the measured ratio C/A versus Q^2. The solid curves display the dispersion-theory prediction for C/A using $F_\pi = (1+Q^2/0.471)^{-1}$ and the value of C/A calculated using the dispersion-theory prediction for C together with the expression (0.025 $sr^{-1})\sigma_{tot}$ for A. The transverse component becomes increasingly important as Q^2 increases.

In conclusion, we have observed that the cross section for the reaction $\gamma_v + p \to \pi^+ + n$ has a strong dependence on ϵ indicating a substantial scalar component. The dispersion theory used to analyze the data in terms of the pion form factor[7] substantially underestimates the contribution of transverse photons at large values of Q^2. The transverse cross section has a much weaker Q^2 dependence than that predicted by the dispersion theory and it is compatible with being the same as that found for the virtual-photon–proton total cross section.

It is known from previous measurements[2,3] that the cross section for single-pion electroproduction has a significant isoscalar component which for fixed W increases with Q^2. The dispersion theory assumes that there is no isoscalar component. The isoscalar component could be contained entirely in the transverse component of the cross section and thus its neglect could partially account for the failure of the dispersion theory to reproduce the observed transverse component.

The data reported here imply that the previous determinations of the pion form factor using dispersion theory are overestimates. The redetermination of the pion form factor and the further analysis of the transverse component will be the subject of a later communication.

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Resonance Criteria and the $^1D_2$ Diproton*

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A relativistic three-body theory of the (2^+) $NN\pi$ system was applied to calculate elastic and inelastic $(^1D_2)$ pp scattering below 1 GeV. Although the amplitude satisfies all standard resonance criteria, it has no pole corresponding to a diproton resonance.

The spectrum of elementary "particles" provides the chief test of strong interaction dynamics. It is therefore critical that resonant phenomena be correctly analyzed and interpreted, i.e., that peaks in mass spectra be properly identified with poles of the S matrix. As noted by Trippe *et al.*,[1] presently acceptable criteria are almost uniquely linked to the behavior of the Argand plot (left-hand circle, "speed" maximizing near the resonance energy), although these criteria are not always applied objectively (e.g., the Z^* controversy[1]). Moreover, these criteria are primarily motivated by simple examples drawn from (multichannel) two-body scattering. Thus, a considerable majority of tabulated inelastic resonances have been analyzed on the basis of a rather crude analogy (related to the isobar model) in which the actual multiparticle structure is entirely suppressed. Paradoxically, this raises the possibility that we are arbitrating fundamental theories on the basis of criteria whose significance is equally in doubt.

Unfortunately, past and present work by this author suggests that multibody effects are not at all negligible in many cases. Recently I showed that the competition of overlapping quasi-two-body channels in the A_1 3π system ($\rho\pi$ vs $\epsilon\pi$) can lead to a true resonance pole on the second sheet, despite the absence of rapid phase behavior usually associated with such a state.[2] The present Letter is motivated by a new and more disturbing result. I show, via a covariant three-body treatment of the (2^+) $NN\pi$ system, that the elastic pp and π^+D amplitudes satisfy the above criteria for a resonance near the $N\Delta$ threshold, *without* an associated S-matrix pole. Together, these results imply that the accepted resonance criteria are neither necessary nor sufficient in the presence of n-body effects.[3]

Aside from the general significance of this result, there is also considerable interest in this particular $NN\pi$ system. Thus, the question of a "diproton resonance" in the $^1D_2$ state has been raised repeatedly by a number of authors.[4] In particular, Suzuki recently applied a very sophisticated N/D technique to this problem and concluded that resonant behavior is "inevitable." Moreover, this state dominates πD scattering throughout the "resonance" region, and is vital to a better understanding of pion dynamics. In this context, a three-body treatment possesses an immense advantage because pion production and NN dynamics are described simultaneously by the same theoretical framework. Although practical relativistic treatments of the Faddeev type require a very specific (separable) form for the off-shell πN t matrix, this can be avoided by employing the quite general boundary condition formalism (BCF) introduced by this author.[2,5]

As noted by Amado,[6] a minimal scheme for unitarizing three-particle amplitudes must take the form of a one-dimensional integral equation. The basic idea of the BCF is to isolate the primary (model-independent) singularities characterizing the associated kernel, and to treat the (relatively smooth) remainder phenomenologically. A great deal of specific information concerning the BCF is now available in the literature[5]; to avoid repetition, we restrict details to the application at hand. However, in this context it should be stressed that, nonrelativistically, it is at least as general as an arbitrary combination of two- and three-body potentials.[7] Less rigorously, one may argue that the covariant version employed here provides an equally comprehensive (effective) description of meson exchange, and hence that our conclusions are not specific to the BCF.

