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Multimuon Production in Deep-Inelastic Muon Scattering*

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We present the characteristics of a model calculation of multimuon production in deep-inelastic muon scattering. In the model the muons are assumed to originate from the production and subsequent weak decay of a pair of hadrons that carry new quantum numbers and have a mass of ~1.8 GeV. The results of the calculation are to be compared with the forthcoming experimental data, and could shed light on the properties of the new hadrons.

Until the recent discovery[1] of a $K\pi$ resonance at 1.86 GeV, the existence of a family of hadrons with a new quantum number[2] has only been indirectly inferred by experiments. Among these is the high-energy neutrino experiment in which dimuon events are observed.[3] Now, with the observation of the 1.86-GeV resonance, the existence of hadrons with a new quantum number is directly confirmed, and the usefulness of the dimuon events in the neutrino experiment will be in the investigation of the weak-interaction properties of these new hadrons. Detailed theoretical studies[4] have been carried out to interpret the dimuon events.

Recently, Chen[5] has reported on the observation of multimuon events in a deep-inelastic muon scattering experiment carried out at Fermilab. These events should provide valuable information on the properties of the new particles. Although the multimuon events in the muon scattering experiment are potentially less informative than the dimuon events in the neutrino experiment, they have certain advantages over the latter. One is that the kinematics of the muon beam is by far better defined than that of the neutrino plus antineutrino beam. Another is that the mechanism for the electroproduction of these new hadrons, at this stage of the development, is perhaps less uncertain theoretically than the production mechanism in the neutrino reactions. The multimuon events in muon scattering could therefore provide valuable information, with good statistics, about the leptonic or semileptonic decay properties of the new hadrons. We report in this Letter on the findings of a theoretical model calculation. They are to be compared with the experimental data, which are forthcoming.

Our model consists of the following assumptions:

(i) There is produced an associated pair of hadrons, charged or neutral, with new quantum numbers, the conservation of which is violated only by weak interactions. These subsequently undergo weak decay into muons. *For definiteness, we shall take the mass to be* $M_c = 1.8$ *GeV*, and refer to the new quantum number as "charm,"[2] in its generic sense.

(ii) We shall interpret the observed apparent positive scaling violation[6] at small $x = Q^2/2M\nu$ as due to charm production. Since the rise[7] of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ had been revealed for the range of center-of-mass energies from 3 to 5 GeV, it was conjectured[8] that a positive scaling violation should take place at small x in electroproduction. Now that the rise of R is established beyond reasonable doubt to be due to charm production,[1] it is natural, and perhaps compelling, to associate likewise with charm production the positive scaling violation observed at small x. To be specific, we assume that charm production takes place predominantly at $x < \frac{1}{15}$. In this region of small x, the structure function $F(x, y)$, in the scaling limit, is assumed to be independent of x, and its dependence on y is taken to be $1 - y + \frac{1}{2}y^2$, corresponding to $\sigma_L/\sigma_T = 0$. There are two more effects to be taken into account. These are the experimentally observed nonscaling behavior at small x, which we approximate by $Q^2/(Q^2 + 1.8^2)$, and the threshold effect. Let $s = (P + q)^2 = 2M\nu - Q^2 + M^2$, which is the square of the hadronic invariant mass. The threshold factor is, in general, a function of $(s - s_0)/s$, where s_0 is the value of s at the charm threshold. For a pair of charmed hadrons to be produced, $s_0 = (M + 2M_c)^2$. To be specific, we take[9] the threshold factor to be $[(s - s_0)/s]^3$. Thus, we assume

$$F_{\text{charm}}(x, y) = A\,\frac{Q^2}{Q^2 + (1.8)^2}\left(\frac{s - s_0}{s}\right)^3 \tfrac{3}{2}(1 - y + \tfrac{1}{2}y^2). \tag{1}$$

In the scaling limit, the integral of this function over y is equal to A, which is indicated by experiment[6] to be on the order of 0.1.

