Nuclear Fireball Model for Proton Inclusive Spectra from Relativistic Heavy-Ion Collisions*

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A simple model is proposed for the emission of nucleons with velocities intermediate between those of the target and projectile. In this model, the nucleons which are mutually swept out from the target and projectile form a hot quasiequilibrated fireball which decays as an ideal gas. The overall features of the proton-inclusive spectra from 250- and 400-MeV/nucleon ^{20}Ne ions and 400-MeV/nucleon ^{4}He ions interacting with uranium are fitted without any adjustable parameters.

In relativistic heavy-ion reactions both projectile fragmentation[1] and target evaporation[2] are, to a certain extent, qualitatively understood. In the intermediate-velocity region, the emission of composite particles can now be understood as the coalescence of nucleons close to each other in momentum space due to final-state interactions.[3] Calculations of the nucleon spectra in this intermediate-velocity region are presently being attempted with cascade models,[4] relativistic hydrodynamics,[5] and classical microscopic scattering models.[6] A very simple model is proposed here consisting of a fireball, formed from the nucleons mutually swept out from the target and projectile, which decays as an ideal gas. This model uses the geometrical concepts of the abrasion model,[7] the free expansion of an ideal gas, and for the extension to higher energies, the statistical thermodynamics of strong interactions by Hagedorn.[8] The Hagedorn thermodynamics has been applied, with a temperature fitting procedure, to medium-energy proton-nucleus reactions,[9] and its use for relativistic heavy-ion collisions has been pointed out by Chapline *et al.*[10] Our model fits the gross features of the existing data on proton inclusive spectra in this intermediate-velocity region. The agreement of this geometrical, thermodynamic model with the data presents a challenge to understand its success microscopically.

The geometrical concepts of this model are illustrated in Fig. 1. It is assumed that the target and projectile are spheres with radius equal to $1.2A^{1/3}$, and that the projectile and target make clean cylindrical cuts through each other,[7] leaving a spectator piece of the target and, if the impact parameter is sufficiently large, also a spectator piece of the projectile. These spectator pieces eventually lead to the products of target spallation and projectile fragmentation, which are not considered further in the present Letter.

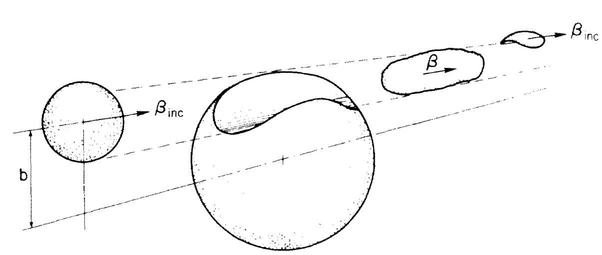

FIG. 1. A neon nucleus is incident on the uranium nucleus with a velocity β_{inc} in the laboratory frame and impact parameter b. The swept-out nucleons from the projectile and target are called the fireball, and their center of mass has the velocity β in the laboratory frame.

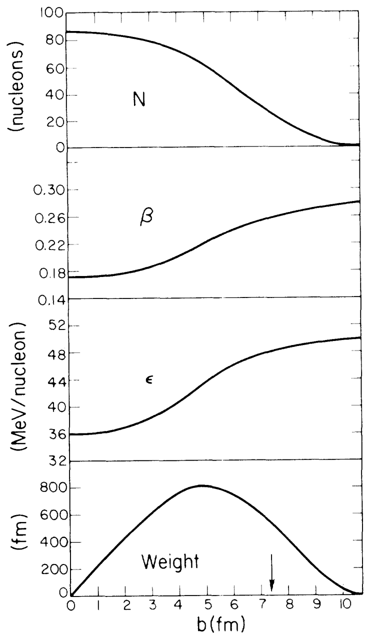

FIG. 2. Calculated quantities as a function of impact parameter b for 250-MeV/nucleon ^{20}Ne on uranium. N is the number of nucleons in the fireball, β is the laboratory velocity of the fireball, ϵ is the available energy per nucleon in the center of mass of the fireball, and "weight" is the number of protons in the fireball times $2\pi b$. The arrow indicates the radius of uranium.

One can calculate, as a function of impact parameter (b), the number (N) of nucleons swept out from both the projectile and target. The laboratory velocity (β) of the center of mass of the swept-out nucleons is then calculated from kinematics. In the center of mass of these nucleons, the available energy per nucleon (ϵ) is assumed to be their kinetic energy minus an 8-MeV binding energy. Figure 2 shows these quantities as a function of impact parameter for the case of 250-MeV/nucleon ^{20}Ne ions impinging on uranium. It is then assumed that the available energy in the center of mass heats up the swept-out nucleons leading to a quasiequilibrated nuclear "fireball." The fireball is treated relativistically as an ideal gas whose temperature (τ) is determined by the available energy per nucleon.[11] At the 250- and 400-MeV/nucleon incident energies $\epsilon \approx (3/2)\tau$, and one has a Maxwell-Boltzman energy distribution. At the higher bombarding energy of 2.1 GeV/nucleon, ϵ is much bigger, of the order of 350 MeV/nucleon. This energy may go into internal excitation of the nucleons to baryon resonances as well as randomized kinetic energy of the nucleons. A different thermodynamics has to be used to include this increase in the density of states. We used the mass spectrum obtained by Hagedorn and Ranft[12] from the bootstrap condition to relate a temperature to the available energy. However, in the calculation of the final proton spectra, the various de-excitation modes of the isobars, including the recoil from pion emission, were neglected.

