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Can Asymptotic Freedom Explain the Neutrino Anomalies?*

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We estimate renormalization effects on $\sigma_{\bar{\nu}}/\sigma_\nu$ and $\langle y\rangle_{\bar{\nu}}$ in the standard four-quark model, improving on a calculation by Altarelli, Petronzio, and Parisi. Even with systematic overestimates, the Weinberg-Salam model does not give a satisfactory account of the data.

The outstanding feature of charged-current reactions at high incident neutrino energies is the unmistakable rise in $R=\sigma_{\bar{\nu}}/\sigma_\nu$ and $\langle y\rangle_{\bar{\nu}}$.[1-3] This suggests that there may be new quarks and new currents. Such possibilities have been studied in the context of the parton model,[4] so successful at describing lower-energy scattering. An alternative explanation has been proposed by Altarelli, Petronzio, and Parisi.[5] They point out that asymptotic freedom implies such effects. These result from scaling violations predicted for any model.[6,7] Their simplified calculation falls shy of accounting for the data but is close enough to merit further consideration.

We have reconsidered the problem. All our approximations are conservative in the following sense: We wish to determine whether the standard four-quark model, with left-handed currents,[8] admits a large enough increase in R and $\langle y\rangle$ with increasing E. Each complication we ignore would only make the predicted increases yet smaller. We conclude that the standard model is inadequate to explain the data.

In parton language, the effects of asymptotic freedom can be described as follows. The parton distribution functions depend weakly on Q^2 as determined by integro-differential equations. These reflect interaction corrections to the impulse approximation.

Our first approximation is to factorize the x and Q^2 dependence. We assume that the u-quark distribution function satisfies

$$u(x,Q^2)\simeq u(x)U(Q^2), \tag{1}$$

where

$$U(Q^2)\equiv\int_0^1 u(x,Q^2)x\,dx, \tag{2}$$

and similarly for $\bar{u}$, d, $\bar{d}$, s, $\bar{s}$, c, $\bar{c}$, and gluon. These Q^2-dependent functions satisfy coupled differential equations, and their sum is constrained to be 1 (the energy-momentum sum rule). All quark and antiquark functions approach a common value as $Q^2\to\infty$. In particular, antiquark functions grow while valence functions decrease. This relative growth of antiquark functions implies an increase in R and $\langle y\rangle$.

Factorization [Eq. (1)] is a conservative approximation because the exact theory implies that the mean x of the structure functions decreases with increasing Q^2. For a given increase in E, the factorization overestimates the increase in average Q^2 because $Q^2=2mExy$. Consequently, we are overestimating the scale-breaking effects.

We need the parton distributions as functions of x to convert Q^2 dependence into E dependence. We assume the standard shapes given by Barger *et al.*[9]

Strictly speaking, we should use ξ, a variable depending on Q^2, E, y, the struck-quark mass, and the final quark mass, rather than x.[6] For production of heavy quarks, we have included the heavy-quark-mass dependence in the appropriate

ξ variable, which summarizes the kinematic threshold effects. A consequence is that the effective threshold for charm production is higher than the nominal threshold by a factor $1/\langle\xi\rangle$, where $\langle\xi\rangle$ is the mean ξ of the struck-quark distribution. Here is another way in which ignoring the decrease of $\langle\xi\rangle$ with Q^2 overestimates the increase in R and $\langle y\rangle$. The m_p^2/Q^2 corrections incorporated in ξ [6,10] can be ignored in getting a conservative final answer because their effect is to reduce R and $\langle y\rangle$. The largest effect to $O(m_p^2/Q^2)$ is

$$xW_3/W_T = 1 - 2x^2 m_p^2/Q^2 + \dots . \quad (3)$$

Over the range of interest, this ratio increases roughly 10% which would cause R to decrease about 15% and $\langle y\rangle$ to decrease about 10%.

Explicit appearance of the gluon function is eliminated using the energy-momentum sum rule. The equations for the quarks can be integrated analytically if we ignore dependence on m_i^2/Q^2, where m_i are various quark masses. Here, m_i^2/Q^2 modifies the powers of $\ln Q^2$ in the evolution of the ith quark function[6] by a factor that behaves like $(1+am_i^2/Q^2)^{-1}$. So dropping the m_i^2/Q^2 dependence in the differential equations is a conservative simplification because in doing so we are overestimating the rate of growth of S and $\bar{S}$ or C and $\bar{C}$ until $Q^2 \gg m_s^2$ or m_c^2. The light-quark functions are virtually unchanged. So the fraction of antiquarks is made slightly higher by this approximation. The resulting equations are

$$\begin{aligned}\Sigma(Q^2) &= \tfrac{3}{7} + [\Sigma(Q_0^2) - \tfrac{3}{7}]\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{-56/75}, \\ \Delta(Q^2) &= \Delta(Q_0^2)\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]^{-32/75},\end{aligned} \quad (4)$$

where $\Sigma(Q^2) \equiv U + \bar{U} + D + \bar{D} + S + \bar{S} + C + \bar{C}$ and Δ is a difference (e.g., $U-D$, $U-\bar{U}$, etc.). Λ determines the gluon coupling via

$$\frac{g^2(Q^2)}{4\pi} = \frac{12\pi}{25}\left(\ln\frac{Q^2}{\Lambda^2}\right)^{-1} + O(g^4(Q^2)). \quad (5)$$

We have chosen $\Lambda = 0.5$ GeV. Were it appreciably larger, Eq. (4) would imply large scaling violations at low energies where scaling is known to be good. Our Λ is twice the favored value of Ref. 5 and is in agreement with estimates from e^+e^- annihilation[11] and e-p scattering.[12] All numerical factors in the above equations are particular to four quark flavors.

