In this latter case, despite the absence of significant facular clusters, the oblateness signal is just as strong as for the remainder of the days.

To correct for the occurrence of faculae, we subtract $c_f F_c$ from the observations, where c_f is obtained by least-squares method and has the values 0.10 ± 0.02, 0.04 ± 0.02, and 0.02 ± 0.02, respectively, for the three limb exposures 17.9″, 11.6″, and 5.3″. But the three oblateness signals are observed to be equal. Moore[6] photographed a small facular cluster repeatedly as it rotated away from the limb. The excess light flux from the cluster was observed to increase strongly with distance (up to 20″ arc), consistent with the above values of c_f.

After correcting Chapman's data for the differences in color filter,[7] computational technique, and in some cases in limb exposure, I find that they are reasonably consistent with the facular signals that we observed for the 8 days with strongest signals. Chapman also observed 6 days without measurable facular signals. However, he seems to have observed for a much smaller fraction of such days and a much larger fraction of days with large signals (a fraction 4 times as great as ours). Apparently his observational days were not completely random in selection but "somewhat" favored days with substantial facular patches at the limb.[8]

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[1]R. H. Dicke, Solar Phys. 37, 271 (1974).

[2]R. H. Dicke, Solar Phys. 47, 475 (1976).

[3]R. H. Dicke and H. Mark Goldenberg, Astrophys. J. Suppl. 27, 131 (1975).

[4]G. A. Chapman, Phys. Rev. Lett. 34, 755 (1975), and references therein.

[5]The facular indicator, F_c, was obtained for 1966 by Chapman and Ingersoll from solar photographs and was tabulated by them as $F_c/100$. See G. A. Chapman and A. P. Ingersoll, Astrophys. J. 175, 819 (1972), and Chapman, Ref. 4, for subsequent discussion.

[6]Ronald Moore, private communication.

[7]G. A. Chapman estimates a correction factor of 0.65 (private communication).

[8]G. A. Chapman, private communication.

COMMENTS

Energy Deposition in Laser-Heated Plasmas

H. Brysk
KMS Fusion, Inc., Ann Arbor, Michigan 48106
(Received 7 June 1976)

A set of relations obtained by Brueckner between a bremsstrahlung spectrum and the underlying electron distribution do not hold if the energy dependence of the Gaunt factor is retained. A different set of relations is then derived. Their practical usefulness for interpretation of laser-heated plasmas appears to be limited.

In a Letter with the above title,[1] Brueckner starts from an expression for electron bremsstrahlung including the Born-approximation differential Gaunt factor which he writes in effect as $3^{-1/2}\pi \ln\Lambda_{\rm rad}$, where

$$\ln\Lambda_{\rm rad} = \ln\left[\frac{\epsilon^{1/2}+(\epsilon-h\nu)^{1/2}}{\epsilon^{1/2}-(\epsilon-h\nu)^{1/2}}\right] = 2\cosh^{-1}\left(\frac{\epsilon}{h\nu}\right)^{1/2}, \quad h\nu \le \epsilon, \tag{1}$$

but proceeds to replace $\ln\Lambda_{\rm rad}$ by the constant 2 (for a Gaunt factor of 1.1) on the grounds that it "is slowly varying with ϵ and $h\nu$." The subsequent manipulations include an integration over ϵ with lower limit $h\nu$, a second such integration (with a distribution function) over the upper limit of the first, then up to two parametric differentiations in $h\nu$. It would be remarkable if the sum of these operations should be insensitive to replacing Eq. (1) by a constant. Indeed, the calculations can be performed without this approximation in elementary fashion[2] (as described below) and the results disagree with all of Brueckner's relations [his Eqs. (9)–(11)].

