follows. Let us assume that D is of the form of a cylinder with axis along the magnetic field. It is clear that as the field gets greater, the ratio of the radius of the cylinder to its length will decrease since the magnetic energy term is proportional to the square of the distance from the cylinder axis. Therefore we would expect this approximation to be worst for small $\boldsymbol{H}$. Even for this case (see below) the error is only 3.6% in γ. I would expect the error in C to get progressively smaller as H increases and finally approach zero as H approaches infinity, but I have not proved this. The advantage of taking this shape for D is that the problem becomes separable and can be solved exactly in terms of the confluent hypergeometric function. (Numerical work is still necessary to solve the resulting transcendental equations.)

The results may be stated as follows. Call the free energy per particle calculated from the cylindrical approximation f_c and write it in the form

$$f_c = (5\pi^2/6b_0^2)g(s), \tag{17}$$

where $s \equiv \omega b_0^2$. $g(s)$ can be obtained numerically but I only give it in the low-field ($s \ll 1$) and high-field ($s \gg 1$) limits:

$$g(s) = 1.054 + 0.002\,76 s^2 + \dots \quad (s \ll 1), \tag{18}$$

$$g(s) = \frac{3s}{5\pi^2} + 1.59\left(\frac{\ln s}{s}\right)^{2/3} + \dots \quad (s \gg 1). \tag{19}$$

The correct value of g for small s is

$$g_{\text{correct}} = 1 + 0.002\,86 s^2 + \dots, \tag{20}$$

so that even at small s where the cylindrical approximation should be poorest, the first coefficient is only 5.4% off and the second 3.6% off.

The susceptibility can be calculated from (16) but, as one sees at once, it goes rapidly to zero as H increases.

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[1]J. M. Luttinger, Phys. Rev. Lett. 37, 609 (1976). I shall refer to this paper as I.

[2]This gives the correct answer if the magnetic field is zero. See R. Friedberg and J. M. Luttinger, Phys. Rev. B 12, 4460 (1975). I believe that the magnetization and susceptibility given by this assumption are actually the rigorous results as $\beta \to \infty$ (not just an inequality), but have not found a rigorous proof of this conjecture.

[3]Apart from a relatively negligible region near the boundary of D.

Double-Peaked Longitudinal and Transverse NMR Spectra in Superfluid ^{3}He-A†

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New NMR resonances in bulk ^{3}He-A have been investigated. These appear in both longitudinal and transverse modes as small satellites on the low-frequency sides of the already-well-studied resonances predicted by theory. We find that the satellite frequencies in the two modes have slightly different temperature dependences. We also find that the satellites are enhanced by rapid temperature changes and are suppressed in a confined geometry.

In 1972, Osheroff *et al.*[1] performed the first NMR experiments on superfluid ^{3}He. Using a Pomeranchuk cell containing a mixture of liquid and solid, they discovered that in addition to a rather large signal coming from the solid which resonates at the Larmor frequency, upon cooling into the A phase a small satellite shifted to higher frequencies. This was subsequently identified as the transverse signature of the superfluid, existing independently of the signal from the solid. Here we present the results of recent experiments on a new generation of effects wherein the now-well-studied[2] superfluid transverse and longitudinal NMR lines themselves exhibit satellites. The longitudinal satellite has previously been observed by Avenel *et al.*[3]

These measurements were performed in a Pomeranchuk cell using conventional cw NMR as in

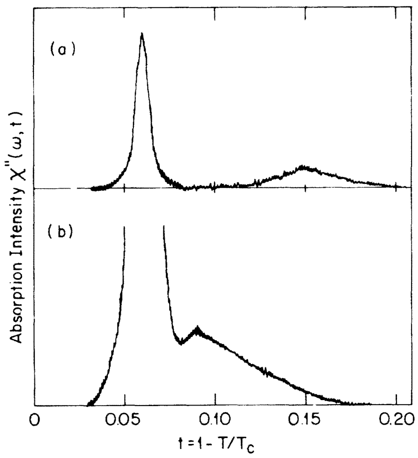

FIG. 1. Temperature sweeps through (a) longitudinal-resonance and (b) transverse-resonance peaks. The vertical axis is the rf level in the spectrometer while the frequency and magnetic field are held constant. The frequencies and fields have been chosen in (a) and (b) so that the main lines appear at the same temperature. This has been done to facilitate comparison of the satellite positions.

