

FIG. 1. The π^+ structure functions for several values of Q^2 at $W \simeq 2.7$ GeV. For these data, $p_T^2 < 0.02$ GeV^2. The solid line is the same in all four figures and it is given by the expression $1.65\exp(-3.25x')$.

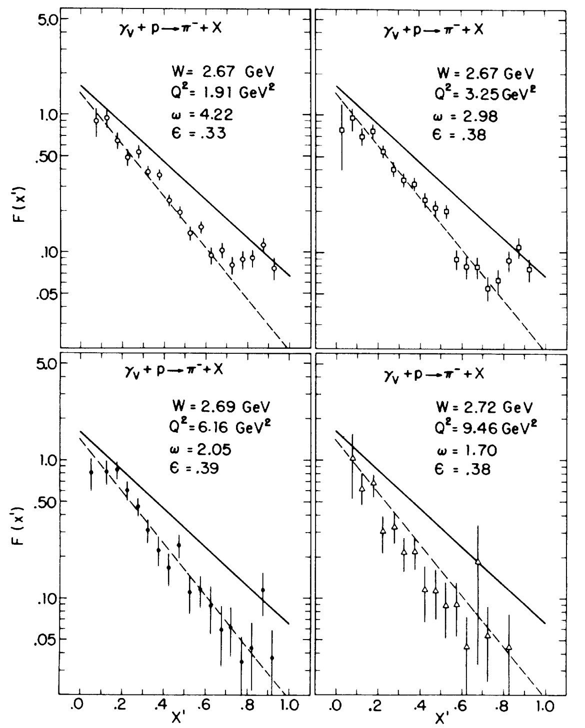

FIG. 2. The π^- structure functions for several values of Q^2 at $W \simeq 2.7$ GeV. For these data, $p_T^2 < 0.02$ GeV^2. The solid and dashed lines are the same in all four figures. The solid line is the same as in Fig. 1 and is given by the expression $1.65\exp(-3.25x')$. The dashed line is a fit to all the π^- data depicted in this figure and is given by the expression $1.43\exp(-4.35x')$.

the correction. Pion decay was included in the Monte Carlo determination of the acceptance. The uncertainties shown in the figures are statistical only and do not include the overall systematic error which is estimated to be less than ± 8%. No radiative corrections have been applied to the data. Calculations indicate that the radiative correction is less than 20% in the range $0 < x' < 0.7$ and it varies by less than 10% over all five data points.

Figures 1, 2, and 3 show the π^+ and π^- structure functions as a function of x' for $p_T^2 < 0.02$ GeV^2. The solid line in the figures is

$$F(x') = 1.65\exp(-3.25x'). \tag{6}$$

The coefficient for x' is taken from the muon-production results of Anderson *et al.*[9] It has been found that this x' dependence also describes the earlier reported data of the Harvard group[3,6,7] and the data of other groups.[10,11] For fixed W, the π^+ data are strikingly constant over the entire Q^2 range. The π^- data, on the other hand, show an increasingly steep x' dependence as Q^2 increases. The dashed line in Figs. 2 and 3 is given by

$$F(x') = 1.43\exp(-4.35x'), \tag{7}$$

which was determined by a fit to all the π^- data for $0 < x' < 0.7$ with $W \simeq 2.7$ GeV.

In order to study the behavior more closely in the region $0 < x' < 0.7$, the structure functions have

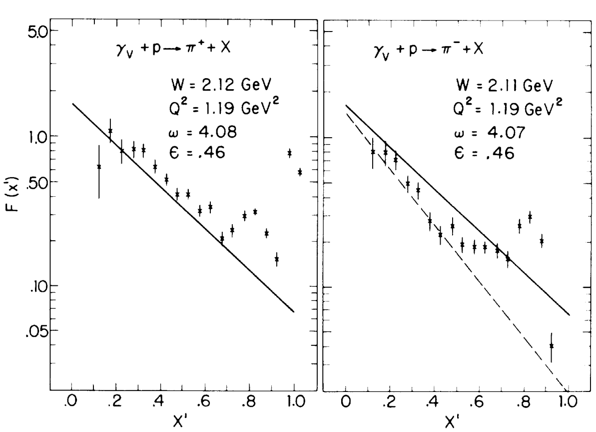

FIG. 3. The π^+ and π^- invariant structure functions for $W \simeq 2.2$ GeV, $Q^2 = 1.2$ GeV^2, and $p_T^2 < 0.02$ GeV^2. The solid and dashed lines are the same as those shown in Figs. 1 and 2.

