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Relaxation Instability in Tokamaks

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A three-dimensional, nonlinear, numerical simulation is described in which saw-tooth oscillations occur as a result of a hydromagnetic relaxation instability. The time evolution of the magnetic topology demonstrates the essential validity of the model suggested by Kadomtsev to explain the saw-tooth oscillations observed in tokamak experiments.

Several tokamak experiments have exhibited an unstable behavior having the form of relaxation oscillations in which the soft-x-ray emission has a saw-tooth time dependence[1,2,3]. In the central region the results indicate a slowly rising temperature followed by a rapid fall. The whole process occurs repeatedly with a period of the order of a millisecond.

The following general explanation of this behavior has gained some acceptance. The inner temperature rises due to Ohmic heating. The resulting increase in conductivity leads to an increase in the current density on axis and as a consequence the safety factor q falls below unity. The plasma then undergoes an instability which transports the energy which has been produced by Ohmic heating out to larger radii. In some way the plasma relaxes back to an axisymmetric state having $q > 1$ and the whole process is then repeated.

The question arises as to precisely how this phenomenon occurs. A model has been suggested by Kadomtsev,[4] the basic elements of which are as follows. When the value of q falls below unity an $m = 1$ instability occurs. As a result the plasma surfaces are displaced to one side and resistivity allows a magnetic island to form on the opposite side of the plasma around the $q = 1$ surface. This island grows and displaces the original set of magnetic surfaces which then decay away. The resulting value of q is greater than unity but the concentration of the current toward the magnetic axis leads to a lowering of q until instability reappears and the whole cycle is repeated.

Our purpose here is to describe the results of a three-dimensional, nonlinear calculation which reproduces relaxation instabilities of the type observed experimentally and which demonstrates the basic features of Kadomtsev's model. The configuration studied is cylindrical and, in order to achieve acceptable computation times, a higher value of β has been used than is obtained in tokamak experiments.

The equations solved are the time-dependent hydromagnetic equations including resistivity, viscosity, Ohmic heating, and an energy loss. The resistivity is taken to be proportional to $T_e^{-3/2}$. The viscosity used is small and is taken to be constant. The Ohm's law is

$$\vec{E} + \vec{v} \times \vec{B} = \eta(T)\vec{j},$$

and the resulting Ohmic heating is included together with an energy-loss term in the energy equation

$$\frac{1}{\gamma - 1}\frac{\partial p}{\partial t} = \frac{1}{\gamma - 1}\nabla\cdot(p\vec{v}) - p\nabla\cdot\vec{v} + \eta j^2 - \frac{1}{\gamma - 1}\frac{p}{\tau(x)}.$$

The form of the energy-loss term is arbitrary but it represents an attempt to describe the more rapid energy-loss rate in the outer region.

The calculations were carried out on a rectangular grid using a generalized form of the Lax-Wendroff method. Relaxation oscillations were first demonstrated on a $10\times10\times7$ grid. The calculations described here were carried out on a more refined $14\times14\times10$ grid and showed the same relaxation oscillations and overall time development. Because of the complexity of the calculation, no claim to precision can be made. However, the main requirement is that the qualitative features be properly described and it is believed that this is the case.

The cross section is square, having sides of length $2a$, and the periodicity length in the direction of the axis is $2\pi a$. The initial configuration has an axial current only, being given by[5]

$$j_z = 1.22 \frac{B_0}{a} \cos\left(\frac{\pi x}{2a}\right) \cos\left(\frac{\pi y}{2a}\right),$$

where B_0 is the initial uniform axial magnetic field. The corresponding value of q on the magnetic axis is 1.64. The total current is maintained constant by applying the appropriate electric field. The boundary condition $\hat{n}\cdot\vec{v}=0$ is applied at the surface, where $\hat{n}$ is the unit normal vector and $\vec{v}$ is the plasma velocity. All quantities are normalized to the uniform initial density ρ_0, to the velocity $V_A = B_0/\rho_0^{1/2}$, and to the length a. In terms of these quantities the following normalized physical properties were used:

resistivity $\eta = 0.0005T^{-3/2}$,

viscosity $\nu = 0.04$,

energy-loss time $\tau = \dfrac{5.2}{r}$,

where r is the distance from the center of the cross section. To prevent a piling up of plasma density at the boundary a small diffusion term $0.01\nabla^2\rho$ was added to the continuity equation.