Following Brueckner's notation, but abbreviating by putting $C \equiv \lambda_Z/2mc^2$ (without inquiring into its numerical evaluation), I find that the counterpart of his Eq. (8) with the dependence of the Gaunt factor on ϵ and $h\nu$ taken into account is the less tidy

$$dE_{\rm rad}/d(h\nu) = 2C\int_{h\nu}^{\infty} n(\epsilon_0)d\epsilon_0 \int_{h\nu}^{\epsilon_0} d\epsilon \cosh^{-1}(\epsilon/h\nu)^{1/2}$$

$$= C\int_{h\nu}^{\infty} n(\epsilon_0)d\epsilon_0[(2\epsilon_0 - h\nu)\cosh^{-1}(\epsilon_0/h\nu)^{1/2} - \epsilon_0^{1/2}(\epsilon_0 - h\nu)^{1/2}]. \tag{2}$$

The limit $h\nu = 0$ of this expression does not exist. (It is well-known that the radiation spectrum has an integrable singularity at the low-energy end.) Subject to modest constraints on the electron distribution [it is sufficient that $n(h\nu)$ be finite and that n vanish as ϵ_0^{-3} or faster for large ϵ_0, and that $n'(h\nu)$ be finite and that n' vanish as ϵ_0^{-4} for large ϵ_0], Eq. (2) can be differentiated with respect to $h\nu$, with n' eliminated by an integration by parts,

$$\frac{d}{d(h\nu)}\left[\frac{dE_{\rm rad}}{d(h\nu)}\right] = C(h\nu)^{-1}\int_{h\nu}^{\infty} n(\epsilon_0)d\epsilon_0[2h\nu \cosh^{-1}(\epsilon_0/h\nu)^{1/2} - \epsilon_0^{1/2}(\epsilon_0 - h\nu)^{1/2}], \tag{3}$$

and the procedure repeated once more,

$$\frac{d^2}{d(h\nu)^2}\left[\frac{dE_{\rm rad}}{d(h\nu)}\right] = C(h\nu)^{-2}\int_{h\nu}^{\infty} n(\epsilon_0)d\epsilon_0\epsilon_0 - h\nu)^{-1/2}. \tag{4}$$

A third differentiation encounters a divergent term in the integration by parts. The only results obtainable in the limit $h\nu = 0$ are thus

$$E_{\rm fast} \equiv \int_0^{\infty} \epsilon_0 n(\epsilon_0)d\epsilon_0 = -C^{-1}\{h\nu[d/d(h\nu)][dE_{\rm rad}/d(h\nu)]\}_{h\nu=0} \tag{5}$$

$$= C^{-1}\{(h\nu)^2[d^2/d(h\nu)^2][dE_{\rm rad}/d(h\nu)]\}_{h\nu=0}. \tag{6}$$

Equations (5) and (6) are not directly useful for data reduction, i.e., it is not practical to differentiate the experimental spectrum and extrapolate the derivative to $h\nu = 0$. One might insert into these equations an analytic form for the spectrum derived from a model, but there is then not much of substance to be learned beyond what is already in the model. If the spectrum is assumed exponential (as in Brueckner's fits in his two figures), the right-hand sides of Eqs. (5) and (6) vanish. A finite (and consistent) result is obtained if the Gaunt factor for a Maxwellian electron distribution[3] is included; the answer is clearly sensitive to the form of the Gaunt factor. The shape of the spectrum with Gaunt factor is not easily distinguished from the exponential at large $h\nu$ (where the data points are), although the small-$h\nu$ extrapolations differ significantly.

The relations involving the spectrum and its derivatives having been exhausted, a supplementary approach is to integrate Eq. (2) over all $h\nu$ to obtain the total energy radiated by the electron distribution. Upon interchanging the order of integrations and then setting $h\nu = \epsilon_0 x^{-2}$, there results

$$E_{\rm rad} = 2C\int_0^{\infty} n(\epsilon_0)\epsilon_0^2 d\epsilon_0 \int_1^{\infty} x^{-3}dx[(2 - x^{-2})\cosh^{-1}x - (1 - x^{-2})^{1/2}] = C\int_0^{\infty} n(\epsilon_0)\epsilon_0^2 d\epsilon_0. \tag{7}$$

The same answer can easily be obtained by integrating Brueckner's Eq. (8); this is unsurprising, since it is known that the total radiation is nearly the same whether calculated with or without the Gaunt factor.[3] In a similar fashion, moments of the spectral distribution of radiation can be related to higher moments of the electron distribution. Again, the practical limitation on the application of Eq. (7) is that most of $E_{\rm rad}$ is contributed by the low-energy radiation below the data, and this contribution is sensitive to the shape of this unobserved part of the spectrum; since $E_{\rm rad}$, the "hard-tail" component, is of order 10^{-3} of the total radiation, the uncertainty can be large.

