tion

$$N(\epsilon_0) = \text{const} \times \exp(-\epsilon_0/\bar{\epsilon}_0)$$

with $\bar{\epsilon}_0 = 10$ keV, and for a silicon plasma where $\langle Z \rangle = 10$. The Gaunt factor [Eq. (2)] for this case is given in Fig. 1 for several values of electron energy. We have evaluated g_{coll} at $n_e = 10^{21}/\text{cm}^3$, $\theta = 1$ keV, and $\epsilon = 10$ keV for which $g_{\text{coll}} = 7.89$. The variations in electron energy and electron density can appreciably alter g_{coll}. If, for example, the fast electrons enter dense plasma at $n_e = 10^{22}/\text{cm}^3$, g_{coll} is reduced by about 15%, which is a measure of the uncertainty of the analysis.

Figure 2 gives the exact and approximate (with $g_{\text{rad}} = 2$) evaluation of Eq. (1), normalized for $g_{\text{rad}} = 2$ to unity at $h\nu = 0$.

The exact calculation is about 20% less than the exact result for $h\nu > 5$ keV. Correspondingly, in fitting an observed x-ray spectrum above the region of thermal emission ($h\nu \gtrsim 5$ keV), an accurate result can be obtained with the assumption of $g_{\text{rad}} \cong 1.6$, increasing the estimated number and energy of fast electrons by 20–30%.

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Phase Velocity of High-Frequency Phonons in Glass below 1 K

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Precision measurements of the phase velocity of sound waves propagating in fused silica at temperatures between 0.03 and 1.0 K and at frequencies between 0.3 and 2.0 GHz reveal deviations from a $\ln T$ temperature dependence when $\hbar\omega/kT \ll 1$ and the existence of a velocity minimum when $\hbar\omega/kT \cong 2.2$. These results are analyzed in terms of the two-level tunneling model and yield quantitative information on the deviation from energy uniformity of the density of scattering states.

The thermodynamic and transport properties of glass at low temperatures are strikingly different from those of pure crystalline insulators. These differences, which have been reviewed recently,[1] have been explained by assuming the presence of broad distributions of two-level atomic tunneling or defect states intrinsic to the glassy phase.[2] In the present Comment, we show that precision measurements of the temperature-dependent phase velocity of sound v_φ in silica glass below 1 K can provide quantitative information on the energy variation of the density of phonon scattering states $\bar{n}(\epsilon)$. We find that a uniform $\bar{n}(\epsilon)$ is *not* consistent with the observed v_φ's and the two-level model at phonon frequencies $\omega/2\pi = \epsilon/h$ between 0.3 and 2.0 GHz when $\hbar\omega/kT \ll 1$. In addition, we are able to study for the first time v_φ in the regime where $\hbar\omega/kT > 1$ and observe that v_φ passes through a minimum when the condition $\hbar\omega/kT \cong 2.2$ is satisfied.[3] Numerical calculations of $v_\varphi(\omega, T)$ based on the two-level model and an energy-dependent $\bar{n}(\epsilon)$ show good agreement with experiment over the entire frequency and temperature domain.

Recent measurements[4] of the temperature-dependent decay length, l, of monochromatic phonons in SiO_2 glass from 0.02 to 0.5 K at 0.5 GHz have shown agreement with predictions of the tunneling model based on resonant phonon absorption by the two-level systems

$$l^{-1}(\omega, T) = l_0^{-1}(\omega) \tanh(\hbar\omega/2kT), \tag{1}$$

$$l_0^{-1}(\omega) = \pi\omega\bar{n}(\omega)\gamma_i^2/\rho v_i^3, \tag{2}$$

using the notation of Golding and co-workers.[4,5] Piché *et al.*[6] have shown that the above expressions lead to a temperature-dependent, but frequency-independent, $v_\varphi(T)$, of the form

$$\Delta v_\varphi/v_\varphi = (\bar{n}\gamma_i^2/\rho v_i^2)\ln(T/T_0) \tag{3}$$

in the regime $\hbar\omega/kT \ll 1$ provided $\bar{n}(\omega) = \bar{n}_0$, a constant. They also suggested that the behavior of v_φ above 1 K due to *relaxational* interaction with two-level systems would be more consistent with their data if $\bar{n}(\omega)$ were a monotonically increasing function of ω, as suggested earlier by Stephens[7] from the excess T^3 specific heat in glass.

The precision of the present relative-velocity measurements, better than $\Delta v_\varphi/v_\varphi \sim 10^{-6}$ above 0.5 GHz, has permitted the observation of deviations from Eq. (3), undetected in the previous experiments. The deviations exist, we believe, not

as a result of the breakdown of the two-level model but from the existence of the energy-dependent $\bar{n}(\epsilon)$ similar in magnitude to the previous suggestions.

Figure 1 shows the phase-velocity changes of longitudinal sound waves in Suprasil fused silica containing approximately 1200-ppm OH ions[5] at four frequencies between 0.03 and 1.0 K. All v_φ changes are referred to the value of v_φ at 1 K and the data for each frequency in Fig. 1 are offset by 200 ppm at 1 K. The significant features of the data are (1) upward curvature of $\Delta v_\varphi/v_\varphi$ on the $\ln T$ plot for all phonon frequencies between 0.2 and 1.0 K, where $\hbar\omega/kT \cong 48f(\mathrm{GHz})/T(\mathrm{mK}) \ll 1$ and (2) a weaker temperature dependence below 0.1 K, evolving into a minimum at $\hbar\omega/kT \cong 2.2$. The minimum is observed in the 1.94-GHz data near 42 mK as shown in the inset of Fig. 1. Previous Brillouin scattering measurements of v_φ by Pelous and Vacher[8] in fused silica were carried out to $\hbar\omega/kT \lesssim 0.8$ and they did not observe the minimum whereas the present measurements extend to $\hbar\omega/kT \lesssim 3.2$.

