Rev. C 13, 147 (1976).
[4]J. V. Maher, M. W. Sachs, R. H. Siemssen, A. Weidinger, and D. A. Bromley, Phys. Rev. 188, 1665 (1969).
[5]A. Gobbi, R. Wieland, L. Chua, D. Shapira, and D. A. Bromley, Phys. Rev. C 7, 30 (1973).
[6]W. Reilly, R. Wieland, A. Gobbi, M. W. Sachs, and D. A. Bromley, Nuovo Cimento 13A, 913 (1973).
[7]G. R. Satchler and W. G. Love, to be published. The effective interaction used was $v(r) = 6315 e^{-4.0r}/4.0r - 1961 e^{-2.5r}/2.5r$ MeV.
[8]G. Bertsch, J. Borysowicz, H. McManus, and W. G. Love, to be published.
[9]R. G. Stokstad, C. B. Fulmer, M. Halbert, S. Raman, A. H. Snell, and P. H. Stelson, Bull. Am. Phys. Soc. 20, 664 (1975), and to be published.
[10]R. G. Stokstad, Ref. 1b, p. 327; D. Shapira, R. G. Stokstad, and D. A. Bromley, Phys. Rev. C 10, 1063 (1974).
[11]F. G. Perey, unpublished; modified by L. W. Owen.
[12]R. A. Chatwin, J. S. Eck, D. Robson, and A. Richter, Phys. Rev. C 1, 795 (1970).
[13]G. J. Michaud and E. W. Vogt, Phys. Rev. C 5, 350 (1972).
[14]It is possible that these small variations in N are manifestations of coupling to the 2^+ state.
[15]L. D. Rickertsen, G. R. Satchler, R. G. Stokstad, and R. M. Wieland, in Proceedings of the Symposium on Macroscopic Features of Heavy-Ion Collisions, Argonne, Illinois, April 1976 (unpublished).
[16]A folding-model analysis of $^{14}N + ^{12}C$ and $^{14}N + ^{16}O$ scattering measured at forward angles only ($\theta_{c.m.} \lesssim 35°$) has been reported by P. J. Moffa *et al.*, Phys. Rev. Lett. 35, 992 (1975). In this case the folded potential is preferred over the Woods-Saxon on the basis of the predicted L dependence of the form factor rather than the depth of the potential in the interior region. In fact, both potentials yielded equivalent fits to the measured elastic scattering.

Calculated Half-Lives of Superheavy Nuclei near $^{354}[126]$†

P. Möller and J. R. Nix
Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545
(Received 2 September 1976)

For nuclei in the region of of 126 protons and 228 neutrons we use the macroscopic-microscopic method to calculate fission barriers, α-decay energies, and β-decay energies, as well as half-lives with respect to spontaneous fission and α decay. The nucleus $^{354}[126]$ is found to be β stable, but its spontaneous-fission half-life is only 39 ms and its α-decay half-life is only 18 yr.

Many theoretical calculations[1-3] have indicated the possibility of an island of relatively stable superheavy elements (half-lives $\approx 10^5-10^{10}$ yr) near the predicted magic proton number $Z = 114$ and magic neutron number $N = 184$. The nucleus $^{294}[110]$ is predicted to be the most stable nucleus in this region when spontaneous fission, α decay, and β decay are all considered. Half-lives have also been calculated for much heavier elements, for example, nuclei near $^{472}[164]$, with somewhat diverse results.[4-6]

Despite considerable efforts, such superheavy elements have not been found in nature or produced in the laboratory.[7,8] However, recently some proton-induced x-ray spectra from microscopic crystalline monazite inclusions in biotite mica were interpreted as evidence for the existence of primordial superheavy elements[9] with $Z = 127$, 126, 124, and 116. Although this interpretation is highly inconclusive,[10] it has nevertheless spurred a theoretical interest[11-14] in the region of $Z = 126$ and $N = 228$, which corresponds to the next magic neutron number beyond 184. (Nuclei with $Z \approx 126$ and $N \approx 184$ lie far above the extrapolated line of β stability and are also expected to decay rapidly by α emission.)

