

limit on the Higgs mass can be reduced below that implied by Eq. (9) to about 3.5 GeV. For the mass range $4.9 \text{ GeV} > M_H > 3.5 \text{ GeV}$, the metastable asymmetric vacuum is essentially stable.

To summarize this example, I state that discovery of a Higgs scalar with any mass above 3.5 GeV would lend strong support for the gauge-theory ideas. If the mass is in the range $3.5 \text{ GeV} > M_H > 4.9 \text{ GeV}$, however, it would suggest vacuum instability; for that case, there follows a "doomsday prediction" since a supercritical vacuum bubble may be created in an ultrahigh-energy collision. Although a reliable calculation is difficult, naive estimates indicate that the required energy density can be closely approached in collisions involving the highest-energy cosmic rays at $\sim 10^{11}$ GeV but is some orders of magnitude greater than the highest artificially generated energy density which is presently attained at the CERN intersecting storage rings.

Upon returning from this model to the general case, it appears that the various field theories,¹ for which the effective potential has been calculated up to the one-loop level, must be re-examined in the light of this new viewpoint. As in the Weinberg-Salam model, it is likely in other quantum field theories that, for a given range of the parameters in the Lagrangian, quite different theories can occur, depending on the arbitrary

choice between absolutely stable and metastable (but practically stable) vacuum states.

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New Determination of the π^- Mass from Pionic-Atom Transition Energies*

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We have measured 29 pionic-atom transitions in eight elements. From the best twelve transitions, we obtain $m_{\pi^-} = 139\,568.6 \pm 2.0 \text{ keV}/c^2$.

Pionic-atom transition energies provide the most accurate determination of the π^- mass (m_{π^-}). By choosing transitions for which correc-

tions to the Klein-Gordon equation are small and relatively well understood, m_{π^-} can be extracted with an accuracy limited principally by statistics

TABLE I. Transition energies and $\Delta m_\pi c^2 = (m_\pi c^2 - 139\,568.6)$ keV for the well-measured transitions.

TRANSITION			THEORY	EXPERIMENT	$\Delta m_\pi c^2$
			(eV)	(eV)	(keV)
1)	Ag	5g-4f	186116.6	186109 ± 3	5.6 ± 2.7
2)		5f-4d	188594.0	188724 ± 90	
3)		4f-3d	406554.6	406662 ± 25	
4)	Cd	5g-4f	194187.0	194187 ± 3	- .05 ± 2.6
5)		5f-4d	196960.2	196954 ± 100	
6)		4f-3d	424494.5	424524 ± 35	
7)	Sn	5g-4f	210844.1	210842 ± 5	1.3 ± 3.6
8)		4f-3d	461610.5	461588 ± 60	
9)	I	6h-5g	128305.2	128315 ± 7	
10)		5g-4f	237140.8	237136 ± 4	2.6 ± 2.8
11)		6g-4f	365065.0	365073 ± 31	
12)	Ba	6h-5g	143339.9	143342 ± 5	-1.4 ± 6.2
13)		5g-4f	265026.8	265023 ± 5	1.7 ± 3.3
14)		6g-4f	407901.5	407965 ± 28	
15)	Au	7i-6h	172237.3	172240 ± 6	-1.8 ± 5.4
16)		7h-6g	173137.3	173064 ± 46	
17)		6h-5g	286853.9	286857 ± 4	-2.2 ± 2.3
18)		6g-5f	290512.9	290620 ± 65	
19)		7h-5g	458304.3	458248 ± 46	
20)	Tl	7i-6h	181136.4	181131 ± 5	3.7 ± 4.5
21)		7h-6g	182130.6	182124 ± 26	
22)		6h-5g	301725.6	301732 ± 4	-3.8 ± 2.2
23)		6g-5f	305918.8	305919 ± 140	
24)		7h-5g	481998.3	481983 ± 49	
25)	Pb	7i-6h	185670.8	185672 ± 4	-1.4 ± 3.7
26)		7h-6g	186715.0	186739 ± 16	
27)		6h-5g	309305.5	309307 ± 7	-1.6 ± 3.4
				309330 ± 7	
28)		6g-5f	313790.7	313476 ± 195	
29)		7h-5g	494072.2	493988 ± 39	

and the precision of the spectrometer used. Shaffer,¹ using a bent crystal spectrometer, reported a value of m_π with an uncertainty of 72 ppm. Backenstoss *et al.*,² using a Ge(Li) spectrometer, achieved an uncertainty of 43 ppm, and recently, Marushenko *et al.*,³ achieved an uncertainty of 15 ppm also using a bent crystal spectrometer. We report here a new determination of m_π based on the precision measurement of twelve transitions with an intrinsic Ge spectrometer and claim for it an uncertainty of 15 ppm. This is achieved by use of recently improved (by an order of magni-

tude) calibration energies (6 ppm), high statistics (6 ppm), and a high-resolution Ge detector.