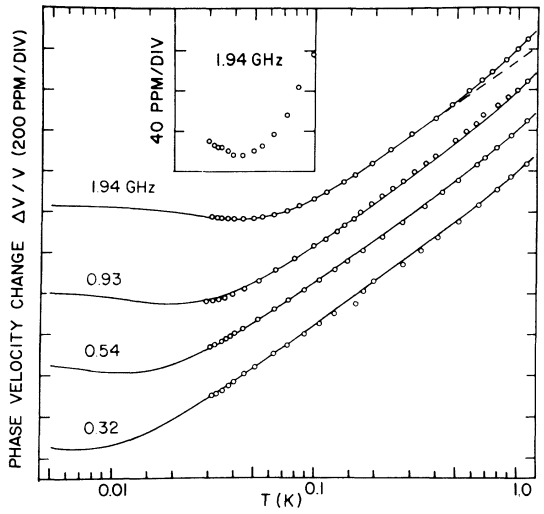

FIG. 1. Relative phase velocity of longitudinal sound waves in Suprasil fused silica glass as a function of frequency and temperature. The dashed line shows the phase velocity expected from the two-level model with a constant density of states; the solid line shows the calculated velocity with a quadratic (in energy) correction term added to $\bar{n}(\epsilon)$. Each frequency is offset by 200 ppm at 1 K. The inset shows the region near the velocity minimum.

In order to understand the phase-velocity temperature dependence, we have calculated $\Delta v_\varphi/v_\varphi$ expected for resonant interaction of sound by two-level systems by numerically evaluating the Kramers-Kronig transform of Eq. (1):

$$\frac{v_\varphi(T)-v_\varphi(T_0)}{v_\varphi} = \frac{\gamma_i^2}{\rho v_i^2}\int_0^\infty \frac{d\omega'\bar{n}(\omega')\omega'}{\omega^2-\omega'^2}\left[\tanh(\omega'/\omega^*)-\tanh(\omega'/\omega_0^*)\right], \tag{4}$$

where $\omega^* = kT/\hbar$ and $\omega_0^* = kT_0/\hbar$ with $T_0 = 1$ K. We initially assumed $\bar{n}(\omega) = \bar{n}_0$ and obtained the results shown by the dashed line in Fig. 1. This calculation is in good agreement with the data below 0.5 K (i.e., the minimum is correctly given), but the data deviate increasingly from the function at higher temperatures. In an attempt to improve agreement above 0.5 K, we considered a frequency-dependent density of states, $\bar{n}(\omega) = n_0[1 + (\omega/\omega_0)^2]$. Equation 4 was evaluated numerically for various values of ω_0 and the amplitude, $\bar{n}_0\gamma_L^2/\rho v_L^2$, to obtain the best fit. The result most consistent with v_φ at the four frequencies was $\bar{n}_0\gamma_L^2/\rho v_L^2 = (312 \pm 6)\times 10^{-6}$ and $\omega_0^{-2} = (6 \pm 2)\times 10^{-24}$ sec^2, as given by the solid lines in Fig. 1. We emphasize that the good agreement found above does not rule out other forms for the correction terms to $\bar{n}_0$. The quadratic form was used here for simplicity (only one additional parameter) and also because it was previously used to explain the excess T^3 specific heat where it was found[7] that $\omega_0^{-2} = 4.6\times 10^{-24}$ sec^2 in good agreement with the present value. This agreement can be interpreted as evidence for believing that it is the density of states, and not the coupling constant γ_i, which is energy dependent. Note, however, sound waves interact effectively with only a subset of the states contributing to the specific heat[5] and it is therefore possible, in principle, to have different distributions of states dominating each of the properties.

We briefly note that the deviation of the temperature dependence of the phonon thermal conductivity κ in glasses from T^2 ($\kappa \sim T^{1.8}$ to $T^{1.95}$ from experiment)[7,9,10] can be a result of the nonuniform $\bar{n}(\omega)$. To quantitatively test this idea, we use Eq. (1) for l_i[6,4] and our present results for $\bar{n}(\omega)$ to calculate κ from the kinetic formula $\kappa = \sum_i \int_0^\omega {}^D C_i(\omega) v_i l_i[\omega, \bar{n}(\omega)]$. The $\bar{n}(\omega)$ described above leads to a conductivity which varies as T^2 near and below 20 mK, with increasing deviation from T^2 above this temperature. The simple power law $\bar{n}(\omega) = \bar{n}_0\omega^\nu$, $0 < \nu < 0.3$, suggested previous-

ly[10] was not used since it leads to an infinite κ from resonant scattering alone. We find an *effective* power law for the silica glass κ which agrees favorably with the $T^{1.95}$ power law obtained by Lasjaunias *et al.*[10] from measurements between between 20 mK and 1 K. For example, the difference between a T^2 and a $T^{1.95}$ power law extrapolated from 20 mK to 0.4 K gives a conductivity ratio of 1.13; our calculation yields a ratio $\kappa(\tilde{n}_0)/\kappa[\tilde{n}(\omega)] = 1.17$. The agreement worsens at higher temperatures, reflecting the inadequacy of the quadratic correction to $\tilde{n}$ in approximating a power law. Nevertheless, we believe this calculation quantitatively supports the assertion that the larger number of scattering states at higher energies leads to a thermal conductivity which varies less rapidly than T^2.

In conclusion, the present results show that the details of sound propagation in a glass at low temperatures can be understood quantitatively over wide frequency and temperature ranges by a particular distribution of two-level scattering states. Alternatively, the present work can be viewed as lending support to the idea of intrinsic two-level systems in glass.

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