To describe a state of three (mass-shell) particles, we let $(\pm)\vec{p}_\alpha$ be the momentum of particles β and γ in their c.m. frame, and let $\vec{q}_\alpha$ be the momentum of particle α in that frame ($\alpha \neq \beta \neq \gamma$). In a partial-wave decomposition we couple $\vec{l}_\alpha(\vec{p}_\alpha)$ and $\vec{\lambda}_\alpha(\vec{q}_\alpha)$ by taking $\vec{j}_\alpha = \vec{l}_\alpha + \vec{\sigma}_\alpha$, where σ_α is the total spin of the $\beta\gamma$ pair, and forming states of definite channel spin $\vec{S}_\alpha = \vec{s}_\alpha + \vec{j}_\alpha$, where s_α is the spin of particle α. We subsequently couple $\vec{S}_\alpha$, $\vec{\lambda}_\alpha$ to form a state of total J. In such a description the physical states correspond to $p_\alpha = \kappa_\alpha(q_\alpha, s)$, where κ_α is the on-shell c.m. momentum defined by the condition $P^2 = s$ (P is the total four-momentum). Two-particle scattering is described by the on-shell amplitude $t_{\alpha l}(\kappa_\alpha)$, which is taken as input to the three-body problem.

However, since κ_α ranges over unphysical values in our integral equation and $t_{\alpha l}(\kappa_\alpha)$ will in general possess a left-hand cut, it is useful to employ the particular representation $t_{\alpha l} = N_{\alpha l}/D_{\alpha l}$, where

$$N_{\alpha l}(\kappa_\alpha) = f_{\alpha l}(\kappa_\alpha^2) j_l(a_\alpha \kappa_\alpha) + a_\alpha \kappa_\alpha j_{l+1}(a_\alpha \kappa_\alpha), \quad D_{\alpha l}(\kappa_\alpha) = i\kappa_\alpha [f_{\alpha l}(\kappa_\alpha^2) h_l(a_\alpha \kappa_\alpha) + a_\alpha \kappa_\alpha h_{l+1}(a_\alpha \kappa_\alpha)]. \tag{1}$$

This is equivalent to an energy-dependent boundary condition at a characteristic range $a_\alpha = |\vec{r}_\beta - \vec{r}_\gamma|$; $f_{\alpha l}(\kappa_\alpha^2)$ is essentially the logarithmic derivative, and is taken to be a meromorphic function of κ_α^2 fitted to the data. Here the πN P_{11} and P_{33} phases were taken from Carter, Bugg, and Carter, and the NN $^3S_1$ phase from MacGregor, Arndt, and Wright (MAW)[8]; these correspond to $a_\alpha = 0.22$, 0.19, and 0.86 fm, respectively. In practice, κ_α^2 ranges from a minimum $\bar{\kappa}_\alpha^2 = -\min(m_\beta^2, m_\gamma^2) \equiv \kappa_\alpha^2(Q_\alpha, s)$ to its physical maximum. Below we use $N_{\alpha l}^c$ to denote $N_{\alpha l}$ with $f_{\alpha l}(\kappa_\alpha^2)$ replaced by $f_{\alpha l}(\bar{\kappa}_\alpha^2) \equiv f_{\alpha l}^c$ (note that $\kappa_\alpha \to \bar{\kappa}_\alpha$ as $q \to Q_\alpha$).

In the present application, we consider three distinct pair-plus-spectator configurations coupled to $J^P = 2^+$, $I = 1$, and labeled by $j = 1, 2, 3$: (1) $^3S_1(NN) + \pi(\lambda = 1)$, (2) $P_{11}(\pi N) + N(\lambda = 2)$, (3) $P_{33}(\pi N) + N(\lambda = 0)$; these correspond to the πD, NN, and $N\Delta$ channels, respectively. Antisymmetrizing the nucleon coordinates yields three coupled integral equations of the form

$$X_{ik}(q_i) = N_{ik}(q_i, q_k^0) + \sum_j \int_0^{Q_j} dq\, q_j^2 N_{ij}(q_i, q_j) t_{jl_j}(\kappa_j) X_{jk}(q_j)/N_{jl_j}^c(\kappa_j), \quad (2)$$

where q_k^0 corresponds to the initial state; $D_{jl_j}(\kappa_j(q_j^0, s)) = 0$. Thus, for an initial pp state $k = 2$ with the πN pair taken at the nucleon mass (the P_{11} t matrix explicitly exhibits the nucleon pole.) Prescriptions for forming physical amplitudes are identical with those of the Faddeev theory[9]; e.g., up to a normalization, $X_{22}(q_2^0)$ is the elastic pp amplitude.

