Vacuum Instability and Higgs Scalar Mass*

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(Received 24 September 1976)

A procedure is given to estimate the decay probability of a metastable vacuum state in quantum field theory. As an application, in the Weinberg-Salam model the lower bound (for $\theta_W = 35°$) on the Higgs mass can be reduced from 4.9 GeV, corresponding to effective stability of a metastable vacuum.

The vacuum state of a quantum field theory is traditionally defined to be that state with zero occupation numbers or, alternatively, that state of lowest energy. The properties of the vacuum state are of special importance when spontaneous breakdown of symmetry occurs, for the symmetry of the vacuum then becomes as significant as the symmetry of the Lagrangian defining the theory.

A convenient method for identifying the vacuum state is to compute the effective potential $V[\varphi]$ which is the generating functional of one-particle irreducible Green's functions for external scalars with vanishing momenta. The vacuum state corresponds to a minimum of the effective potential.

The identification of the vacuum becomes ambiguous when the effective potential has more than one minimum; the standard procedure in the literature[1] for removing this ambiguity is to assert that the correct vacuum must correspond to an absolute minimum and that any nonglobal minimum corresponds to an unstable and hence unacceptable candidate for the vacuum. Our present aim is to query this procedure—for if the probability for decay is sufficiently small (even allowing that the decay can be triggered by quantum fluctuations at an arbitrary space-time point), then a nonglobal minimum of $V[\varphi]$ appears to represent as good a candidate for the vacuum as any other.

Let the effective potential have two (or more) minima and let us suppose that the vacuum state realized by nature is a nonglobal minimum. Then there is a finite probability for quantum tunneling from this vacuum to one of lower energy. In more physical terms, quantum fluctuations may spontaneously generate a "bubble" whose interior is built upon the lower-energy vacuum state. Such a bubble clearly has a negative volume energy but it has a positive surface energy because of the potential barrier involved. Thus, there is a critical size; below this size the bubble will shrink, but above it, there is classical instability with respect to rapid and unlimited expansion.

In order to minimize the surface energy for a given volume, the bubble must be spherical. But we can go further[2] by noting that to calculate the barrier tunneling it is convenient to treat it as the classical motion in the imaginary time direction. The advantage of such a Euclidean description is that the equations of motion become O(4)-symmetric. Thus, we may treat the growth from zero radius to the critical radius by considering a four-dimensional hypersphere. If the radius of the hypersphere is denoted by R, the relevant bubble action is given by

$$A = -\tfrac{1}{2}\pi^2 R^4 \epsilon + 2\pi^2 R^3 S_1, \tag{1}$$

where ϵ and S_1 are the volume and surface energy densities, respectively. The stationary value of this action is

$$A_m = \tfrac{27}{2}\pi^2 S_1^4/\epsilon^3 \tag{2}$$

corresponding to the critical radius $R_m = 3S_1/\epsilon$. Assuming R_m is large compared to the surface thickness (this assumption can be justified *a posteriori* in the example which we consider below), the action per unit hypersurface, S, is given[2] by the action for tunneling through the potential barrier $V(\varphi)$. In imaginary time this is the classical action for motion in the corresponding potential well, $-V(\varphi)$. By considering an infinitesimal change in R, it is easy to see that S_1 is also the surface energy per unit area of the three-dimensional spherical bubble. The energy density ϵ is obtained directly from the difference between the effective potential evaluated for the two vacua.

The number N of such tunnelings that have already occurred in the past will be estimated by

$$N = (V_u \Delta^4)\exp(-A_m), \tag{3}$$

where V_u is the space-time volume of the backward light cone and Δ is the appropriate mass scale of the theory. To determine Δ accurately entails evaluating the one-loop corrections to the WKB approximation, but for the case that we consider it is sufficiently precise to set $\Delta = R_m^{-1}$, the mass whose Compton wavelength is the critical bubble radius.

Although the technique is of general applicability, it is best illustrated by a specific example —the well-known Weinberg-Salam theory of weak interactions. In this theory a metastable asymmetric vacuum state occurs when the Higgs scalar mass is sufficiently small. The effective potential for this model, when the gauge vector loop corrections are included, is given by[3]

$$V[\varphi] = \tfrac{1}{2}\mu^2\varphi^2 - \lambda\varphi^4 + B\varphi^4 \ln(\varphi^2/V^2), \tag{4}$$

where V is the vacuum value of the Higgs field given by

$$V = (G\sqrt{2})^{-1/2} = 248 \text{ GeV} \tag{5}$$

and

$$B = \frac{3}{64}\alpha^2\left(\frac{2+\sec^4\theta_W}{\sin^4\theta_W}\right) = 9.7\times 10^5 \tag{6}$$

for the empirical value of the mixing angle $\theta_W = 35°$. The Higgs mass is given by

$$M_H^2 = V''(v) = 4Bv^2 - 8v^2(\lambda - B). \tag{7}$$

The difference in energy densities between the asymmetric and symmetric vacua is

$$\epsilon = V(v) - V(0) = (\lambda - B)v^4. \tag{8}$$

It follows that if $B > \lambda$, the asymmetric vacuum is absolutely stable; this occurs if the Higgs scalar mass satisfies the inequality

$$M_H > M_{cr} = (4Bv^2)^{1/2} = 4.91 \text{ GeV}. \tag{9}$$

This is the lower bound advocated recently by Weinberg.[3]

We now investigate the stability of the asymmetric vacuum state when this bound is violated, that is,

$$\delta M^2 = M_{cr}^2 - M_H^2 = 8\epsilon > 0. \tag{10}$$

To do this, we need the WKB approximation to the one-dimensional tunneling problem between the two potential minima. The philosophy that we adopt here is to regard $V[\varphi]$ of Eq. (4) as the real potential and calculate the classical motion in $-V[\varphi]$; as previously mentioned, a slightly more accurate estimate for both the tunneling probability and the mass scale Δ in Eq. (3) may be obtained by a more complete treatment of the one-loop corrections, but for small couplings the corrections will also be small and will not alter our general conclusions.