(iii) Since the electroproduction of hadrons at small x is similar in many characteristics[10] to hadron production in hadron-hadron scattering, hadron phenomenology will be assumed for charm production in muon scattering. Specifically, we shall assume a factored longitudinal (defined by the virtual photon) and transverse momentum distribution for the normalized inclusive production cross section of the charmed hadron (with longitudinal momentum $p_\parallel$ and transverse momentum $p_\perp$ in the target frame), of the form

$$f(p) = \mathfrak{N} \exp(-az) \exp(-bp_\perp^2), \tag{2}$$

where $z = p_\parallel/\nu$, and $\mathfrak{N}$ is a normalization factor. *We shall choose*[11] $a = 3$ *and*[12] $b = 6/(Q^2 + 1)$.

(iv) For the decay of the charmed hadron (with momentum p) into a muon (with momentum k), we take

$$E_k \frac{1}{\Gamma}\frac{d\Gamma}{d^3k} = \begin{cases} \pi^{-1}\delta((p-k)^2) \quad \text{(leptonic)}, & \text{(3a)} \\[2ex] F^{-1}(p\cdot k)\dfrac{[(M_c^2 - M_K^2) - 2(p\cdot k)]^2}{M_c^2 - 2(p\cdot k)} \quad \text{(semileptonic}^{13}\text{)}, & \text{(3b)} \end{cases}$$

where F is a normalization factor and depends only on the masses. We understand that (3a) applies to the purely leptonic decay of a vector particle and (3b) applies to the semileptonic decay of a charmed meson or baryon.

With these assumptions, the differential cross section for trimuon production, via

$$\begin{aligned}\mu^+(E) + N(P) \to \mu^+(E_f) + \bar{U} + U + X,&\\ &\hookrightarrow \mu^-(k) + \cdots\\ &\hookrightarrow \mu^+(k') + \cdots\end{aligned} \tag{4}$$

is given by (B being the branching ratio)[14]

$$\frac{d\sigma}{dx\,dy\,d^3k\,d^3k'} = ME\,\frac{8\pi\alpha^2}{Q^4}\,F_{\rm charm}(x,y)\int d^3p\,d^3p'\,f(p)f(p')\left(\frac{B}{\Gamma}\frac{d\Gamma}{d^3k}\right)\left(\frac{B}{\Gamma}\frac{d\Gamma}{d^3k'}\right). \tag{5}$$

Some of the characteristic distributions which are obtained on the basis of (5) *for beam energy* $E = 150$ *GeV* are given in Figs. 1–4. The numerical computation takes into account the constraints $x < \frac{1}{15}$ and $Q^2 > 1$ GeV^2, as well as kinematical constraints such as $(q+P-p)^2 \geq (M+M_c)^2$, which insures the production of both U and $\bar{U}$. To compare the experimental data, constrainsts imposed by experimental acceptance and data cuts will have to be incorporated.

In Fig. 1 we display the y distribution. The structure in the small-y region reflects the interplay between $1 - y + \frac{1}{2}y^2$ and the threshold factor $(s - s_0)^3/s^3$ in (1).

In Fig. 2 we display the energy distribution of the muon (k), which carries an opposite charge to the incident muon. Figure 3 displays the E_k/E_f distribution, which expresses detailed energy asymmetry between the primary (E_f) and secondary (E_k) muons. These distributions are sensitive to the value of a in (2), and therefore, to a large extent, reflect the production mechanism of the hadrons. Only at small energies do the two decay mechanisms (3a) and (3b) make a difference in the energy distribution. It is seen from Fig. 3 that there is considerable asymmetry between the primary muon (E_f), which is fast, and the secondary muon (E_k), which is slow. We have also calculated the ratio of the average energies $\langle E_f\rangle/\langle E_k\rangle$, which is given by

$$\langle E_f\rangle/\langle E_k\rangle = \begin{cases} 75/17 = 4.4 \text{ (leptonic)},\\ 75/12 = 6.3 \text{ (semileptonic)}.\end{cases} \tag{6}$$

In Fig. 4 we display the perpendicular-momentum ($k_\perp$) distribution of the secondary muon, where the perpendicular direction is defined to be perpendicular to the plane of the incident (E) and scattered (E_f) muons, and the transverse-momentum (k_t) distribution, where the transverse direction is defined with respect to the virtual photon. It is seen that these distributions, and especially the $k_\perp$ distribution, are quite sensitive to the decay mechanism and are therefore useful for revealing whether the purely leptonic or the semileptonic decay is the dominant mechanism. *If a purely leptonic decay mechanism is determined by experimental data to be dominant or important, then a decaying vector particle is necessarily implied.* This in turn is likely to imply that vector states are lighter than pseudoscalar states in the family of charmed hadrons. On the other hand, if a semileptonic distribution is favored by experi-