So far, the discussion has dwelt on formal relations of the type sought by Brueckner for arbitrary electron distributions. The practical significance of inclusion of the Gaunt factor is most readily demonstrated by an illustrative example, namely, the insertion of the distribution

$$n(\epsilon_0)d\epsilon_0 = n_0 \exp(-\epsilon_0/\theta)d\epsilon_0 \tag{8}$$

into Eq. (2). Repeated integration by parts yields

$$dE_{\rm rad}/d(h\nu) = Cn_0\theta^2 \exp(-h\nu/2\theta)K_0(h\nu/2\theta) \tag{9}$$

upon recourse to the integral tables.[4] Equation (9) is readily differentiated:

$$[d/d(h\nu)][dE_{\rm rad}/d(h\nu)] = -\tfrac{1}{2}Cn_0\theta \exp(-h\nu/2\theta)[K_0(h\nu/2\theta) + K_1(h\nu/2\theta)]. \tag{10}$$

According to Brueckner's derivation (suppressing the Gaunt factor), the $h\nu = 0$ limit of Eqs. (9) and (10) is purported to yield the energy and number of the fast electrons, respectively. The results are

$$E_{\rm fast} \to \infty, \quad N_{\rm fast} \to -\infty. \tag{11}$$

Thus, even though the Gaunt factor makes only a moderate difference over most of the radiation spectrum, its strong impact near $h\nu = 0$ completely invalidates Brueckner's Eqs. (10) and (11).

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Energy Deposition in Laser-Heated Plasmas

Dale B. Henderson and Michael A. Stroscio
Theoretical Division, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545
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A simple connection between the x-ray spectrum and the number and energy distribution of suprathermal electrons in laser-heated plasmas has been suggested by Brueckner. This relation is shown to depend sensitively on an inaccurate approximation to an integral occurring in the model.

Brueckner[1] has calculated the number and energy distribution of suprathermal electrons produced in laser-heated plasmas using only the experimental x-ray spectrum. His derivation is, however, in error and his formulas should not be applied. In particular, conclusions to be drawn from his analysis applied to experimental examples are inconsistent with a proper analysis.

We begin with Eq. (5) of Ref. 1 (which is based on the unstated assumption that the collisional drag uniformly dominates the other energy losses),

$$\frac{d\epsilon_{\rm rad}}{d(h\nu)} = \frac{2}{3\pi}\frac{e^2}{\hbar c}\frac{\langle Z^2\rangle}{mc^2\langle Z\rangle}\int_{h\nu}^{\epsilon_0}\frac{\ln\Lambda_{\rm rad}}{\ln\Lambda_{\rm coll}}\,d\epsilon, \tag{1}$$

where[2]

$$\Lambda_{\rm coll} = (3/2e^3)(\theta^3/\pi n_e)^{1/2}$$

and

$$\Lambda_{\rm rad} = \frac{\epsilon^{1/2} + (\epsilon - h\nu)^{1/2}}{\epsilon^{1/2} - (\epsilon - h\nu)^{1/2}}.$$

At this point it was argued that $\ln\Lambda_{\rm coll}$ and $\ln\Lambda_{\rm rad}$ are slowly varying functions. The approximation of $\ln\Lambda_{\rm rad}$ by a constant, 2.0, is insufficient. It varies from zero to infinity over the range of integration, as $h\nu/\epsilon_0$ goes to zero. Also, the evaluation of $\ln\Lambda_{\rm coll}$ was numerically incorrect; it should have been 13.1 rather than 7.85 at 10 keV and $10^{21}/{\rm cm}^3$.

Actually the indicated integration may be performed analytically. We have from Ref. 1,

$$\frac{d\epsilon_{\rm rad}}{d(h\nu)} = \frac{\lambda_z}{mc^2}\frac{1}{2}\int_{h\nu}^{\epsilon_0}\ln\left[\frac{\epsilon^{1/2} + (\epsilon - h\nu)^{1/2}}{\epsilon^{1/2} - (\epsilon - h\nu)^{1/2}}\right]d\epsilon, \tag{2}$$

where

$$\lambda_z = \frac{4}{3\pi}\frac{e^2}{\hbar c}\frac{\langle Z^2\rangle}{\langle Z\rangle}\frac{1}{\ln\Lambda_{\rm coll}}.$$

Here we have only to change variables to $X^2 = (1$