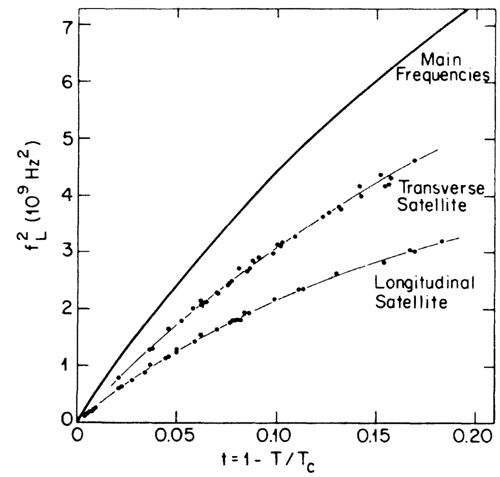

FIG. 2. Longitudinal satellite frequency and the transverse satellite shift frequency as a function of temperature. The main longitudinal and transverse shift frequencies, shown in the heavy line, are identical to previous results.

Ref. 2. The NMR region is defined by a vertical solenoid 1 cm in diameter, 1.3 cm tall, and is open at both ends. Longitudinal experiments are performed with the static magnetic field vertical, parallel to the rf field; during transverse experiments, the static field is horizontal. Our only procedural difference from Ref. 2 is that in transverse (as well as *longitudinal*) experiments we have swept the *temperature* of the liquid through resonance, rather than the magnetic field. This eliminates background contributions from spurious sources such as the protons in the surrounding epoxy walls. Schematic drawings of absorption versus temperature are given in Fig. 1.

The first property we have investigated is the temperature dependence of the longitudinal and transverse satellite frequencies. Near T_c these frequencies vary as $t^{1/2}$ (where $t = 1 - T/T_c$) as does the main longitudinal line, as shown in Fig. 2. It is therefore useful to consider the ratio of the satellite frequency to the main line's frequency, which removes the strong $t^{1/2}$ factor and reveals a more subtle temperature dependence. We find that in the longitudinal case the ratio $R_L(t) = \Omega_{sat}(t)/\Omega_{main}(t)$ is not $1/\sqrt{2} = 0.707$ as suggested by Avenel *et al.*,[3] but rather varies from about 0.74 at T_c to 0.67 at the B' transition, as shown in Fig. 3.

Matters concerning the transverse spectrum are more delicate. The main transverse frequency is related to the main longitudinal frequency by the condition[1]

$$\omega_{main}^2 = (\gamma H_0)^2 + \Omega_{main}^2(t).$$

It is therefore reasonable to *assume*[4] that the transverse satellite (t.s.) obeys an analogous condition with the term $\Omega_{main}^2(t)$ replaced by $\Omega_{t.s.}^2(t)$, a new transverse shift frequency. If the longitudinal and transverse satellites are real resonances in bulk A phase we would naively expect

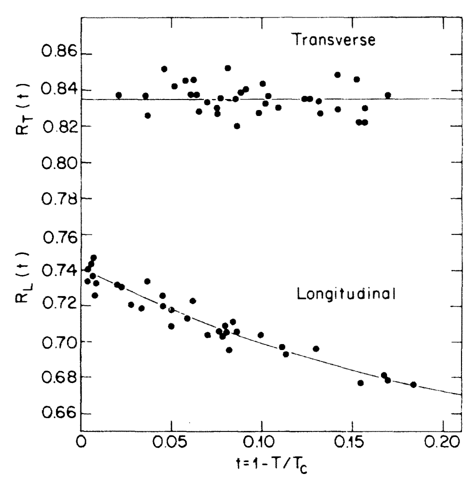

FIG. 3. Ratio of the longitudinal and transverse satellite frequencies to the main longitudinal frequency. This figure reveals the different temperature dependences of the two modes. The lines are intended only as a guide to the eye.