For each magnetic surface, the value of q is obtained by calculating the number of periodicity lengths traversed per encirclement of the nested magnetic axis. The inner region has flux surfaces which are almost circular and will therefore be unstable to $m=1$ modes if the axial value of q is less than unity.[6] This is essentially the same condition for the $m=1$ mode as that obtained in toroidal geometry for similar configurations.[7,8]

Figure 1 shows the time dependence of the pressure at the center of the minor cross section. It is seen that the pressure follows a sequence of relaxation oscillations similar to the experimentally observed saw teeth. In the present calculation the oscillations decay away and the instability finally takes the form of a stationary helix. (For other choices of energy-loss function, total current, and viscosity more regular oscillations can occur.) Initially q is greater than unity every-

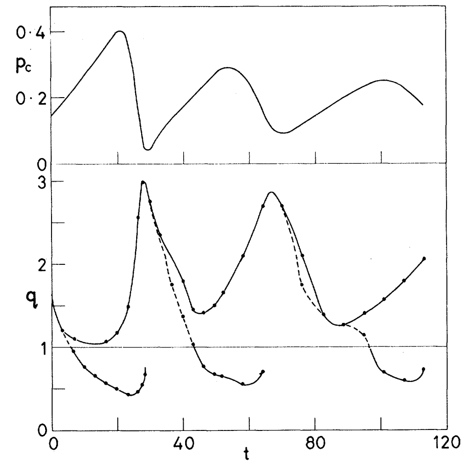

FIG. 1. The upper graph gives the pressure at the center of the minor cross section plotted against time. The lower graph shows the value of q on the magnetic axes (full lines) and that at the position of minimum q (dashed line).

where. As the plasma is Ohmically heated the temperature increases more rapidly toward the axis and the resulting concentration of the current in this region takes the value of q on the magnetic axis below unity. The configuration is now unstable and the small perturbation included in the initial conditions begins to grow, deforming the plasma into a helix. In each cross section the plasma moves to one side and as a result the central pressure falls. The behavior up to this point is straightforward. The way in which the plasma now restores itself from this helical configuration back to a symmetric configuration is best understood by considering the time development of the magnetic field structure during the cycle.

In order to obtain a clear picture of the magnetic field structure it is convenient to "unwind" the helix by transforming to the magnetic field $\vec{B}^*$ where

$$B_r^* = B_r \text{ and } B_\theta^* = B_\theta - (2\pi r/L)B_z,$$

r and θ are polar coordinates based on the center of the minor cross section, z is the coordinate along the length, and L is the periodicity length. The magnetic field B^* is then used to calculate the transformed trajectories of the magnetic field lines in the r, θ plane.

The number of encirclements of the nested magnetic axis per periodicity length now differs by unity from that in the untransformed geometry. Consequently, in the initial state, the lines on

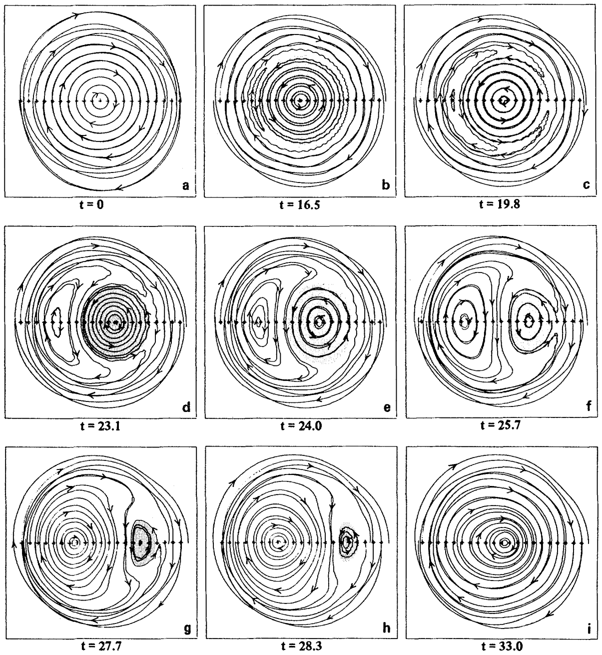

FIG. 2. Plots of the transformed magnetic field line trajectories during the first cycle. The regions with $q<1$ are shown shaded. The initially stable configuration becomes unstable as a result of current concentration. An island is formed (having $q>1$) which grows and displaces the original unstable "island." The island with $q<1$ decays away.

the $q=1$ surface now become points whereas the lines on either side rotate in opposite directions, $q>1$ resulting in a clockwise rotation and $q<1$ in an anticlockwise rotation. The resulting computer-produced diagrams giving the magnetic-field-line trajectories for a sequence of times are shown in Fig. 2. The regions having $q<1$ are shown shaded.

Some comment is needed on the apparent irregularities in the field-line trajectories. The waviness of the lines arises from the fact that the diagnostic unwinding procedure applied to the trajectories is based on a circular geometry whereas the region is square. Thus each traversal of a corner introduces a spurious deflection. It is clear, however, that this does not affect the actual results or their interpretation.

From Fig. 2(a) it is seen that, at $t=0$, $q>1$ everywhere. The subsequent concentration of current leads to $q<1$ in an inner region [Fig. 2(b)]. The resulting instability is apparent in Fig. 2(c) where the original magnetic axis has started to move to one side and a new axis has arisen on the opposite side of the plasma where a magnetic island has formed around the $q=1$ surface. As the instability grows still further the region of plasma enclosed by the island becomes comparable with that of the original system [Fig. 2(d)] and they can now be regarded as two island systems —the original one with $q<1$ and a new one with $q>1$. In subsequent Figs. 2(e)–2(h) the new island displaces the old and an axisymmetric configuration is formed once again, the value of q now being greater than unity [Fig. 2(i)]. The plasma is now returned to an essentially axisymmetric configuration.