These recent studies investigated possible mechanisms which could make these superheavy elements sufficiently stable to be observed in nature. For example, Wong[12] considered the possibility of toroidal nuclei, and Andersson *et al.*[13] considered the possibility that the nuclear surface diffuseness readjusts itself to produce a large proton gap at $Z = 126$. However, with the exception of Ref. 13, half-lives for nuclei in this new region were not calculated. Furthermore, because of the neglect in Ref. 13 of the large macroscopic restoring force against changes in the surface diffuseness, the half-lives calculated there should be regarded as extreme upper limits.

Here we take an alternative approach and calculate in a conventional way the half-lives of superheavy nuclei in this new region. For this purpose we use a macroscopic-microscopic model that has been successful[15,16] in calculating fission barriers, ground-state deformations, and masses

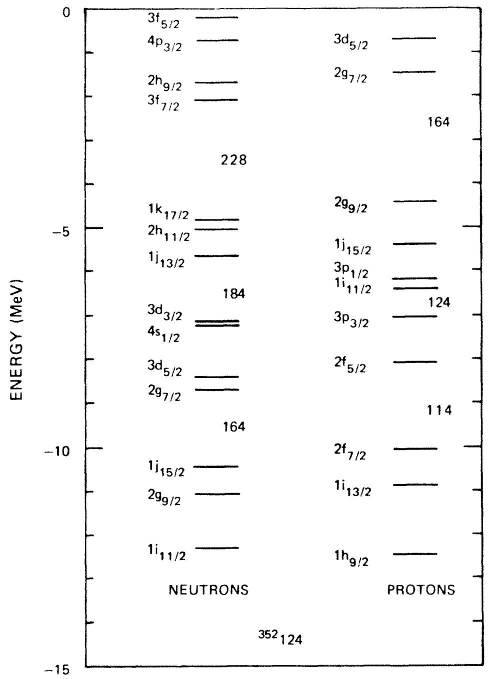

FIG. 1. Calculated single-particle levels for the spherical nucleus $^{352}[124]$. The 228-neutron gap is 2.8 MeV wide, but there is no shell closure at 126 protons. Instead, a gap of 0.6 MeV occurs for 124 protons.

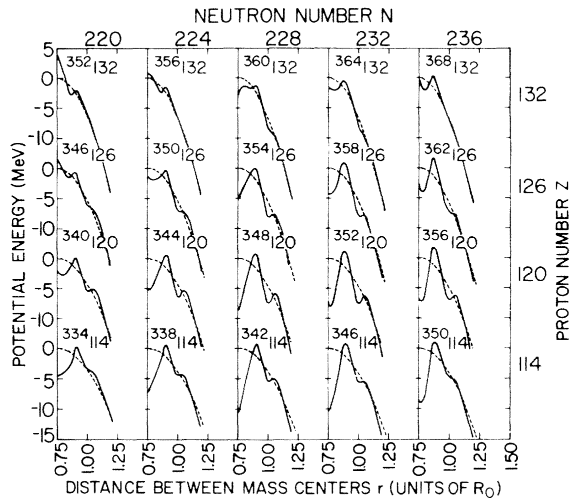

FIG. 2. Calculated fission barriers for superheavy nuclei near $^{354}[126]$, plotted as functions of the distance between the centers of mass of the two halves of the system. The dashed curves give the droplet-model contributions, and the solid curves give the total potential energies.

in the actinide region.

The potential energy of deformation is calculated for a sequence of y-family shapes[17] leading from a spherical ground-state shape to prolate saddle-point shapes and beyond. The potential energy is given by the sum of a macroscopic part and microscopic shell and pairing corrections.

For the macroscopic part we use the droplet model of Myers and Swiatecki,[18] which should be more satisfactory than the liquid-drop model[19] for extrapolating into new regions of nuclei. The reason is that the droplet model reproduces such properties as nuclear sizes and the dependence of actinide fission-barrier heights upon neutron number more satisfactorily than does the liquid-drop model.

