Jackiw for calling these references to our attention.

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Radio-Frequency Atomic Beam Measurement of the $(2^2S_{1/2}, F=0)$-$(2^2P_{1/2}, F=1)$ Lamb-Shift Interval in Hydrogen

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The $(2^2S_{1/2}, F=0)$-$(2^2P_{1/2}, F=1)$ interval in atomic hydrogen has been measured in zero magnetic field by scanning a radio-frequency perturbation through the atomic resonance. The measured interval was found to be 909.904 ± 0.020 MHz, which is consistent with that reported by Lundeen and Pipkin, 909.940 ± 0.020 MHz. Our deduced value of the Lamb shift 1057.862 ± 0.020 MHz is in good agreement with Mohr's calculated value 1057.864 ± 0.014 MHz but not with Erickson's value of 1057.912 ± 0.011 MHz.

In this Letter we report a measurement of the $(2^2S_{1/2}, F=0)$-$(2^2P_{1/2}, F=1)$ interval in atomic hydrogen and from it deduce a value of the hydrogen $n=2$ Lamb shift. Our method differs from that of a previous determination of Lundeen and Pipkin[1] in that we have used a slower beam (21 keV) and a single microwave region in the form of a 50-Ω transverse transmission line to induce the transition.

The separated-field[2] approach of Lundeen and Pipkin would appear to have an important advantage over the single-field technique[3] because it produces an interference signal whose linewidth can be made significantly narrower than that obtained by the single-field method. However when one of the atomic states can decay by spontaneous radiation, "interference narrowing" of the resonance is obtained at the expense of signal strength as can be seen in Fig. 3 of Ref. 1 in which Lundeen and Pipkin report a threefold narrowing. The significant loss in signal strength and increased complexity of the line shape and apparatus are major disadvantages which should be weighed against the advantages of having a much reduced linewidth. In our experiment we have chosen to use a single-field region in the form of a 50-Ω transverse slab line[4] and a slower beam (21 keV) which together ensure an adiabatic switch-on and -off of the perturbation. Since the solutions of Maxwell's equations for slab-line geometry (see Ref. 4) can be obtained analytically, a precise description of the spatial distribution of the field exists, so it is possible to give an accurate description for the atomic and instrumental line shape.

Figure 1 shows the main features of the apparatus in which the metastable hydrogen beam is produced by charge exchange on molecular hydrogen of a 100-μA 21-keV proton beam extracted from a radio-frequency ion source.[5] Noting that the natural linewidth of the $2P$ state is ≈100 MHz then, from Fig. 2, it can be seen that an oscillator tuned to about 1120 MHz will simultaneously drive both the β and γ resonances, thus quenching the $2^2S_{1/2}, F=1$ level by $\Delta M_F = 0$ transitions. Radio-frequency state selection of the metastable

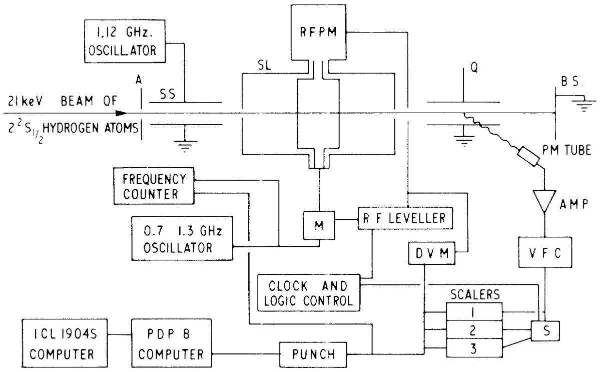

FIG. 1. Schematic diagram of the apparatus. A, aperture; SS, radio frequency state selector; SL, transverse microwave slab line; RFPM, radio frequency power meter (HP435A); Q, dc quench region; BS, beam stop; M, *p-i-n* diode attenuator/modulator; S, switch.

