Analysis of $^{12}C + ^{12}C$ Scattering and the Depth of the Real Potential

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An optical-model analysis of $^{12}C + ^{12}C$ elastic scattering from $E_{c.m.} = 35$ to 63 MeV indicates a sensitivity of the scattering cross section at large scattering angles ($\theta_{c.m.} \sim 50°$–$90°$) to a region of internuclear separation extending in to $r \sim 2$ fm. Real nuclear potentials obtained with the folding model and a realistic interaction fit the data remarkably well. This suggests that the nuclear potential for $^{12}C + ^{12}C$ in the region of large overlap is significantly deeper than would be expected on the basis of previous analyses of systems such as $^{16}O + ^{16}O$.

The subjects of heavy-ion elastic scattering measurements and the shape and depth of the real nucleus-nucleus potential have received considerable study during the past decade.[1] The present work bears on two important features of heavy-ion reactions which have emerged during this period. First, in systems with $\eta \gtrsim 10$ ($\eta = Z_1 Z_2 e^2/\hbar v$) it has been seen that the elastic scattering is sensitive to the value of the real potential only over a narrow region of internucleus separation centered at the strong-absorption radius.[2,3] Second, an analysis of the scattering of $^{16}O + ^{16}O$ at energies $E_{c.m.} \lesssim 40$ MeV ($\eta > 4.6$) led to the introduction of very shallow Woods-Saxon real potentials ($V \sim -17$ MeV, $r_0 \sim 1.35$ fm, $a \sim 0.56$ fm)[4,5] which were then used to fit a variety of other heavy-ion systems, $^{12}C + ^{12}C$ included, at comparable bombarding energies.[6] In our analysis of $^{12}C + ^{12}C$ elastic scattering at energies from $E_{c.m.} = 35$ to 63 MeV ($\eta = 2.4 \rightarrow 1.8$), we find a marked sensitivity of the scattering to the value of the real potential over the large region $r \simeq 2$–8 fm. Most remarkably, the real potential in the interior region deduced for $^{12}C + ^{12}C$ appears to be much deeper than was found earlier[4] for the $^{16}O + ^{16}O$ system. In fact, it is the "deep" potentials predicted by the double-folding model[2] with a realistic interaction[7,8] which are successful in reproducing the $^{12}C + ^{12}C$ elastic scattering.

The experimental data were obtained at the Oak Ridge isochronous cyclotron facility and consist of angular distributions measured at fourteen bombarding energies.[9] They will be described in detail in a separate publication. Figure 1 shows representative angular distributions at six of these energies. The high bombarding energies minimized contributions from compound elastic scattering[10] such that an analysis of individual angular distributions (as opposed to energy-averaged excitation functions) was possible. The optical-model code HIGENOA[11] was used with real potentials either of the Woods-Saxon type or from the folding model. In both cases, the imaginary potentials were Woods-Saxon. Regardless of the type of real potential, we found that the best fits to the data required an imaginary potential with a central depth in the range of 10–20 MeV.

The absorptive strength given by the above imaginary potential (corresponding to a mean free path of about 2 fm), together with the small values of η, accounts for the dependence of the predicted angular distributions on the value of the real potential in the interior (~ 2–6 fm) as well as the exterior (6–8 fm) region. This dependence was demonstrated by the method[2] in which the interior portion of the real potential (full curve, Fig. 2) is arbitrarily held constant at the value $V(r')$ over the region $0 \leq r < r'$. Calculations of the angular distribution were then made and compared for values of r' extending from 1 to 7 fm. A crucial point is that a sensitivity to the interior region is only apparent for the cross sections at large angles ($50° \lesssim \theta_{c.m.} \lesssim 90°$). Attempts to fit the data using an imaginary potential sufficiently deep to eliminate this sensitivity were unsuccessful.