The experiment was done at the Space Radiation Effects Laboratory (SREL).⁴ The spectrometer system, with a 3.1-cm² intrinsic Ge diode,⁵ had a resolution of 870 eV at 316 keV. The energy calibration was based on twelve standard γ rays,⁶ adjusted for the new measurement by Deslattes *et al.* of the ¹⁹⁸Au (412 keV) line. A quadratic relation between γ -ray energy and channel number was adequate to give a rms deviation for the twelve lines of 3 eV and a χ^2/N of 1.2. Full details of the experimental techniques will be given in a forthcoming paper on precision measurements of muonic-atom transitions.

In all, 29 transitions in the energy range 100 to 500 keV, distributed among eight elements, were measured. Some weak transitions do not contribute significantly to the mass determination but are included for completeness. In Table I we give for each transition the theoretical transition energy based on the best value of m_π from this work, the measured value of the energy, and the differences between the individual pion-mass determinations and their mean.

The theoretical values (Table II) are calculated with use of the techniques described in Watson and Sundaresan.⁷ For the screening we used Vogel's effective potential⁸ obtained from a self-consistent calculation for the electrons (as opposed to the treatment by Tauscher²). The error due to unfilled levels is estimated to be 2 eV; and there is an additional error (assumed to be 1%) due to the polarization of the electron cloud by the pion. Since a number of different transitions were used, this error is treated as statistical.

TABLE II. The Klein-Gordon transition energy and theoretical corrections in eV for the well-measured transitions.

	Klein-Gordon	Uehling	$\alpha^2(Z\alpha)$	$\alpha(Z\alpha)^3$	Finite size	Elec. scrn.	Strong int.	Nucl. polar.
(1)	185 372.6	749.9	5.1	-5.7	-0.1	-12.4	6.3	0.8
(4)	193 397.4	794.3	5.4	-6.2	-0.1	-12.9	8.1	1.0
(7)	209 958.4	887.8	6.1	-7.3	-0.1	-14.0	12.2	1.2
(10)	236 095.1	1039.6	7.1	-9.4	-0.2	-15.9	22.5	2.1
(12)	142 894.5	475.8	3.3	-6.1	-0.0	-28.0	0.1	0.3
(13)	263 800.2	1205.8	8.2	-11.8	-0.5	-18.0	40.4	2.5
(15)	285 696.3	1238.5	8.4	-27.0	-0.0	-69.9	5.3	2.3
(17)	171 772.7	569.7	3.9	-15.5	-0.0	-93.9	0.0	0.4
(20)	180 638.5	611.8	4.2	-17.3	-0.0	-101.4	0.0	0.5
(22)	300 486.9	1325.7	9.0	-30.1	-0.1	-75.9	7.3	2.7
(25)	185 156.1	633.6	4.4	-18.3	-0.0	-105.4	0.0	0.5
(27)	308 025.0	1370.6	9.3	-31.8	-0.1	-79.1	8.6	2.9

The strong-interaction shifts were calculated using the parametrization of Krell and Erickson⁹ as modified by Tauscher.¹⁰ We have used the nuclear-shape parameters c and t from Engfer *et al.*¹¹ for these and the finite-size corrections. Since the strong-interaction shifts may vary by as much as 20% depending on which set of parameters are used, we have taken the theoretical error to be 20% of the correction. The strong-interaction shift depends sensitively on t , which is poorly determined and may vary randomly. Hence we treat this error as statistical. A careful search, revealing no systematic discrepancies between theory and experiment, justifies this treatment.

The overall error was estimated as follows: For each of the well-measured transitions we have calculated a statistical error by adding in quadrature the experimental statistical error, the screening error, and the strong-interaction error. Using this for weighting, we have calculated a best value of m_π with a statistical error Δm_π^{st} . We then added the experimental systematic error to each measured transition energy and calculated an upper estimate m_π^u from the weighted mean. The final quoted error is

$$\Delta m_\pi = [(m_\pi^u - m_\pi)^2 + (\Delta m_\pi^{\text{st}})^2]^{1/2}.$$

The value of χ^2/N is 0.7. The total error comes from calibration (0.9 keV/c²), fitting (1.2 keV/c²), angle effect (0.6 keV/c²), statistics (0.7 keV/c²) and theory (0.6 keV/c²). Using all the transitions or treating the theoretical error as systematic shifts the mass by at most 0.3 keV/c² and increases the error marginally. To obtain m_π we have used the lines with experimental errors less than 8 eV, ignoring the I 6*h*-5*g* transition, which is outside our normal calibration range, and some of the data for Pb 6*h*-5*g*, which is inconsistent with the fitted mass.