beam by this method is desirable since it simplifies the composite line shape and reduces the overall linewidth. However, to be effective the state selector must be placed well down-beam from the charge exchange cell to prevent repopulation of the $F=1$ level by cascade transitions from higher states. In the apparatus there is about 0.5 μs of free flight between the two. The microwave slab line[4] consists of a 1 in.-diam aluminnum bar between two flat aluminum plates, the spacing of which is such as to form a 50-Ω transmission line. Compensated step discontinuities at each end allow the slab line to be terminated in precision General Radio 900 connectors. A precision 10-dB General Radio attenuator and a Hewlett-Packard power sensor type 8481A absorb the radio-frequency power. When so terminated, the measured voltage standing-wave (VSWR) into either port was found not to exceed 1.006 over the frequency range 0–1.5 GHz. The VSWR calibration and determination of the microwave power response of the attenuator/power meter system as a function of frequency were made at the microwave standards laboratory Aquila.[6] The metastable beam is arranged to travel perpendicular to the direction of the radio-frequency wave, parallel and close (2.5 mm) to the surface of one of the plates in order to reduce rapid changes in the direction of the microwave electric field.

The position of the α resonance was determined by recording the quench at seventeen different frequencies in the range 0.7–1.28 GHz for constant radio-frequency power as measured by the power meter. The oscillator power was stabilized by means of a servo loop incorporating a *p-i-n* diode attenuator, to better than three parts in 10^4; and in order to reduce the effect of fluctuations in the metastable beam strength, the microwave power is chopped at 120 Hz by means of the *p-i-n* diode, while at the same time scalers 2 and 3 are synchronously switched to record beam intensity. The metastable beam strength was determined by quenching with a dc electric field (~1 kV/cm), and measuring the light produced with an uv photomultiplier. First-order Doppler shifts (~0.010 MHz) were eliminated by reversing the microwave connections after each frequency scan.

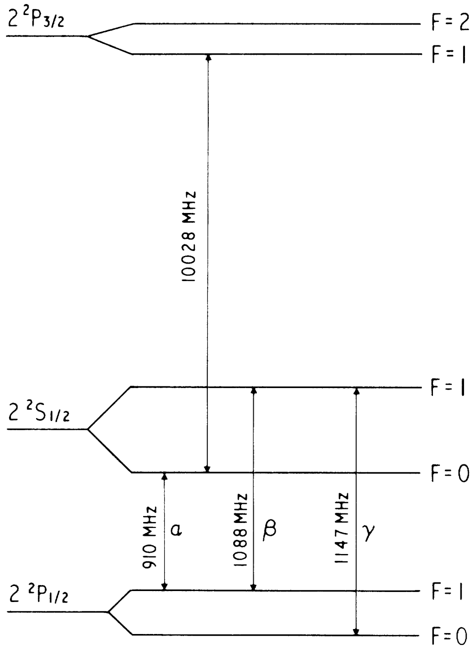

FIG. 2. Fine and hyperfine structure of the $n=2$ levels in atomic hydrogen. The frequency intervals are taken from Brodsky and Parsons (Ref. 9) and are rounded to the nearest MHz.

The line shape thus obtained was fitted by an empirical line shape of the form $\sum_i a_i F_i(\nu)$, where i runs from 1 to 3 over the three hyperfine components, and

$$F_i(\nu)=[Q_i+Q_i']+a_6[Q_i(Q_i-1)(Q_i-\tfrac{1}{2})+Q_i'(Q_i'-1)(Q_i'-\tfrac{1}{2})],$$

$$Q_i=1/(1+\lambda_i^2),\quad \lambda_i=(\nu-a_4-h_i)/a_5,$$

$$Q_i'=1/(1+\lambda_i'^2),\quad \lambda_i'=(\nu+a_4+h_i)/a_5.$$

h_i are the calculated hyperfine additions; a_1 to a_3, the fitted hyperfine resonance amplitudes;