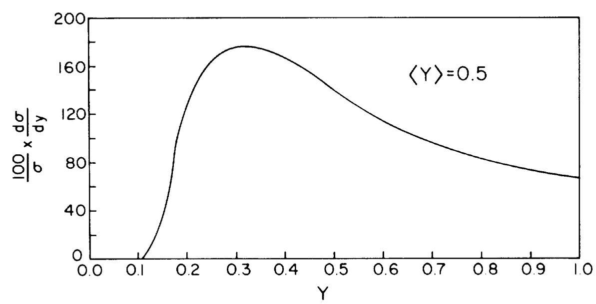

FIG. 1. y dependence of the model.

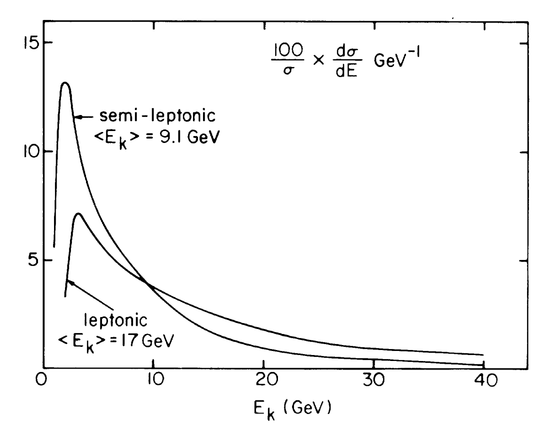

FIG. 2. Normalized energy spectrum of the secondary muon resulting from leptonic or semileptonic decays.

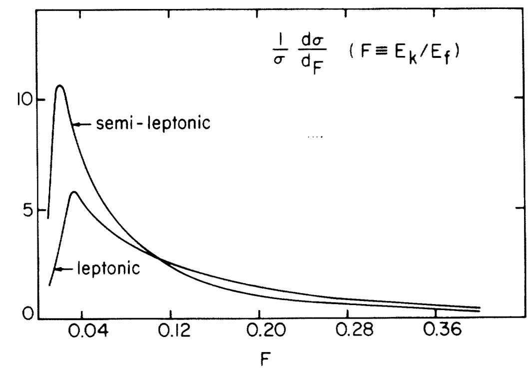

FIG. 3. Distribution of E_k/E_f, where E_k and E_f are, respectively, the energies of the secondary and scattered primary muons.

ment, one would not be able to draw firm conclusions, although a decaying pseudoscalar meson would be a more natural candidate.

We have also calculated the two-muon invariant-mass distributions. They do not seem to contain any distinctive information, and are not shown. They essentially serve as background distributions for resonance searches in the two-muon channels.

We conclude with the following remarks: (a) Our calculation obviously applies to the case where the secondaries are electrons instead of muons. (b) The observation of multimuon events in electroproduction at small $x = Q^2/2M\nu$ strongly supports the interpretation that the positive scaling violation observed at small x values is due to charm production. It is perhaps less natural to attribute the violation to asymptotic freedom.[15] (c) According to present theoretical thinking, the neutral-current weak interaction should not be important for the multimuon production in muon experiment.

A more detailed discussion of the model and the calculation will be published elsewhere.