The microscopic corrections are calculated by use of Strutinsky's method[20] from single-particle levels in a diffuse-surface single-particle potential of the folded Yukawa type.[17] The values of the single-particle potential parameters are taken from Ref. 15; they have been determined by means of statistical-model calculations[21] and adjustments to experimental single-particle levels in actinide nuclei. Because the potential radius and depth determined in this way are consistent with known nuclear properties, they can be extrapolated to new regions of nuclei with greater confidence than can parameters whose values are determined solely by adjustments to experimental single-particle levels.

The calculated single-particle levels for the spherical nucleus $^{352}[124]$ are displayed in Fig. 1. Our levels are very similar to those calculated by Wong,[12] although his 228-neutron gap is only 2.0 MeV wide, whereas ours is 2.8 MeV wide. However, for $N = 228$ Wong calculates a shell correction of only -1.48 MeV, whereas we obtain -8.61 MeV. Neither we nor Wong obtains a proton shell closure at $Z = 126$. The closure occurs instead at $Z = 124$, with a gap of only 0.6 MeV in our calculation and 0.4 MeV in Wong's. Our proton shell correction is slightly positive for values of Z near 124; it decreases monotonically with decreasing Z until we reach $Z = 114$, for which it is -4.8 MeV.

These results for the proton levels are significantly different from those obtained by Petrovich *et al.*,[11] who find a 1.3-MeV proton gap at $Z = 126$. The primary reason for this difference is that the interaction strength and radius of the spin-orbit potential used by Petrovich *et al.* are substantially smaller than those commonly used.

Calculated fission barriers for twenty nuclei in this region are plotted in Fig. 2 as functions of r, the distance between mass centers of the two halves of the system.[15] The highest barrier occurs for the magic numbers $Z = 114$ and $N = 228$ and is 11.8 MeV high. (We assume that the ground state lies at an energy of 0.5 MeV above the minimum to take into account the zero-point energy in the fission direction.) As we either increase or decrease the proton or neutron number from these values, the barrier height decreases. The barrier for $Z=126$ and $N=228$ is 5.2 MeV high.

In a one-dimensional WKB (quasiclassical) theory the spontaneous-fission half-life is related to the penetrability P through the barrier by[1,2]

$$T_{sf}=10^{-28.04}\ \mathrm{yr}/P,$$

where we use the value $\omega_0=1\ \mathrm{MeV}/\hbar$ for the frequency of assaults on the barrier. The probability P of penetrating the barrier $V(r)$ at a penetration energy E_0 is given by[22]

$$P(1+\exp K)^{-1},$$

where

$$K=2\int_{r_2}^{r_2}\left\{\frac{2B_r(r)}{\hbar^2}[V(r)-E_0]\right\}^{1/2}dr.$$

The limits of integration r_1 and r_2 are the points of entrance and emergence, respectively, into and from the barrier, at a penetration energy E_0 that lies 0.5 MeV above the minimum in the barrier. The function $B_r(r)$ is the inertia with respect to r associated with motion in the fission direction.

We relate the inertia B_r to the inertia B_r^{ir} corresponding to irrotational flow by[2]

$$B_r-\mu=k(B_r^{\mathrm{ir}}-\mu),$$

where μ is the reduced mass of the final fragments and k is a semiempirical constant. The calculated irrotational inertia[2] is approximated by

$$B_r^{\mathrm{ir}}-\mu=\frac{17}{15}\mu\exp\left[-\frac{128}{51}\left(\frac{r}{R_0}-\frac{3}{4}\right)\right].$$

This is similar to the approximation used by Randrup and co-workers,[3,23] but unlike Randrup and co-workers we choose the coefficient of the exponential to reproduce the leading term in the irrotational inertia for spheroidal distortions. The radius R_0 of the spherical nucleus is taken to be[18]

$$R_0=(1.175\ \mathrm{fm})A^{1/3}.$$

The constant k is determined to reproduce optimally the spontaneous-fission half-lives of the five nuclei ^{244}Pu, ^{244}Cm, ^{246}Cf, ^{246}Fm, and ^{252}Fm. For the value $k=16$, the resulting root-mean-square deviation between the logarithms of the calculated and experimental half-lives is 2.5.