These twelve most significant measurements give

$$m_\pi = 139\,568.6 \pm 2.0 \text{ keV}/c^2.$$

This value for m_π is consistent with those given by Shafer (139 566 ± 10) and Backenstoss *et al.* (139 569 ± 6). The more accurate measurement by Marushenko *et al.* gave $m_\pi = 139\,565.7 \pm 1.7$. However, after correcting the $W k_\alpha$ to the Au standard⁶ (11 ppm), using improved electron screening⁸ (6 ppm), and folding in the theoretical error, this latter measurement gives $m_\pi = 139\,568.1 \pm 2.2$ keV/c², in excellent agreement with our result.

It is apparent from an examination of the weak-

er transitions in addition to the lines used above that the Klein-Gordon equation adequately predicts the level energies and, in particular, their l -splittings. Using the recently remeasured value of the muon momentum in $\pi \rightarrow \mu\nu$ decay by Daum *et al.*¹² (29 787 ± 5 keV/c) and assuming the π^+ and π^- masses to be identical (CPT), we find the square of the μ neutrino mass to be $0.22 \pm 0.40 \text{ MeV}^2/c^4$. The contribution to the errors from the uncertainty in p_μ is about 3 times that due to the uncertainty in m_π . Our upper limit of the μ neutrino mass (at 90% confidence level) is $0.86 \text{ MeV}/c^2$.

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Exact Solution for the Influence of Laser Temporal Fluctuations on Resonance Fluorescence

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The influence of the laser fluctuations on resonance fluorescence is treated exactly within the framework of multiplicative stochastic processes. For the phase diffusion model of the laser, I find that, in the limit of strong fields, the ratios of the width and height of the central peak to the side peak are, respectively, $2/3x$ and $3x$, where $x = (\gamma + \gamma_c)/(\gamma + 2\gamma_c)$. The envelope of the intensity correlations is found to decay at the rate $(3\gamma + \gamma_c)/2$.

The theory of resonance fluorescence in single-mode laser fields with definite amplitude and phase is by now fairly standard¹⁻³ and has recently been investigated by several experimental workers.^{4,5} An important problem here having direct bearing on experiments is how the temporal fluctuations of the laser beam affect the characteristics of the resonance fluorescence. In a recent paper,⁶ Eberly has developed a theory to take into account the effect of the amplitude fluctuations of the laser beam on the spectrum of resonance fluorescence. In this Letter, I report an exact theory which takes into account the effect of temporal fluctuations of the laser beam on the spectrum of resonance fluorescence, the antibunching effects, and the evolution of the atomic populations, dipole moment, etc. The theory which I present here is for a single two-level atomic system and is easily generalized to multi-

level systems,⁷ to cases when other relaxation mechanisms, such as collisional relaxation,⁸ affect strongly the scattering. The approach in the present work is also applicable to experiments on level crossing with partially coherent light. The theory should also have applications to laser-induced chemical reactions, since the underlying dynamical equations have very similar structure. From the viewpoint of statistical mechanics, my work provides one example of the very few exactly soluble models.

A fully quantum-electrodynamical theory of resonance fluorescence in nonfluctuating fields is presented in Ref. 2 (see also Refs. 1b, 3). The spectrum calculated there is in agreement with the one calculated by Mollow^{1a} earlier. In this approach, the atomic dynamics of a two-level atom (with energy separation ω) interacting with the zero-point fluctuations and an external laser beam is described by

$$\frac{\partial \rho}{\partial t} = -i(\omega - \omega_0)[s^z, \rho] - \gamma(s^+ s^- \rho - 2s^- \rho s^+ + \rho s^+ s^-) + \frac{1}{2}id[s^+ \mathcal{E}(t) + s^- \mathcal{E}^*(t), \rho], \quad (1)$$

where $\mathcal{E}(t)$ is the slowly varying part of the electric field of the laser beam of frequency ω_0 , 2γ is equal to Einstein A coefficient. The density operator ρ is in a frame rotating with the frequency of the external field. For fluctuating laser beams, the field $\mathcal{E}(t)$ is a stochastic variable and, thus, I have a case of stochastic Liouvillian; and such cases have been treated in a number of approximate ways in the literature.⁹

Here I treat the phase diffusion model of the laser and obtain exact results using the techniques of multiplicative stochastic processes.¹⁰ In this model the temporal phase fluctuations of the laser beam