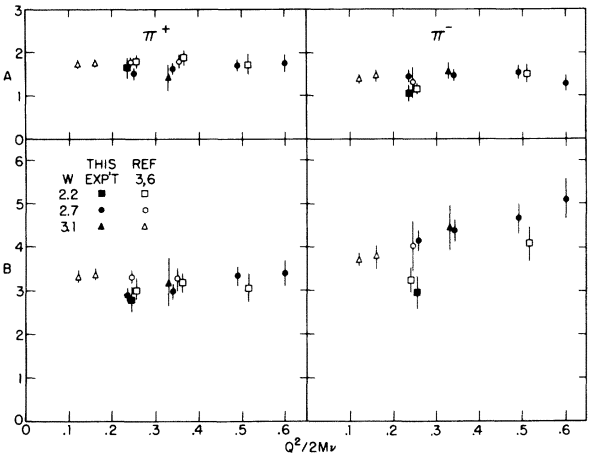

FIG. 4. A plot versus ω of the fits of the π^+ and π^- invariant structure functions to the form $A\exp(-Bx')$.

been fitted with the form

$$F(x') = A\exp(-Bx'). \tag{8}$$

We have restricted ourselves to $x' < 0.7$ to avoid the exclusive final states $\pi^+ n$, $\pi^+\Delta^0$, and $\pi^-\Delta^+$. Figure 4 summarizes the results of this fit as a function of ω for the five (W,Q^2) points reported here and for the earlier Harvard data.[3,6,7] For the π^+ data the A and B coefficients are to a remarkable degree independent of Q^2 and ω. Thus the π^+ invariant structure function displays trivial Bjorken scaling and the x' dependence is not a function of Q^2. This indicates that the π^+ invariant cross section scales in ω in the same manner as the total virtual photoproduction cross section. On the other hand, the π^- structure function changes with ω and is consistent with becoming a universal nontrivial function of ω as W increases. The A and B parameters for the 2.7-GeV and 3.1-GeV data lie on the same curve indicating no W dependence for $W \gtrsim 2.6$ GeV. Thus the π^- structure function appears to display Bjorken scaling at high W ($W > 2.6$ GeV). It should be noted that the (W,Q^2) points (2.2 GeV, 1.2 GeV^2) and (2.7, 2.0) have the same value of ω but differ from each other. This gives further evidence for the residual W dependence.

For $W > 2.6$ GeV, we have fitted the parameter A of Fig. 4 to a constant for the π^+ and π^- data with the results

$$A(\pi^+) = 1.70 \pm 0.04 \quad (\chi^2/\mathrm{DOF} = 5.5/8), \tag{9}$$

$$A(\pi^-) = 1.42 \pm 0.03 \quad (\chi^2/\mathrm{DOF} = 2.0/7) \tag{10}$$

(DOF is degrees of freedom). We have also fitted the parameter B to a constant and to a constant plus a $1/\omega$ term. The results are

$$B(\pi^+) = \begin{cases} 3.26 \pm 0.06 \quad (\chi^2/\mathrm{DOF} = 8.6/8), & \text{(11a)} \\ (3.31 \pm 0.14) - (0.23 \pm 0.50)1/\omega \\ \qquad (\chi^2/\mathrm{DOF} = 8.4/7), & \text{(11b)} \end{cases}$$

$$B(\pi^-) = \begin{cases} 4.08 \pm 0.15 \quad (\chi^2/\mathrm{DOF} = 15.9/7), & \text{(12a)} \\ (3.35 \pm 0.05) + (2.90 \pm 0.18)1/\omega \\ \qquad (\chi^2/\mathrm{DOF} = 0.4/6). & \text{(12b)} \end{cases}$$

From the χ^2, it is clear that $B(\pi^+)$ is consistent with being a constant and that $B(\pi^-)$ requires the additional $1/\omega$ dependence.