The laboratory distributions are calculated assuming isotropic decay in the rest frame of the fireball, transforming relativistically to the laboratory frame, and summing over all impact parameters. The importance of each impact parameter is given by the number of protons in the fireball times $2\pi b$, and this weight is shown at the bottom panel of Fig. 2 for ^{20}Ne on uranium. The characteristics of the fireball for the impact parameter at the maximum weight are listed in Table I.

The experimental proton spectra obtained in our previous experiment[3] are shown in Fig. 3. The spectra at the lower proton energies probab-

TABLE I. Calculated properties of the fireball at the impact parameter with the maximum weight (b_{mw}) on a uranium target.

Projectile (MeV/nucleon)		β_{inc}	b_{mw} (fm)	N	ϵ (MeV/nucl.)	τ (MeV)	β
250	^{20}Ne	0.61	4.8	64	44	29	0.22
400	^{20}Ne	0.71	4.8	64	74	49	0.27
400	^{4}He	0.71	4.7	25	51	34	0.17
2100	^{20}Ne	0.95	4.8	64	363	92	0.56
2100[a]	^{20}Ne	0.95	4.8	64	133	66	0.14, 0.91

[a]This assumes that there are separate, equal-temperature, projectile and target fireballs and that only 25% of their original relative longitudinal momentum in their center of mass was dissipated into heat (ϵ). According to the equation of state used, about 28 MeV/nucleon of the value of ϵ is used in making baryon excited states.

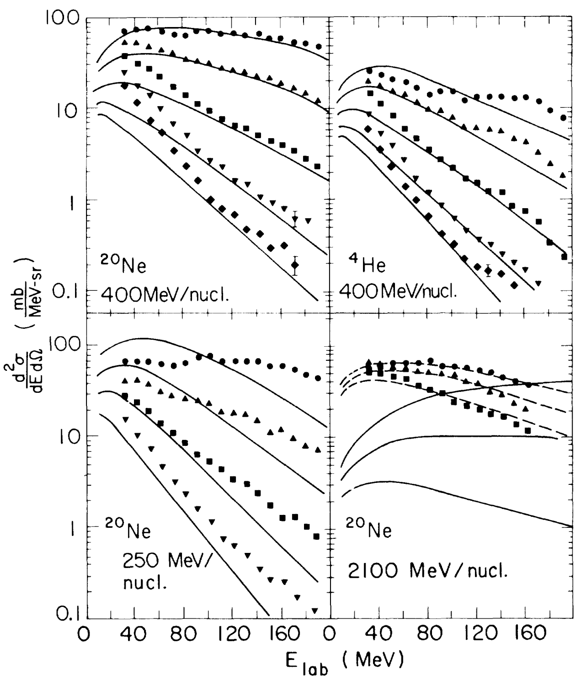

FIG. 3. Measured proton inclusive spectra from a uranium target at 30°, 60°, 90°, 120°, and 150° in the laboratory arranged in decreasing order. When the less then five sets of data are shown, the backward angles are missing. The solid lines are calculated with the fireball model. For the case with 2.1-GeV/nucleon ^{20}Ne on uranium, the data normalization is uncertain by a factor of 2. The dashed lines for this case, which have been raised by a factor of 2.5 to fit the data, are calculated assuming that there are separate, equal-temperature, projectile and target fireballs and that only 25% of their original relative longitudinal momentum in their center of mass was dissipated into heat.

ly include a contribution from the proton evaporation from the target residue, but exclude those protons which have coalesced into composite particles.[3] Also at 250-MeV/nucleon incident energy the most forward angle probably includes some contribution from fragmentation of the projectile residue. Considering these points, the overall agreement with the 250- and 400-MeV/nucleon data is good.[13] Notice that the considerably lower cross sections with the He projectile are also described and that there are no adjustable parameters in the calculation.

At 2.1 GeV/nucleon the calculation, even including the Hagedorn mass spectrum, fails to describe the data as shown by the solid lines in Fig. 3 because it predicts values of τ and β which are too large. One way to describe the shape of the data is to assume (dashed curves) that there are separate projectile and target fireballs and that only 25% of their original relative longitudinal momentum in their center of mass is randomized and dissipated into heating both to the same temperature. In contrast, at the lower bombarding energies it was found that a poor fit to the data was obtained when one assumed that less than 75% of the relative momentum was randomized.

In the present model it has been assumed that the fireball consists of only the nucleons in the swept-out region. However, for small impact parameters, one can imagine that the projectile never penetrates through the target and thus the available energy is shared among all the nucleons in the target and projectile. A straightforward calculation for 400-MeV/nucleon ^{20}Ne on uranium gives $\tau = 13.5$ MeV and $\beta = 0.076$ for the entire system. This "target explosion" could account for a large part of the discrepancy at the lower proton energies.

The surprising success of the simple model for proton inclusive spectra indicates the importance of the use of thermodynamic concepts in relativistic heavy-ion reactions. The large number of swept-out nucleons combined with an anticipated, fairly large number of interactions per particle is presumably responsible for a quasiequilibrated system—the fireball—which can then be described in terms of mean values and statistical distributions.

The present model also enables us to predict nucleon multiplicities and, with some modification, pion inclusive spectra and multiplicities. However, the model does not give any answer as to how the system evolves from the original pieces of cold nuclear matter to the heated fireball. We have assumed in the present work that energy stored in other modes during this process (e.g., in a compression mode) is fully transformed into thermal energy by the time the nucleons no longer feel any mutual interaction. Hopefully, more detailed or more exclusive measurements in the future will reveal possible effects of these modes. However, where previously it was not even known if sufficient equilibration was established in relativistic heavy-ion reactions to use the concept of an equation of state, such an approach now seems more promising.

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