The procedure followed in fitting an angular distribution was first to require that the more regular oscillations at forward angles ($\theta \lesssim 40°$) be well reproduced. Further modifications to the potentials to improve the quality of the fit at larger an-

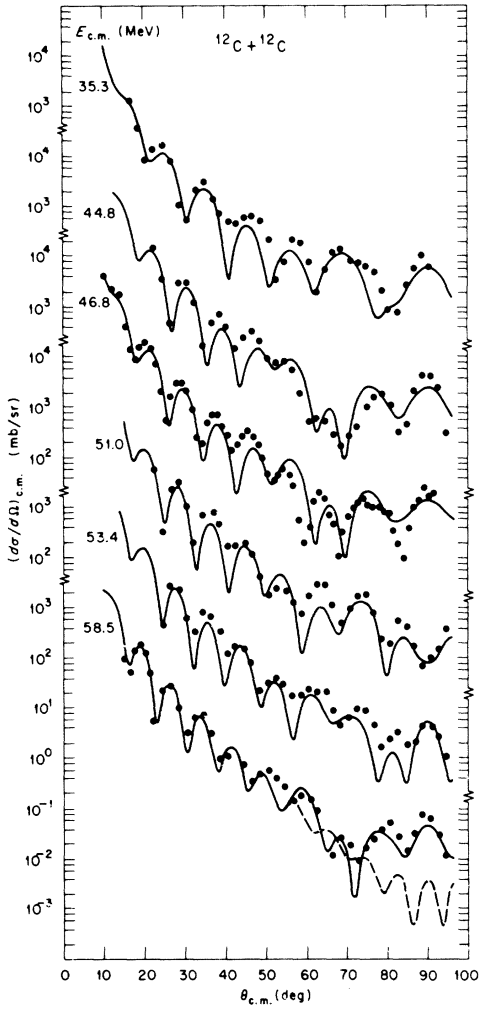

FIG. 1. Typical measured angular distributions for elastic scattering and folding-model calculations as described in the text. The dashed line corresponds to a real Woods-Saxon potential as shown in Fig. 2 and the same imaginary potential as used for the folding-model prediction. These two calculations also illustrate the sensitivity to the interior region.

gles were subject to this first requirement. Minimization of χ^2 was used in parameter searches, but final judgments on the quality of a fit were made by visual evaluation.

In the case of the shallow Woods-Saxon potentials, we were generally unsuccessful in finding for any bombarding energy a set of six parameters which could fit the data in the large-angle region. At most bombarding energies, the magnitude of the cross sections at larger angles was underestimated. An example of this is shown by the dotted line in Fig. 1 for $E_{c.m.} = 58.5$ MeV. In some cases the general magnitude might be correct, but the phase of the phase of the structures in the region 75°–90° would be incorrect. In the few cases where a search on a particular angular distribution produced a reasonable fit over the whole angular range, attempts to extend this type of potential to fit data at other energies were unsuccessful. The potential of Gobbi *et al.*[5] (which has a small imaginary diffuseness and can simulate the effects of l depencence[12]), somewhat deeper Woods-Saxon real potentials ($-V \sim 50$–100 MeV), and potentials having a soft repulsive core[13] were tried.

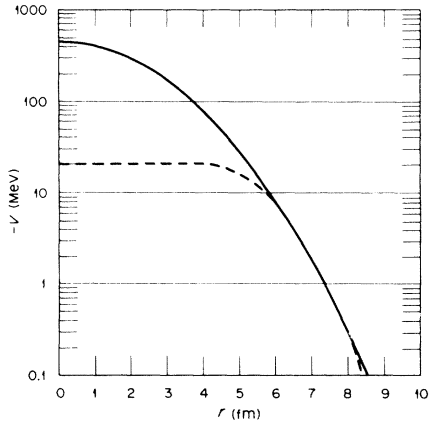

FIG. 2. Comparison of the folding-model potential and a typical shallow Woods-Saxon real potential ($V = -20$ MeV, $r_0 = 1.25$ fm, and $a = 0.52$ fm).