In the present calculation the behavior in subsequent cycles differs somewhat from the first. The minimum value of q occurs away from the magnetic axis. A new magnetic island is formed on the surface of minimum q when q falls below unity. This island does not remove the original island but is itself ultimately expelled by a resurgence of the original island having $q>1$. The complete time behavior of the q values is shown in Fig. 1 where the values of q on the magnetic axes are given by full lines and the value of q at the minimum is shown by a dashed line.

For computational reasons the conductivities used in the calculations are much lower than those in experiments. It is nevertheless of interest to attempt a quantitative appraisal of the validity of Kadomtsev's description. The time, t_c, for the growth of the instability and the reclosure of the magnetic surfaces is very short. In the first cycle it occurs between times 10 and 20 so that $t_c \simeq 10$. The time which would characterize simple magnetic diffusion in the region inside the $q=1$ surface at radius r_s is $\tau_R=\sigma r_s^2$, where σ is some average over the region. Because of the temperature variation the value of σ varies considerably over this region, the limiting values giving $\tau_R \sim 1000$ at the center, and $\tau_R \sim 30$ at the $q=1$ surface. The appropriate value would lie between these two.

Kadomtsev's[4] estimate of t_c for the observed type of behavior predicts that, rather than being characterized by τ_R, the time t_c is given by (in rationalized units)

$$t_c \sim (\tfrac{1}{4}\tau_R\tau_H)^{1/2},$$

where $\tau_H = r_s/c_*$, c_* being the characteristic Alfvén speed in the transformed magnetic field B_*. Since $\tau_H \simeq 1$ the above values of τ_R give t_c between 16 and 3. These span the observed value, $t_c \simeq 10$.

It must be recognized, however, that while there is no discrepancy with Kadomtsev's estimate for the time scale, the uncertainties over the appropriate time and space averages involved are such as to leave some doubt. It is hoped to resolve this by further computations.

Summarizing, we see that the computation described above exhibits similar features to those observed in tokamak experiments and also gives support to Kadomtsev's interpretation of these observations.

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Startup of a Neutral-Beam-Sustained Plasma in a Quasi-dc Magnetic Field*

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Warm plasma has been injected along field lines into a quasi-steady-state magnetic mirror in the 2XIIB experiment. This has provided a suitable target plasma for density buildup and heating with neutral-beam injection. With the injection of 310 A equivalent of 14-keV deuterium atoms, the density exponentiated to 3.7×10^{13} cm^{-3}, the average ion energy increased to 12 keV, and the β reached 0.4. A rate equation describing the density buildup is given.

The initial production and maintenance of a dense, energetic plasma in a steady-state magnetic mirror field has been a major experimental problem in the field of plasma physics. A number of possible solutions have been pursued, most of which involve the creation of a target plasma suitable for buildup and heating by neutral-beam injection. Only recently has neutral-beam current greatly exceeding the equivalent of 10 A become available[1] to test these methods. Previous experiments, which were limited to evaluating the major problems, are summarized below. Creation of plasmas by Lorentz ionization of energetic neutrals[2-6] has been limited by instabilities to densities up to 10^{10} cm^{-3}. Electron-cyclotron-resonance breakdown and heating of a background gas resulted in stable hot-electron plasmas with β's of 0.5.[7] (β is defined as the ratio of the plasma pressure to the vacuum magnetic field pressure.) However, the densities have been limited to less than 10^{12} cm^{-3}. Experiments in which plasmas are produced by laser irradiation of a pellet are in progress.[8,9] Other target plasmas that have been tried or suggested include arc discharges[10,11] and an injected plasma heated and trapped by electron-cyclotron resonance.[12]

Dense, high-energy and high-β, mirror-confined plasmas have been achieved in the 2*X* series of experiments.[13,14] These experiments used fast-pulsed magnetic mirrors for trapping and heating of an injected plasma. We have achieved densities $n_e \leq 10^{14}$, ion energies $E_i \leq 10$ keV, mean target diameters $10 \leq l \leq 19$ cm, and ratios of plasma density to external neutral density $n_e/n_0 \geq 100$. Calculations have shown that such plasmas can be sustained by trapping hot ions from neutral beams to balance losses, primarily charge exchange on cold gas and Franck-Condon neutrals.[15] In fact, this plasma was demonstrated to be a suitable target for buildup with neutral beams in 2*X*IIB if stabilized with streaming plasma.[16] The major disadvantage of this technique is the difficulty of combining pulsed trapping fields with dc magnetic field reactors.

This paper discusses experiments in 2*X*IIB which show that the streaming plasma used pre-