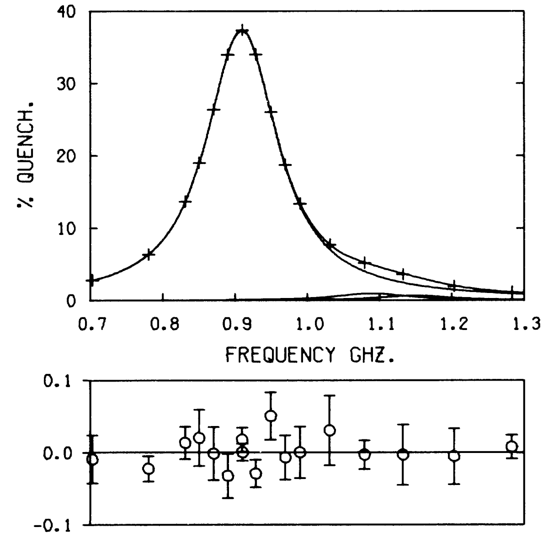

FIG. 3. Typical experimental resonance with fitted α, β, and γ component line shapes, and remaining residuals after fitting. (Run H is shown.)

and a_4, the derived (uncorrected) value for the Lamb shift. This fitting function was shown to fit accurately (± 0.005 MHz) simulated line shapes derived by numerically integrating the Schrödinger equation with use of the theoretical expression for the rf electric field. For a typical experimental run (see Fig. 3), a_4 is determined to ~0.040 MHz, and a_1 to a_3 to better than 5 parts in 10^4.

TABLE I. Summary of experimental runs (rf normal).

Run No.	Measured Line Centre -1050(MHz)	Bloch Siegert Shift (MHz)[a]	Corrected Line Centre -1050(MHz)	1σ error(MHz)
9B	7.875	0.086	7.789	0.056
9C	8.018	0.086	7.932	0.112
10B	7.955	0.086	7.869	0.052
10C	7.983	0.086	7.897	0.075
12B	7.917	0.086	7.831	0.034
12C	7.894	0.095	7.799	0.035
15A	7.910	0.048	7.862	0.097
15D	7.896	0.086	7.810	0.036
16A	7.938	0.086	7.852	0.053
16D	7.853	0.086	7.767	0.063
18B	7.939	0.095	7.844	0.048
18D	7.978	0.095	7.883	0.042
18F	7.882	0.095	7.787	0.027
G	7.915	0.095	7.820	0.033
I	7.920	0.095	7.825	0.029
13B	7.890	0.086	7.804	0.101
Mean of runs (weighted as $1/\sigma^2$)			1057.823	0.011

[a] See D. A. Andrews and G. Newton, J. Phys. B 9, 1453 (1976).

TABLE II. Summary of experimental runs (rf reversed).

Run No.	Measured Line Centre -1050(MHz)	Bloch Siegert Shift(MHz)[a]	Corrected Line Centre -1050(MHz)	1σ error(MHz)
8A	8.018	0.086	7.932	0.076
9A	7.831	0.086	7.745	0.090
9D	7.841	0.086	7.755	0.112
10D	8.022	0.086	7.936	0.081
12A	7.981	0.086	7.895	0.039
15B	7.917	0.048	7.869	0.086
15C	7.876	0.086	7.790	0.047
16B	7.978	0.086	7.892	0.032
16C	7.920	0.086	7.834	0.051
17B	7.875	0.086	7.789	0.066
18C	7.912	0.095	7.817	0.037
18E	7.915	0.095	7.820	0.030
F	7.965	0.095	7.870	0.036
H	7.913	0.095	7.818	0.034
J	7.913	0.095	7.818	0.042
13A	7.939	0.086	7.853	0.070
Mean of runs (weighted as $1/\sigma^2$)			1057.843	0.011

[a] See Table I, footnote a.