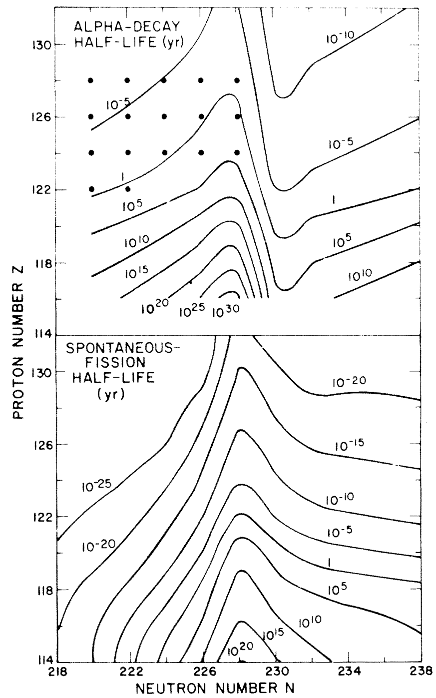

FIG. 3. Contour plots of calculated spontaneous-fission and α-decay half-lives for even-even superheavy nuclei near ^{354}[126]. The solid points indicate even-even nuclei in the limited region $Z\leq 128$, $N\geq 220$ that are calculated to be β-stable.

In a similar study performed with a modified harmonic-oscillator single-particle potential,[23] it was found that the absolute mean deviation between the logarithms of the calculated and experimental half-lives of thirty actinide nuclei is 1.7 (the analogous root-mean-square deviation was not calculated).

The spontaneous-fission half-lives calculated in this way are shown for even-even nuclei in the lower part of Fig. 3. The maximum value of $10^{20.2}$ yr occurs for $Z=114$ and $N=228$. The addition or subtraction of either protons or neutrons relative to these values decreases the calculated spontaneous-fission half-lives. This decrease is most gentle in the direction of increasing neutron number. The calculated spontaneous-fission half-life for $Z=126$ and $N=228$ is only 38 ms. This short half-life arises partly because the barrier for ^{354}[126] is only 5.2-MeV high, but also because the barrier is relatively thin. In particular, in terms of the coordinate r, the barrier for ^{354}[126] is only 0.5 as thick as the barrier for ^{342}[114].

The upper part of Fig. 3 shows α-decay half-lives for even-even nuclei, which are calculated from Viola's and Seaborg's semiempirical relationship[24]

$$\ln(T_\alpha/\mathrm{sec}) = A_Z(Q_\alpha/\mathrm{MeV})^{-1/2} + B_Z,$$

where

$$A_Z = 2.113\,29Z - 48.9879,$$

and

$$B_Z = -0.390\,040Z - 16.9543.$$

The α-decay energy Q_α is determined from the ground-state masses of the nuclei involved, which are calculated for y-family distortions.[17] As expected, for a fixed number of neutrons the α-decay half-lives decrease with the addition of protons. Also, the α-decay half-lives of nuclei with $N > 228$ are substantially smaller than those of nuclei with $N \leq 228$. The calculated α-decay half-life for $Z = 126$ and $N = 228$ is 18 yr.

Finally, we show by the solid points in the upper part of Fig. 3 the even-even nuclei in the limited region $Z \leq 128$, $N \geq 220$ that are calculated to be β-stable. For these nuclei the calculated β-decay energy Q_β and electron-capture energy Q_{ec} are both negative. As seen in the figure, the nucleus with $Z = 126$ and $N = 228$ is calculated to be β-stable, although the extrapolated line of β stability passes somewhat to its left.

The uncertainties associated with the half-lives shown in Fig. 3 are moderately large. For example, for the nucleus $^{354}[126]$ the calculated spontaneous-fission half-life is changed by a factor of $10^{\pm 4}$ if the ground-state energy is changed by 1 MeV, and by a factor of $10^{\pm 3}$ if the inertial constant k is changed from its value of 16 by 6 units. Similarly, its calculated α-decay half-life is changed by a factor of $10^{\pm 5}$ if the α-decay energy Q_α is changed by 1 MeV.