to find $\Omega_{t.s.}(t)=\Omega_{sat}(t)$, whereas we observe in Fig. 2 that the frequencies are quite different. Moreover, we find the rather surprising result that the transverse ratio $R_T(t)=\Omega_{t.s.}(t)/\Omega_{main}(t)$ is apparently temperature independent. The fact that the longitudinal and transverse satellites have different temperature dependences may suggest that the two are *not* directly related to one another.

The second property we have investigated is the spectral weight of the satellite. In the longitudinal case, earlier investigations[2] found that without a satellite, the static susceptibility of the superfluid calculated from the Kramers-Krönig sum rule was only 0.55 (in units of χ_N, the static susceptibility of the normal liquid), whereas according to existing theory[5] it is expected to be unity. Therefore, it is important to determine the weights both of the main line when the satellite is present and of the satellite itself. We find that the weight of the main line in these experiments is equal to that in previous work to within a few percent. The satellite's weight is only about $0.13\chi_N$, so the total susceptibility residing in these two resonance peaks still falls far short of the expected value. The missing susceptibility thus remains one of the outstanding problems of the A phase.

The reason the satellites were not discovered earlier is probably that with the then-existing sensitivities it was simply too small and broad to be seen. The height of the longitudinal satellite is roughly temperature independent at about $1.5\chi_N$, declining slightly both near T_c and at low temperatures. This behavior is not reflected in the temperature dependence of the height of the main line which over most of the A phase increases slowly with decreasing temperature to about $13\chi_N$, except near T_c where it rapidly drops to near zero.[2] The satellite's width is also roughly temperature independent and is typically 6 kHz, in contrast to 3 to 4 kHz for the main line.

The transverse satellite is an entirely different matter. Its height and width are strongly temperature-dependent but this may be simply a reflection of similar behavior by the main line. Because of relaxation effects, the width ($\Delta\omega_{tr}$) of the main line is related to the roughly temperature-independent linewidth ($\Delta\Omega$) of the main longitudinal line by[2]

$$\Delta\omega_{tr}=\frac{\Omega_{main}^2(t)}{(\gamma H_0)^2+\Omega_{main}^2(t)}\Delta\Omega(t).$$

For large fields this can be quite narrow, but at the low frequencies of the present experiment (80 to 120 kHz) γH_0 and $\Omega_{main}(t)$ are comparable, so the transverse linewidth is temperature-dependent and becomes quite large; at the same time, the hight drops to maintain an approximately constant area. As can be seen in Fig. 1, a temperature sweep through the lines makes the satellite appear at lower temperatures than the main line, further compounding the relaxation damping. As a consequence, the transverse satellite is extremely small. The ratio between the heights of the main and satellite lines varies between 10 and 100 in these experiments. We are prevented from examining the satellite very close to T_c because the separation of the two lines is comparable to the linewidths at the low frequencies that we have been using. Simply going to higher frequencies in order to reduce the linewidths might make such a study possible.

In an effort to understand the origins of the satellites, we have performed numerous experiments on the longitudinal satellite, hoping to destroy or enhance it convincingly. We have applied static magnetic fields from zero to 1 kOe. Above 20 Oe, there is no effect on either line, while below this field strength, both decrease in magnitude. (This behavior, in the case of the main line, has been previously observed[6] and is qualitatively understood in terms of the anisotropic order parameter becoming randomly distributed when the orienting strength of the magnetic field is reduced.) We have varied the power level in the rf coil by as much as three orders of magnitude in one experiment (i.e., $2H_1$ varied from 3 to 100 mOe at 42 kHz) without affecting either line.[7] We have applied heat above the NMR region of as much as 300 nW without effect. We have varied the amount of solid formed in the cell by varying the starting temperature of a compression from 18 to 37 m°K, again without effect.[8]