For the barrier penetration, the action is

$$S_1 = \int_{\varphi_1}^{v} d\varphi\,\{2(V[\varphi] - \epsilon)\}^{1/2}, \tag{11}$$

where the lower integration limit is prescribed by $V[\varphi_1] = \epsilon$. Changing variables to

$$r = \delta M^2/M_{cr}^2, \tag{12}$$

$$x = \varphi^2/v^2, \tag{13}$$

one finds

$$S_1(r) = v^3(2B)^{1/2}I(r), \tag{14}$$

where

$$I(r) = \int_{x_1(r)}^{1} dx\tfrac{1}{2}\left[(1+r) - (1+\tfrac{1}{2}r)x + x\ln x - \frac{r}{2x}\right]^{1/2} \tag{15}$$

in which $x_1(r)$ is the solution of the transcendental equation

$$1 + r - (1+\tfrac{1}{2}r)x_1 + x_1\ln x_1 - (r/2x_1) = 0. \tag{16}$$

In terms of the integral $I(r)$, the critical radius $R_m(r)$ is given by

$$R_m(r) = 24(2B)^{1/2}vI(r)/\delta M^2. \tag{17}$$

Finally, in computing the number N [Eq. (3)], we need the space-time volume V_u of the backward light cone; corresponding to a time 10^{10} y, this is $V_u = 10^{164}$ fm^4. Thus, if R_m is in fermis and S_1 in (fermi)$^{-3}$, one has the expression

$$N(r) = \left(\frac{10^{164}}{R_m^4(r)}\right)\exp\left(-\frac{\pi^2}{2}R_m^3(r)S_1(r)\right). \tag{18}$$

The criterion that the metastable asymmetric minimum be a viable candidate for the vacuum will be that $N \ll 1$.

The results of evaluating numerically the integral $I(r)$, and the corresponding values of the critical radius R_m and the probable number, N, of supercritical bubbles created in the backward light cone, are given for several values of M_H in the following table:

M_H (GeV)	$I(r)$	R_m (fm)	N
4.91	0.21	∞	0
4.0	0.064	0.13	$\sim 10^{-80\,000}$
3.6	0.041	0.060	$\sim 10^{-365}$
3.5	0.034	0.047	$\sim 10^{-39}$
3.45	0.032	0.043	$\sim 10^{+20}$
3.4	0.030	0.039	$\sim 10^{+63}$
3.0	0.017	0.018	$\sim 10^{+164}$

From these results, we conclude that the lower limit on the Higgs mass can be reduced below that implied by Eq. (9) to about 3.5 GeV. For the mass range 4.9 GeV $> M_H >$ 3.5 GeV, the metastable asymmetric vacuum is essentially stable.

To summarize this example, I state that discovery of a Higgs scalar with any mass above 3.5 GeV would lend strong support for the gauge-theory ideas. If the mass is in the range 3.5 GeV $> M_H$ $>$ 4.9 GeV, however, it would suggest vacuum instability; for that case, there follows a "doomsday prediction" since a supercritical vacuum bubble may be created in an ultrahigh-energy collision. Although a reliable calculation is difficult, naive estimates indicate that the required energy density can be closely approached in collisions involving the highest-energy cosmic rays at $\sim 10^{11}$ GeV but is some orders of magnitude greater than the highest artificially generated energy density which is presently attained at the CERN intersecting storage rings.

Upon returning from this model to the general case, it appears that the various field theories,[1] for which the effecitve potential has been calculated up to the one-loop level, must be re-examined in the light of this new viewpoint. As in the Weinberg-Salam model, it is likely in other quantum field theories that, for a given range of the parameters in the Lagrangian, quite different theories can occur, depending on the arbitrary choice between absolutely stable and metastable (but practically stable) vacuum states.

The author wishes to acknowledge the hospitality of the Aspen Center for Physics where this work has initiated and thanks the participants there and theoretical colleagues at the University of California at Los Angeles for interesting discussions.

*Work supported in part by the National Science Foundation.

[1]For a review, see P. H. Frampton, University of California at Los Angeles Report No. UCLA/76/TEP/15, 1976 (unpublished).

[2]We thank S. Coleman (private communication) for valuable discussions about the O(4)-symmetric formulation of this problem. The formulation will be spelled out in detail in S. Coleman (to be published), and C. Callan and S. Coleman (to be published). Previous authors have discussed unstable vacua, but without quantitative calculations of the transition probability; see, in particular, T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974); Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun, Zh. Eksp. Teor. Fiz. 67, 3 (1974) [Sov. Phys. JETP 40, 1 (1974)], and Phys. Lett. 50B, 340 (1974).

[3]S. Weinberg, Phys. Rev. Lett. 36, 294 (1976); A. D. Linde, Pis'ma Zh. Eksp. Teor. Fiz. 23, 64 (1976) [JETP Lett. 23, 73 (1976)].

New Determination of the π^- Mass from Pionic-Atom Transition Energies*

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We have measured 29 pionic-atom transitions in eight elements. From the best twelve transitions, we obtain $m_{\pi^-} = 139\,568.6 \pm 2.0$ keV/c^2.

Pionic-atom transition energies provide the most accurate determination of the π^- mass (m_π). By choosing transitions for which corrections to the Klein-Gordon equation are small and relatively well understood, m_π can be extracted with an accuracy limited principally by statistics