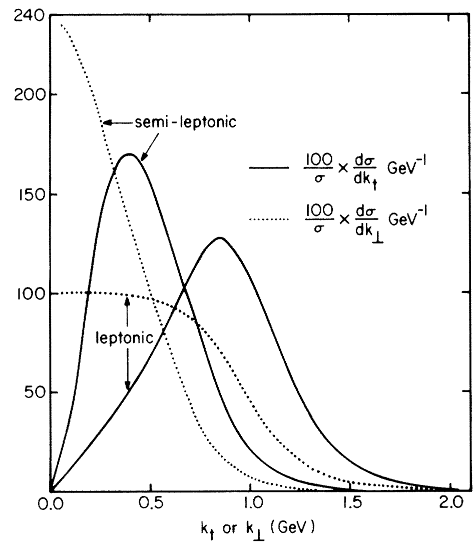

FIG. 4. Perpendicular-momentum ($k_\perp$) and transverse-momentum (k_t) distributions of the secondary muon resulting from leptonic or semileptonic decays of a charmed hadron. The perpendicular direction is defined to be perpendicular to the plane of the incident and scattered primary muons, while the transverse direction is defined with respect to the virtual photon direction (longitudinal). The averages are $\langle k_\perp \rangle = 0.5$ GeV (leptonic); $\langle k_\perp \rangle = 0.4$ GeV (semileptonic); $\langle k_t \rangle = 0.8$ GeV (leptonic); $\langle k_t \rangle = 0.6$ GeV (semileptonic).

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[11]Our choice for the value of a is guided by the recent results of a muon experiment at Fermilab. See W. A. Loomis *et al*. Phys. Rev. Lett. 35, 1483 (1975); H. L. Anderson *et al*., Phys. Rev. Lett. 36, 1422 (1976). Our variable z and the variable x' used in these two papers are essentially the same for not too small values of z.

[12]Our choice for b is guided by the experimental results of Ref. 10, and reflects the thinking that the transverse size of the virtual photon shrinks with Q^2. See, e.g., J. D. Bjorken, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Ithaca, New York, 1971*, edited by N. B. Mistry (Cornell University, Ithaca, N. Y., 1971).

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Comparison of π^+ and π^- Electroproduction at 0° and 90° Center-of-Mass Angles*

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We report measurements of the ratio of positively charged to negatively charged pions electroproduced at 0° and 90° in the virtual photon-nucleon center-of-mass system. The π^+/π^- ratio is studied as a function of W, Q^2, ω, x_T, and x in the range 2.1 GeV $\leq W \leq$ 3.2 GeV and 1.2 GeV$^2 \leq Q^2 \leq$ 9.5 GeV^2.

The Feynman parton-quark model[1] predicts that for pions produced along the direction of the virtual photon in virtual-photon–proton collisions the ratio of the number of positively charged to negatively charged pions should be a function only of ω as W and Q^2 vary, and that it should increase as $1/\omega$ increases. Available data for forward produced pions are consistent with either an ω or W dependence but are not of sufficient quality or range to make a clear choice.[2,3] Although there are no explicit predictions for the charge ratio at 90° in the virtual-photon–nucleon center-of-mass system, it is presumably given by the constituent-interchange model.[4] It is important to determine experimentally the π^+/π^- ratio over a wide kinematic range to test these models further and to compare the behavior at 90° with that at 0°.

We report here new measurements of the π^+/π^- ratio for the electroproduction reaction

$$e + p \to e + \pi^{\pm} + \text{anything} \tag{1}$$

for pions produced at 0° and 90° in the virtual-photon–proton center-of-mass system, and for the reaction

$$e + n \to e + \pi^{\pm} + \text{anything} \tag{2}$$

for pions produced at 0°. Reactions (1) and (2) are analyzed in terms of the virtual photoproduction reaction[5]

$$\gamma_v + N \to \pi^{\pm} + \text{anything}. \tag{3}$$

The square $-Q^2$ of the mass, energy ν, direction, and polarization parameter ϵ of the virtual photon are tagged by the scattered electron. The π^+/π^- ratio is studied as a function of W (the total energy of the virtual-photon–target-nucleon system), Q^2, $\omega = 2M\nu/Q^2$, and the center-of-mass variables $x_T = p_T/p_{\max}^*$ and $x = p_{\parallel}^*/p_{\max}^*$. Here p_T and $p_{\parallel}^*$ are the components of the pion momentum transverse and parallel to the virtual photon direction and $p_{\max}^*$ is the maximum momentum kinematically allowed for the π^+ reaction.

Data were taken with the two-arm spectrometer system described earlier.[6] Shower counters were used to identify electrons. Pions were identified by a combination of threshold Cherenkov counters and time of flight. The uncertainties in the figures are statistical only. Systematic corrections to the π^+/π^- ratios are estimated to be less than 1%.

Figure 1 shows the x_T dependence of the π^+/π^- ratio for the eight (W, Q^2) points taken for pions produced near 90° in the virtual-photon–proton center-of-mass system with a hydrogen target.