We consider only isoscalar targets ($U = D$ and $\bar{U} = \bar{D}$) and assume an SU(3) symmetric sea ($\bar{U} = \bar{D} = \bar{S} = S$). This is a slight overestimate of S and $\bar{S}$, hence an overestimate of charm production. Then Eq. (4) implies that

$$\begin{aligned} U(Q^2) &= \tfrac{1}{4}\left[\tfrac{3}{14} + (U_0 + 2S_0 + C_0 - \tfrac{3}{14})L^{-56/75} + (3U_0 - 2S_0 - C_0)L^{-32/75}\right], \\ S(Q^2) &= \tfrac{1}{4}\left[\tfrac{3}{14} + (U_0 + 2S_0 + C_0 - \tfrac{3}{14})L^{-56/75} + (2S_0 - U_0 - C_0)L^{-32/75}\right], \\ C(Q^2) &= \tfrac{1}{4}\left[\tfrac{3}{14} + (U_0 + 2S_0 + C_0 - \tfrac{3}{14})L^{-56/75} + (3C_0 - 2S_0 - U_0)L^{-32/75}\right], \end{aligned} \quad (6)$$

where $L = \ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)$ and $U_0 = U(Q_0^2)$, etc.

Computing R and $\langle y\rangle$ involves integrating over Q^2 in the range $0 \le Q^2 \le 2mEy$. However, Eq. (6) is certainly unreliable for $Q^2 \lesssim \Lambda^2$ because it follows from perturbation theory in a coupling grown large. We have no well-founded expectation for low Q^2, but we make the following hypotheses: We use Eq. (6) for $Q^2 \ge 1$ GeV^2. For $Q^2 \le 1$ GeV^2, we assume $U(Q^2) = U(1\text{ GeV}^2)$, $S(Q^2) = Q^2 S(1\text{ GeV}^2)$, and $C(Q^2) = 0$. $C(Q^2 \le 1) = 0$ is chosen to mimic the Q^2/m_c^2 suppression of heavy sea quarks at low Q^2 and serves as a boundary condition on $C(Q^2)$.

We adjust the ratio S_0/U_0 to best fit the data. If we ignored the Q^2 dependence of the distribution functions, the absolute magnitude of U and S would not enter in computing R and $\langle y\rangle$. However, absolute magnitudes do affect the Q^2 dependence because the closer the quark functions are to their asymptotic value, the slower is their subsequent evolution. Since total cross sections are difficult to measure in neutrino scattering, we take the overall normalization from electroproduction. Stanford Linear Accelerator Center (SLAC) measurements suggest that

$$\begin{aligned}\int dx\,(F_2^{\text{proton}} + F_2^{\text{neutron}}) &= 0.28 \pm 0.04 \\ &= \tfrac{5}{9}(U + \bar{U} + D + \bar{D}) + \tfrac{2}{9}(S + \bar{S}). \end{aligned} \quad (7)$$

We have ignored the possibility of charm production in Eq. (7) because SLAC measurements of $x \le 0.1$ are at $Q^2 \lesssim 1$ GeV^2. This estimate represents an integral over all available Q^2, subject to experimental cuts. The mean x which enters the determination of 0.28 is around $\frac{1}{3}$. The mean Q^2 for $x = \frac{1}{3}$ is about 4 GeV^2.[13] So we use Eq. (7) as a boundary condition at $Q_0^2 = 4$ GeV^2. This is an underestimate because of the expected shape

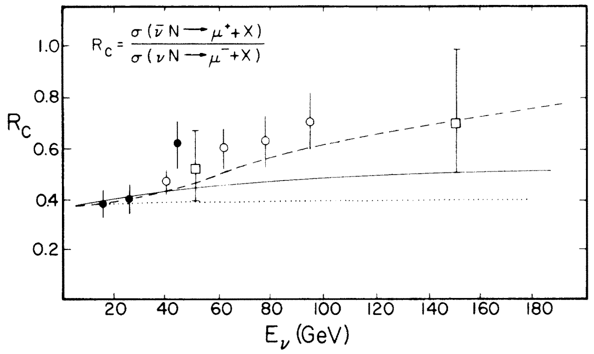

FIG. 1. R_c versus the incoming-neutrino lab energy. The dotted (solid) curve is for the four-quark model without (with) asymptotic-freedom corrections. The dashed curve is for a model with a coupling $(u, b)_R$, where b is a new quark of mass 5 GeV; asymptotic-freedom corrections are included. The data are taken from Ref. 1 (circles) and Ref. 2 (squares).