The line center derived from 32 runs, 15 reversals, are given in Tables I and II, and shown in Fig. 4. Shifts and distortions to the line shape due to the overlapping β and γ resonances were investigated by varying the amount of state selector power and hence (a_2 and a_3) relative to the α componant (a_1). By plotting the line centers as a function of the ratio $(a_2 + a_3)/a_1$, and fitting a straight line, the $a_2 + a_3 = 0$ intercept obtained dif-

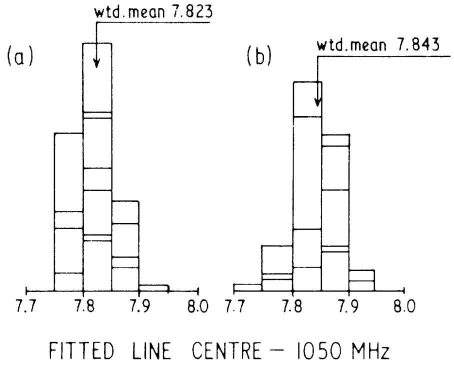

FIG. 4. Weighted histogram of experimental runs listed in Tables I and II with rf direction (a) normal and (b) reversed. The height of each segment is proportional to its $1/\sigma^2$ weighting.

TABLE III. Results and systematic corrections in units of MHz.

	Magnitude (± 1 σ)
Weighted mean of experimental runs	1057.823 ± .011 (Normal)
(including individual Bloch-Siegert	1057.843 ± .011 (Reversed)
shift and hyperfine structure addition[a])	
	1057.833 ± .008
Transverse Doppler Shift	+ 0.020 ± .000
Power meter and attenuator	
frequency dependence	− 0.001 ± .001
Frequency calibration	− 0.001 ± .001
VSWR of slab line	+ 0.002 ± .014
VSWR of attenuator	0.000 ± .010
Zeeman effect	+ 0.002 ± .000
Residual effect of F = 1 resonances	+ 0.004 ± .004
Effect of n = 4 resonances	+ 0.007 ± .005
Stark Shift due to residual 2μA proton beam	− 0.000 ± .000
(v x B) Stark Shift	− 0.004 ± .000
Final Lamb Shift	1057.862 ± .020
Hyperfine addition[a]	− 147.958
$2^2S_{\frac{1}{2}}$ F = 0, $2^2P_{\frac{1}{2}}$ F = 1 interval	909.904 ± .020

[a] The hyperfine addition was calculated using Eqs. (6), (8), and (10a) of Ref. 7, which include anomalous moment and relativistic corrections. We note that our result differs from that given in Ref. 1 by 0.005 MHz.

fers from the simple weighted mean by +0.004 ±0.004 MHz.

Table III gives an analysis of the systematic corrections applied. The shifts due to the VSWR of the line were derived by computer simulations, using the measured values of the VSWR. Shifts due to the possible contribution of $n=4$ metastable atoms to the resonance signal were investigated by pooling the experimental residuals and fitting these to a sum of theoretical $n=4$ line shapes. The only significantly nonzero contributions were found to arise from the $4^2S_{1/2}$-$4^2P_{3/2}$ single quantum resonance at 1240 MHz and the $4^2S_{1/2}$-$4^2D_{5/2}$ double quantum resonance at 840 MHz. Their combined effect is to shift the line center by 0.007 MHz.

Our measured value of the $(2^2S_{1/2}, F=0)$-$(2^2P_{1/2}, F=1)$ interval is some 36 kHz lower than that reported by Lundeen and Pipkin,[1] but is consistent with their measurement. The Lamb shift that we deduce from our measurement is in good agreement with that calculated by Mohr[7] but is 50 kHz lower than Erickson's[8] theoretical value. The main, and at present limiting, sources of error in our method are due to uncertainty in the magnitude and phase of the microwave reflections occurring at impedance mismatches within the slab line and attenuator. To reduce their combined effect to less than, say, 9 kHz (the statistical error) would probably require improvements in the available microwave technology.

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