The function N_{ij} is expressed as $N_{ij}(q_i, q_j) = N_{ij}^s(q_i, q_j) + A_{ij}(q_i, q_j)$, where N_{ij}^s is completely specified by the two-body input in terms of the a_j, $f_j(\kappa_j^2)$ parametrization, and contains the primary three-body singularities. Conversely, A_{ij} is a smooth, real-valued function which is analytic in some strip $|\mathrm{Im}q_i| < \mu$, $|\mathrm{Im}q_j| < \mu$, where μ is a mass characteristic of the exchange forces. Thus, although in general A_{ij} is a complicated functional of the interaction dynamics, its structure is suppressed in the vicinity of the real q_i, q_j axes, and it may be represented by a relatively simple phenomenological form. In this case I took

$$A_{ij}(q, q') = \Lambda_{ij}(s) g_i(q) g_j(q') N_{jl_j}^c(\kappa_j')^{-1};$$
$$g_j(q) = j_{\lambda_j}(q/2\mu_j)(1 + q^2/\mu_j^2)^{-1}, \quad (3)$$

with $\mu_1 = 3m_\pi$, $\mu_2 = \mu_3 = 2m_\pi$. This choice embodies the general features deduced in Ref. 7, but is otherwise arbitrary. However, our purpose here is not to explore a specific dynamical model, but to construct a set of unitary, analytic amplitudes *consistent* with a plausible dynamics. Thus, effects corresponding to different choices of $g_j(q)$, more general forms of A, and neglected channels were simulated by adjusting the (real) coefficients $\Lambda_{ij}(s)$.

As in previous work, the physical amplitudes connecting the various asymptotic states (pp, π^+D, $NN\pi$) were found to be highly correlated, and hence all parametrizations which generated (for example) the $^1D_2$ phase in the elastic region, and identical values for $\sigma(pp \to \pi^+D)$ at $T_L = 375$ MeV and $T_L = 650$ MeV, produced very similar values of *all* observables for $T_L < 900$ MeV. It thus suffices to report the results for the choice $\Lambda_{21} = 2\Lambda_{12} = -\gamma/\sqrt{5}$ and $\Lambda_{31} = 2\Lambda_{13} = \gamma$ (suggested by the properties of N_{ij}^s). Here $\gamma(s) = \alpha(W - W_0)(W + W_0 - 2W_1)$, where W is the c.m. kinetic energy. Parameter values for three examples are given in Table I; corresponding values for the $^1D_2$ phase shift, reflection parameter (η), and the 2^+ contribution to $\sigma(pp \to \pi^+D)$ are plotted in Fig. 1. Experimental points for $\delta(^1D_2)$ were taken from the energy-independent analysis of MAW,[8] while points for $\sigma(pp \to \pi^+D)$ were taken from the compilation of Richard-Serre *et al.*[10] The theoretical curves agree very well with the former except at the highest energies, where the data are relatively incomplete. With respect to the latter, one must keep in mind that contributions from other J^P states are not entirely negligible. In fact, both a similar three-body calculation by this author[11] and an independent estimate[12] indicate that the s-wave πD channel is sizable, and suggest that the middle (solid) curve provides the best description

TABLE I. Parameters defining the A operator described in the text.