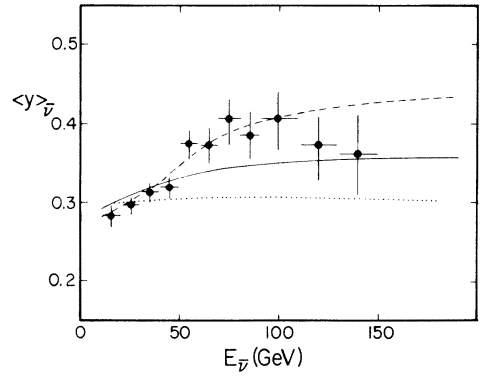

FIG. 2. $\langle y \rangle$ in antineutrino charged-current scattering versus E. The dotted (solid) curve is for the four-quark model without (with) asymptotic-freedom corrections. The dashed curve is for a model with a coupling $(u, b)_R$, where b is a new quark of mass 5 GeV; asymptotic-freedom corrections are included. For comparison with the data, all curves exclude events with $E_\mu < 4$ GeV or with $\theta_\mu < 0.225$ rad or with $Q^2 < 1.0$ GeV^2 *and* $W < 1.6$ GeV. The data are taken from Ref. 3.

variations with Q^2. The measurement in Eq. (7) includes high-x points measured at $Q^2 > Q_0^2$ and low-x points measured at $Q^2 < Q_0^2$. If F_2 could be measured at fixed Q_0^2, its integral is predicted to be larger, probably by a factor of 5–10%. And a larger integral would give a slower rise in $S(Q^2)/U(Q^2)$.

This discussion can be translated into a determination of the *gluon fraction.* For typical values of S_0/U_0, we find that the *gluons carry about 44% of the total energy-momentum at* $Q^2 \simeq 4$ *GeV*^{2}. The shape corrections in Eq. (7) would lower this to 41%–42%. The gluon fraction rises to 57% as $Q^2 \to \infty$ (with four quarks), but it is only 36% at $Q^2 = 1$ GeV^2!

In Figs. 1 and 2 we display the results for R and $\langle y \rangle$ versus E. The solid lines are for $U_0 = 0.22$ and $S_0 = 0.26$ (at $Q^2 = 4$ GeV^2), the best fit from the standard four-quark model. With these values and $\Lambda = 0.5$ GeV, $C(Q^2 = 1) = 0$ implies that $C_0 = C(Q^2 = 4) = 0.010$. For comparison, the dashed lines show the best fit for models[14] with right-handed currents involving at least one new heavy quark b of charge $-\frac{1}{3}$ coupling to the u quark. The powers in Eqs. (4) and (6) increase from 32/75 and 56/75 for $Q^2 \ll m_{b,t}^2$ to 32/63 and 68/63 for $Q^2 \gg m_{b,t}^2$, respectively, where b and t are the fifth and (perhaps) sixth quarks.

The standard four-quark model is not unequivocally excluded by the data. It is never more than 2 standard deviations from experiment. But the shape is systematically in disagreement. Furthermore, at each stage we overestimated the effect. Our calculation shows $\langle y \rangle$ rising from 0.29 at 10 GeV to 0.36 at 150 GeV. Inclusion of the neglected effects could well lower $\langle y \rangle$ at 150 GeV to 0.32.

Our estimates of asymptotic-freedom effects are only slightly smaller than those of Ref. 5. We wish to note in what ways we differ from and have improved upon the earlier estimates. We use a larger coupling constant (virtually as large as possible without contradicting observed approximate scaling). This makes renormalization effects larger, but it is compensated by two features. We use the appropriate ξ variable to describe charm production, which raises the effective threshold energy. And we have a smaller estimate of the gluon fraction because we take account of the fact that the mean SLAC value for Q^2 is larger than 1 GeV^2, and the increase in the sea is proportional to the gluon distribution. Instead of using a single effective Q^2 for each incident energy, we use reasonable shapes for u and s distributions, which then appropriately weight the allowed range in Q^2 for each distinct process. Finally, we have argued that each approximation we make overestimates the effect, but we still do not seem to get something large enough. These effects are, however, too large to be ignored in future analyses.

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Multimuon Production in Deep-Inelastic Muon Scattering*

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We present the characteristics of a model calculation of multimuon production in deep-inelastic muon scattering. In the model the muons are assumed to originate from the production and subsequent weak decay of a pair of hadrons that carry new quantum numbers and have a mass of ~1.8 GeV. The results of the calculation are to be compared with the forthcoming experimental data, and could shed light on the properties of the new hadrons.

Until the recent discovery[1] of a $K\pi$ resonance at 1.86 GeV, the existence of a family of hadrons with a new quantum number[2] has only been indirectly inferred by experiments. Among these is the high-energy neutrino experiment in which dimuon events are observed.[3] Now, with the observation of the 1.86-GeV resonance, the existence of hadrons with a new quantum number is directly confirmed, and the usefulness of the dimuon events in the neutrino experiment will be in the investigation of the weak-interaction properties of these new hadrons. Detailed theoretical studies[4] have been carried out to interpret the dimuon events.

Recently, Chen[5] has reported on the observation of multimuon events in a deep-inelastic muon