Only two experiments have influenced the satellites: (1) placing the liquid into differently shaped containers and (2) varying the compression (or decompression) rate. In "open" geometries, where the vertical size of the cell is 1 cm or more and the horizontal dimension 1 mm or more, the satellites were always present in both longitudinal and transverse modes. In this case, low compression rates of 1 or 2 μ°K/sec did not visibly affect either the main or satellite line. At higher rates the height of the main line decreased, apparently tending to zero at rates of order 20 to 50 μ°K/sec, depending upon frequency. The satellites, however, *increased* in height by as much as a factor of 2 at the highest rates. At all compression rates, the positions of the main and satellite lines were unaffected. Our contrasting "closed" geometry was a series of horizontal flat plates 0.9 mm apart which were placed into our vertical NMR solenoid with only tiny gaps at the edges to maintain pressure equilibrium. (A drawing of this cell appears in Ref. 2.) Here, the satellites did not appear at low compression rates. They could be created, however, by sweeping the temperature rapidly. In fact, the NMR signal could be temporarily increased by suddenly stepping the temperature of the liquid a small amount either up or down. This last effect occurs not only at the position of the satellites but at any temperature below that of the main line while still in the *A* phase.

As for the problem of understanding why the satellites exist we have no solution. It is our hope that this report will stimulate theoretical efforts in this direction. We have studied many possible explanations of which there are two that we have not eliminated and would like to mention here. The first rests on the observation that, solely by topological considerations, the *A* phase in any experiment must possess singularities. It is possible that the superfluid within these singular regions will change continuously from the axial to other states, with the concomitant change in NMR properties. (However, other superfluid states in *bulk* do not possess the required frequencies.[5]) If these singular regions are sufficiently dense, it may even provide an explanation for the missing longitudinal susceptibility. The second possibility is that superflow, driven by the thermal gradient which always exists in our Pomeranchuk cell,[6] may distort the superfluid state, causing it to suffer an anomalous frequency shift as suggested by Takagi[9] and Fetter.[10] We must report, however, that we have been unable to find a reasonable flow configuration which would provide for a double-peaked NMR spectrum.

We would like to thank W. J. Gully for assistance in the early stages of this work and useful comments on the results. We also thank O. Avenel, M. E. Bernier, and E. J. Varoquaux for a discussion of their experiment.

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[1]D. D. Osheroff, W. J. Gully, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 29, 920 (1972).

[2]W. J. Gully, C. M. Gould, R. C. Richardson, and D. M. Lee, J. Low Temp. Phys. 24, 563 (1976). For further details see W. J. Gully, thesis, Cornell University, 1976 (unpublished).

[3]O. Avenel, M. E. Bernier, E. J. Varoquaux, and C. Vibet, in *Proceedings of the Fourteenth International Conference on Low Temperature Physics, Otaniemi, Finland, 1975*, edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975), Vol. 5, p. 429.

[4]It is important to realize that there is no *a priori* reason that the field and temperature dependences should separate so cleanly. In fact, the simple sum-of-squares relation appears to be unique to the axial state [A. J. Leggett, Ann. Phys. (N.Y.) 85, 11 (1974)]. It is only by examining the satellite in a variety of frequencies from 80 to 120 kHz, and reduced temperatures in the range $0.02 \lesssim t \lesssim 0.17$, that we have been convinced of the validity of the analogous condition.

[5]Leggett, Ref. 4.

[6]Gully, Ref. 2.

[7]The fact that the amplitudes of the lines are directly proportional to the applied rf field places a useful constraint on possible theoretical explanations: The satellite must be the solution to a *linearized* theory.

[8]This argues against the possibility of the satellites being resonances in the solid. Furthermore, the frequencies go to zero at the temperature of the *A* transition in the liquid.

[9]S. Takagi, J. Phys. C 8, 1507 (1975).

[10]A. L. Fetter, Phys. Lett. 54A, 63 (1975), and J. Low. Temp. Phys. 23, 245 (1976).