The different ω behavior of the π^+ and π^- structure functions indicates that the origin of the increase in the π^+/π^- ratio as $1/\omega$ increases is a decrease in the number of forward negative pions. To a good approximation, Eqs. (9) through (12) give for the π^+/π^- ratio

$$\frac{N_{\pi^+}}{N_{\pi^-}} = \frac{A(\pi^+)}{A(\pi^-)}\exp[(2.90 \pm 0.18)x'/\omega]. \tag{13}$$

This equation gives the explicit dependence of the π^+/π^- ratio on x' and ω. Such an x' dependence is observed in the data.[12] It should be noted that for $1/\omega = 0$, which corresponds to photoproduction,

$$B(\pi^-) \simeq B(\pi^+)$$

and

$$\frac{N_{\pi^+}}{N_{\pi^-}} = \frac{A(\pi^+)}{A(\pi^-)} = 1.20 \pm 0.04.$$

Not only is the π^+/π^- ratio not a function of x' but the ratio agrees with the value 1.20 ± 0.10 obtained in photoproduction experiments.[13]

We have examined the p_T dependence of the structure function over the larger but still limited range, $p_T^2 < 0.2$ GeV^2. There was no evidence (< 10%) for a change in the p_T dependence of the structure function as Q^2 was varied.

In conclusion, we find that at fixed W, the invariant structure function for π^+ production shows no dependence on Q^2 and thus the π^+ invariant cross section displays the same ω scaling behavior as the total virtual photoproduction cross section. At fixed W, the π^- structure function decreases more rapidly with x' as Q^2 increases and thus the π^- invariant cross section displays a different ω scaling behavior from that of the total virtual photoproduction cross section. This latter behavior is the origin of the observed increse of the π^+/π^- ratio as Q^2 increases at fixed W.

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Yang-Mills Theory on the Mass Shell*

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Gauge-invariant mass-shell amplitudes for quantum electrodynamics (QED) and Yang-Mills theory are defined by dimensional regularization. Gauge invariance of the mass-shell renormalization constants is maintained through interplay of ultraviolet and infrared divergences. Quark renormalizations obey the same simple Ward identity as do the electron renormalizations in QED, while the gluon contributions to gluon renormalizations are identically zero. The simplest amplitude finite in QED, the magnetic moment, is gauge-invariant but divergent in Yang-Mills theory for both external gluon and external photon.

It is traditional to treat ultraviolet (uv) and infrared (ir) divergences of quantum electrodynamics (QED) as separate problems. uv divergences are associated with the internal topology of Feynman diagrams, and they can be removed by a (possibly intermediate) renormalization. ir divergences are controlled by the external momenta, and they are traditionally regularized by the introduction of a photon mass, i.e., an abandonment of gauge invariance in the intermediate stages of calculation of physical cross sections. Even though ir and uv divergences thus appear quite unrelated, there are hints to the contrary. A systematic analysis of Feynman parametric integrals reveals a close connection between the two types of divergences[1]; furthermore, these divergences can be transmuted one into another by a change of gauge (for example, from Landau to Yennie gauge).

The breaking of gauge invariance through the introduction of a photon mass is acceptable for QED, but unacceptable[2] for Yang-Mills[3,4] theories. However, the dimensional regularization[5,6] makes it possible to regularize both uv and ir divergences of QED while keeping the photon strictly massless. I shall here first reconsider QED in this approach, concentrating on the regularization of ir divergences, and then use the same regularization procedure to give an unambiguous definition of Yang-Mills amplitudes on the mass shell. To stress the analogy with QED, I shall refer to the class of Yang-Mills theories considered here (symmetric, with all quarks of equal mass $m \neq 0$ and strictly massless gluons) as quantum chromodynamics (QCD). The details of the calculations will be published elsewhere.[7]

QED.—As their first example of dimensional regularization Bollini and Giambiagi[5] have computed one-loop contributions to electron vertex and wave-function mass-shell renormalizations $Z_1=(1+L)^1$ and $Z_2=(1-B)^{-1}$ in $4-\epsilon$ dimensions,

$$L=-B=-\frac{\alpha}{4\pi}\Gamma\left(\frac{\epsilon}{2}\right)\frac{-3+\epsilon}{1-\epsilon}, \qquad (1)$$

where $\alpha \equiv [e_0^{\,2}/(4\pi)^{1-\epsilon/2}]m^{-\epsilon}$, and throughout this paper I set $m=1$. It can be verified by calculation in the generalized Landau gauge $[g^{\mu\nu}k^2-(1-a)k^\mu k^\nu]$ that this is gauge-independent.[7,8] That