A different situation was found when a much deeper potential, having a magnitude and shape as given by the folding model,[2,7,8] was used. This potential is compared with a typical shallow Woods-Saxon potential in Fig. 2. Figure 1 shows theoretical angular distributions obtained when the normalization of the real potential, N, and the depth, W, of the imaginary potential were adjusted at each energy to provide a best overall fit to the data. The geometry of the imaginary potential is $r_0 = 1.22$ fm, $a = 0.54$ fm in all cases. The theoretical curves have been averaged over an angular region of 2° (c.m.), corresponding approximately to the experimental angular resolution. These "two-parameter" fits are seen to be quite good, reproducing the magnitude and phase of the structure in the large-angle region as well as at forward angles. Qualitatively similar results are obtained at the other energies not shown in the figure. The values of N and W obtained from these analyses at each energy are shown in Fig. 3. We note that all values of N are within ± 5% of an average value having a slight linear energy dependence. The mean deviation is 2.4%. The depth

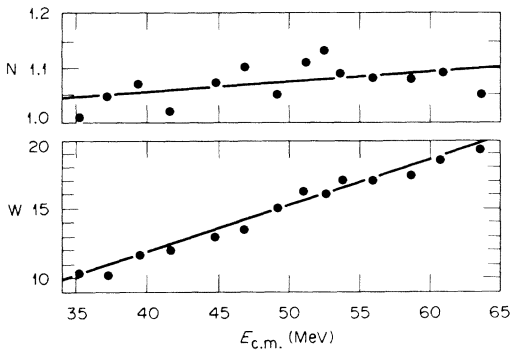

FIG. 3. The values of the normalization factor, N, for the folded potential and the depth, W(MeV), of the imaginary potential which best fit the angular distributions at each energy.

of the imaginary potential changes more rapidly and is well reproduced by a linear energy dependence. If potentials with depths given by the straight lines in Fig. 3 are used the general magnitude of the cross sections in the large-angle region is unaffected, but the "phase" of the oscillations changes in some cases, particularly in the region of 75°–90°. This effect is associated mainly with changes in the real potential.[14]

Since the inelastic scattering (single and mutual excitation of the first 2^+ state) is as intense as the elastic over the energy range studied here, one must inquire to what extent the above results would be modified by the inclusion of coupling between the elastic and inelastic channels. We have made coupled-channels calculations at $E_{c.m.}$ = 51.05 and 58.55 MeV in which the first 2^+ states in each nucleus are coupled to all orders of excitation, thus including mutual excitation as well as reorientation effects.[15] For the folding model, form factors were obtained by folding transition densities with the effective nucleon-nucleon interaction.[7,8] The static and transition densities used reproduced the known rms radius and $B(E2)$ value for ^{12}C. The inclusion of coupling required the imaginary potential to be weaker by ~60% but the real folded potential to be deeper by ~25% in order to fit the forward-angle scattering. In the case of the shallow real potential, the predicted elastic scattering remained generally low at large angles whereas the folding model continued to give overall agreement for the magnitude of the cross sections. Comparable success was obtained in reproducing the experimental results for the single and mutual excitation.

Although cost considerations prevented a detailed adjustment of potentials in the coupled-channels calculations, it appears to us that the inclusion of coupling to the first 2^+ state does not invalidate our main result: viz., deep potentials can reproduce the elastic scattering remarkably well. Considering this result and the apparent inability of shallow Woods-Saxon potentials to provide good fits to the data, we suggest that the real potential for the scattering of $^{12}C + ^{12}C$ is significantly deeper (see Fig. 2) than would have been expected on the basis of previous analyses of other reactions in this mass region and at lower energies. An important question for future study, therefore, is whether the folding model (i.e., deep potentials) will be as successful in reproducing the experimental data for $^{16}O + ^{16}O$ and other such heavy-ion systems.