In summary, by use of conventional methods of extrapolation we find that although the nucleus $^{354}[126]$ is β-stable, its spontaneous-fission half-life is only 39 ms and its α-decay half-life is only 18 yr, with uncertainties that are several powers of ten. In order for the half-life of this nucleus to be sufficiently long for it to be observed in nature ($\gtrsim 10^8$ yr), major changes would be required in either the single-particle potential, the droplet model, or the nuclear inertia that we have used. This is certainly a possibility, but to us a far more likely explanation for the spectra of Ref. 9 is γ rays and/or K x rays from contaminants.

†Work supported by the U. S. Energy Research and Development Administration.

[1]S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafsson, I. L. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. A131, 1 (1969).

[2]E. O. Fiset and J. R. Nix, Nucl. Phys. A193, 647 (1972).

[3]J. Randrup, S. E. Larsson, P. Möller, A. Sobiczewski, and A. Łukasiak, Phys. Scr. 10A, 60 (1974).

[4]J. Grumann, U. Mosel, B. Fink, and W. Greiner, Z. Phys. 228, 371 (1969).

[5]R. Bengtsson, CERN Report No. CERN 70-30, 1970 (unpublished), Vol. 2, p. 646.

[6]A. Łukasiak and A. Sobiczewski, Acta Phys. Pol. B 6, 147 (1975).

[7]G. Herrmann, in *Radiochemistry,* edited by A. G. Maddock (Butterworths, London, 1975), Ser. 2, Vol. 8, p. 221.

[8]C. E. Bemis, Jr., to be published.

[9]R. V. Gentry, T. A. Cahill, N. R. Fletcher, H. C. Kaufmann, L. R. Medsker, J. W. Nelson, and R. G. Flocchini, Phys. Rev. Lett. 37, 11 (1976).

[10]J. D. Fox, W. J. Courtney, K. W. Kemper, A. H. Lumpkin, N. R. Fletcher, and L. R. Medsker, Phys. Rev. Lett. 37, 629 (1976).

[11]F. Petrovich, R. J. Philpott, D. Robson, J. J. Bevelacqua, M. Golin, and D. Stanley, Phys. Rev. Lett. 37, 558 (1976).

[12]C. Y. Wong, Phys. Rev. Lett. 37, 664 (1976).

[13]G. Andersson, S. E. Larsson, G. Leander, S. G. Nilsson, I. Ragnarsson, and S. Åberg, to be published.

[14]H. Schultheis and R. Schultheis, to be published.

[15]P. Möller and J. R. Nix, in *Proceedings of the Third International Atomic Energy Agency Symposium on the Physics and Chemistry of Fission, Rochester, New York, 1973* (International Atomic Energy Agency, Vienna, 1974), Vol. I, p. 103.

[16]P. Möller and J. R. Nix, Nucl. Phys. A229, 269 (1974).

[17]M. Bolsterli, E. O. Fiset, J. R. Nix, and J. L. Norton, Phys. Rev. C 5, 1050 (1972).

[18]W. D. Myers and W. J. Swiatecki, Ann. Phys. (N.Y.) 84, 186 (1974).

[19]W. D. Myers and W. J. Swiatecki, Ark. Phys. 36, 343 (1967).

[20]M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. 44, 320 (1972).

[21]W. D. Myers, Nucl. Phys. A145, 387 (1970).

[22]N. Fröman and P. O. Fröman, *JWKB Approximation* (North-Holland, Amsterdam, 1965), Chap. 9, Sect. 1, pp. 92–97.

[23]J. Randrup, S. E. Larsson, P. Möller, S. G. Nilsson, K. Pomorski, and A. Sobiczewski, Phys. Rev. C 13, 229 (1976).

[24]V. E. Viola, Jr., and G. T. Seaborg, J. Inorg. Nucl. Chem. 28, 741 (1966).