		$\gamma(s)$ parameters	
Model	α	W_0 (fm^{-1})	W_1 (fm^{-1})
1	−1.973	−0.318	0.353
2	−2.389	−0.279	0.329
3	−2.531	−0.304	0.279

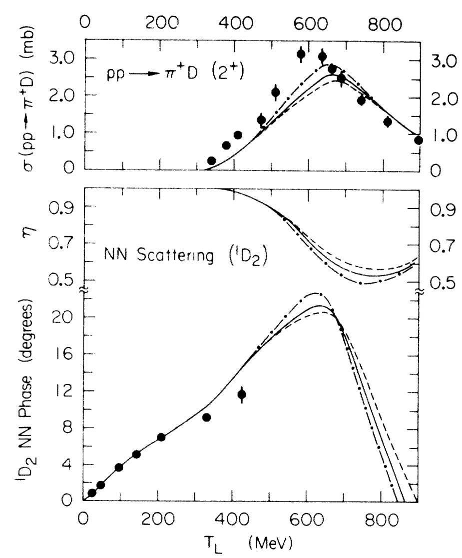

FIG. 1. $^1D_2$ phase parameters and $\sigma(pp \to \pi^+D)$. The dashed, solid, and dash-dotted curves correspond to models 1, 2, and 3 of Table I, respectively. Experimental points shown in the lower (upper) figure were taken from Ref. 8 (10).

of the 2^+ contribution.

Remarkably, the results shown in Fig. 1 bear a striking resemblance to the very different calculation of Suzuki[4] (involving some fourteen fitted parameters). In fact, if the Λ_{ij} parameters are adjusted to saturate $\sigma(pp\to\pi^+D)$, the resulting curves for δ and η are virtually identical to those of Suzuki (under the same assumption). In this respect it should be noted that he also fitted $\sigma(pp\to N\Delta)$; because of the correlations built into the three-body approach, this channel is automatically correct once the $pp\to pp$ and $pp\to\pi^+D$ reactions are fitted. Similarly, the dominant (P_2) πD elastic amplitude is predicted correctly (e.g., a phase of 6° at 48 MeV in agreement with the result of Thomas,[9] and differential cross sections at 182 and 234 MeV reported earlier[11]).

Suzuki's calculation demonstrates that a quasi-two-body description is not only appealing physically, but can (with sufficient finesse) yield correct *physical* amplitudes over some domain. However, the underlying simplification is also reflected in the analytic structure of the amplitudes. Thus, Suzuki was led to predict an associated resonance pole, whereas the present work unequivocally does not. *Nevertheless, both approaches yield* $^1D_2$ *Argand plots which satisfy the resonance criteria.*[13] The reason is simply that the richer singularity structure of the three-body description permits an alternative, nonresonant, representation of the amplitude. In the present case, a resonance pole corresponds to a complex zero of the Fredholm determinant $D(s)\equiv|1-K|$, which has a three-body branch point at $W=0$ in addition to the NN, πD, and $N\Delta$ thresholds. Although $\mathrm{Im}D(s)$ does vanish near $W=200$ MeV, $\mathrm{Re}D(s)$ rises from unity near $W=0$ to a *maximum* of about 1.6 near $W=182$ MeV (the $N\Delta$ threshold). Explicit analytic continuation via Eq. (2) verifies that no nearby zero is present. Here it should be noted that $D(s)$ is quite distinct from the corresponding determinant in Suzuki's work; e.g., if we express the calculated amplitudes as $\tau_{ij}=M_{ij}/D(s)$, the numerator function M_{ij} has a quite different structure in the two approaches (and is *complex* in our case even for a one-channel problem).

A detailed study of this example (and the correlated πD state) suggests that what the multichannel approach misses is the strong *energy-dependent* mixing of the two inelastic channels; i.e., at the three-body level, the πD and $N\Delta$ configurations of the 2^+ $NN\pi$ system are remarkably similar over an extended region for energies s near the $N\Delta$ threshold. Geometrically, since the deuteron is so lightly bound and almost a pure s state, the p-wave πD state is almost equivalent to individual πN p waves, and the (average) spin and isospin projections of the three particles are identical. In the Faddeev theory, this "overlap" would be calculated in terms of bound-state (resonance) form factors for the $D(\Delta)$ systems, folded over a free three-body propagator. Taken on-shell, this overlap constitutes the lowest order contribution to the $N\Delta\leftrightarrow\pi D$ transition amplitude, and is (logarithmically) *singular* near the $N\Delta$ threshold. As I noted previously, this rescattering (or Peierls) singularity dominates πD scattering and is the origin of what is erroneously called the (3, 3) "resonance" in nuclei.[14] Physically, the rescattering singularity can be visualized in terms of an almost asymptotic $N\Delta\leftrightarrow\pi D$ transition via nucleon exchange.[15] Although present in each partial wave, the strength of this effect is dependent on the effective overlap, as is illustrated by the dominance of the 2^+, as compared to the 1^+, πD p wave.