The result of our analysis need not be expressed in terms of one particular parametrization since it is possible to fit the elastic scattering with other formulations (e.g., square of the Woods-Saxon potential), provided the parameters can be adjusted such that $V(r)$ approximates the deep potential shown in Fig. 2 in the region 2–8 fm. Although it is satisfying that the model which does fit the data also has some independent theoretical basis, the approximations presently made in the derivation of the folding model are questionable in the interior region.[16] The results of the present analysis thus should be valuable for future theoretical studies of the nucleus-nucleus potential at large overlap.

In conclusion, the present analysis thus reveals a situation in which a heavy-ion reaction is sensitive to the value of the real potential in the interior region and suggests that this potential is significantly deeper than recently supposed.

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[1b]*Proceedings of the International Conference on Reactions Between Complex Nuclei, Nashville, Tennessee, 1974*, edited by R. L. Robinson, F. K. McGowan, J. B. Ball, and J. H. Hamilton (North-Holland, Amsterdam, 1974), Vol. 2.

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[16]A folding-model analysis of $^{14}N+^{12}C$ and $^{14}N+^{16}O$ scattering measured at forward angles only ($\theta_{c.m.} \lesssim 35°$) has been reported by P. J. Moffa *et al.*, Phys. Rev. Lett. 35, 992 (1975). In this case the folded potential is preferred over the Woods-Saxon on the basis of the predicted L dependence of the form factor rather than the depth of the potential in the interior region. In fact, both potentials yielded equivalent fits to the measured elastic scattering.

Calculated Half-Lives of Superheavy Nuclei near $^{354}[126]$†

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For nuclei in the region of of 126 protons and 228 neutrons we use the macroscopic-microscopic method to calculate fission barriers, α-decay energies, and β-decay energies, as well as half-lives with respect to spontaneous fission and α decay. The nucleus $^{354}[126]$ is found to be β stable, but its spontaneous-fission half-life is only 39 ms and its α-decay half-life is only 18 yr.

Many theoretical calculations[1-3] have indicated the possibility of an island of relatively stable superheavy elements (half-lives $\approx 10^5$–10^{10} yr) near the predicted magic proton number $Z = 114$ and magic neutron number $N = 184$. The nucleus $^{294}[110]$ is predicted to be the most stable nucleus in this region when spontaneous fission, α decay, and β decay are all considered. Half-lives have also been calculated for much heavier elements, for example, nuclei near $^{472}[164]$, with somewhat diverse results.[4-6]

Despite considerable efforts, such superheavy elements have not been found in nature or produced in the laboratory.[7,8] However, recently some proton-induced x-ray spectra from microscopic crystalline monazite inclusions in biotite mica were interpreted as evidence for the existence of primordial superheavy elements[9] with $Z = 127$, 126, 124, and 116. Although this interpretation is highly inconclusive,[10] it has nevertheless spurred a theoretical interest[11-14] in the region of $Z = 126$ and $N = 228$, which corresponds to the next magic neutron number beyond 184. (Nuclei with $Z \approx 126$ and $N \approx 184$ lie far above the extrapolated line of β stability and are also expected to decay rapidly by α emission.)

These recent studies investigated possible mechanisms which could make these superheavy elements sufficiently stable to be observed in nature. For example, Wong[12] considered the possibility of toroidal nuclei, and Andersson *et al.*[13] considered the possibility that the nuclear surface diffuseness readjusts itself to produce a large proton gap at $Z = 126$. However, with the exception of Ref. 13, half-lives for nuclei in this new region were not calculated. Furthermore, because of the neglect in Ref. 13 of the large macroscopic restoring force against changes in the surface diffuseness, the half-lives calculated there should be regarded as extreme upper limits.

Here we take an alternative approach and calculate in a conventional way the half-lives of superheavy nuclei in this new region. For this purpose we use a macroscopic-microscopic model that has been successful[15,16] in calculating fission barriers, ground-state deformations, and masses