The suggestion, therefore, is that this strongly s-dependent mixing is shadowed even in the virtual $N\Delta$ and πD states relevant to NN elastic scattering, and is poorly represented by the multichannel approach. Although this effect is especially important in this example due to the small deuteron binding, it should be noted that the A_1 analog ($\rho\pi\leftrightarrow\rho\pi$) occurs at $\sqrt{s}=1100$ MeV, and that in the Q system one has both ($K^*\pi\leftrightarrow K^*\pi$) at 1180 MeV, and ($K^*\pi\leftrightarrow\rho K$) at 1280 MeV. The implication for resonance analysis is clear: It is essential to employ more sophisticated techniques which do better justice to the n-body degrees of freedom. A reliable analysis should thus include at least the following ingredients: (1) The rescattering terms should be explicitly included; (2) a unitary n-body representation of the amplitude should be constructed in terms of fitting parameters which are not themselves strongly s dependent; and (3) the representation should permit explicit analytic continuation to verify the existence of a pole. In contrast, phase criteria should be abandoned.

With regard to NN and πN dynamics, it is worth noting that the A operator required to modify the inelastic pp cross section (Fig. 1) effectively alters the $N\Delta/\pi D$ overlap; in a Faddeev theory this would require a modification of the off-shell P_{33} amplitude. Thus, *inelastic* pion scattering data *below* the (3, 3) "resonance" should prove particularly valuable in probing this basic interaction mechanism. Although *all* πD cross sections are consistently raised (or lowered) by this modification, elastic data at or above resonance are far less sensitive.[11]

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SU(4) Multiplet Mixing

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The consequences of mixing between SU(4) multiplets of the same J, P, and C are examined. Within the proposed theoretical framework, the presently available data indicate that there should be substantial mixing between the pseudoscalar multiplets (P and P') while the vector multiples (V and V' containing ψ and ψ') remain largely unmixed. The ratio $F_P/F_{P'}\sim 2$ of the pseudoscalar decay constant leads to the suppression of the decays $\psi\to\eta_c\gamma$ and $\psi'\to\eta_c\gamma$.

While the SU(4) approach to the charm interpretation of the J/ψ family of particles has had both quantitative and qualitative success,[1-7] there have been several notable problems. They are the following:

(1) The symmetry-breaking Hamiltonian[1]

$$\mathcal{H}=U_0+U_8+aU_{15}\,, \tag{1}$$

for the masses demands[2] that the usual SU(3) mixing angle θ be 35.3° if $\psi(3100)$ or its pseudoscalar analog is to be pure $(c\bar{c})$. Thus, the mixing angles obtained by a fit to the pseudoscalar meson masses predict[2,7] gross leakage of $(c\bar{c})$ into η and X^0.

(2) The estimates for the mass of the recently discovered[8] charmed pseudoscalar $D(1865)$ are too high by several hundred MeV for the quadratic mass formula.[1-3,7]

(3) There must be substantial SU(4) breaking of coupling constants[4-6] $g_{VP\gamma}$ in order to suppress the decay $\psi\to\eta_c(2800)\gamma$.

Because these problems have not yet arisen with the baryons, but only with the pseudoscalar mesons, we must seek a solution within SU(4). Rather than go the route of introducing extra symmetry-breaking terms in the mass matrix[9] or coupling constants,[4-6] we choose to examine the effects of SU(4) multiplet mixing.

There now appear to be two pseudoscalar multiplets,[10] π, K, η, X^0, $\eta_c(2800)$ (which we denote by P) and $K'(1400)$,[11] $E(1420)$, $\eta_c'(3455)$[12] (P', partially complete); and two vector multiplets, ρ, $K^*(892)$, ω, φ, ψ (denoted by V) and $\rho'(1600)$, $\psi'(3700)$ (V', partially complete). Since the mass splitting within a given multiplet is greater than the difference in average masses of the multiplets, there could be significant effects due to intermultiplet mixing.[13]

Now, the mass matrix elements between states of the same multiplet, say multiplet 1, generated by Eq. (1) contain both the symmetry-breaking parameter a and four reduced matrix elements M_1, $M_1{}^0$, A_1, and B_1 (see Refs. 1 and 2 for notation). Similarly M_2, $M_2{}^0$, A_2, B_2 are introduced for the second multiplet, and T, T^0, A_T, B_T for the cross terms. We set $T=-A_T(1$