Determination of the ^{3}He Superfluid-Density Tensor for the A and B Phases*

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The classic technique of Andronikashvili has been used to determine the superfluid density of the A and B phases of superfluid ^{3}He. In the anisotropic A phase, the texture of the fluid is controlled through the interaction between the boundary condition on the $\vec{l}$ vector and an externally applied magnetic field. Comparison of measurements made for different textures clearly establish the anistropy of the superfluid density tensor for ^{3}He-A.

Since the discovery in 1972 of the superfluid phases of liquid ^{3}He,[1] a number of different experimental approaches have been used to determine the superfluid density. The initial vibrating-wire experiments of Alvesalo *et al.*[2] and the fourth-sound measurements of Kojima, Paulson, and Wheatley[3] and Yanof and Reppy[4] clearly established the superfluid nature of the new phases. More recently Main *et al.*[5] have reported superfluid density measurements made by the torsion pendulum method of Andronikashvili.[6]

In accord with our present understanding of superfluid ^{3}He,[7] the superfluid density is expected, in general, to be a tensor quantity. For ^{3}He-A, the principal axes are set by the direction of the orbital angular momentum vector $\vec{l}$. Near the transition temperature, T_A, the component $\rho_{s\perp}$, associated with superflow perpendicular to $\vec{l}$, is expected to have twice the value of the component, $\rho_{s\parallel}$, associated with superflow parallel to $\vec{l}$. In contrast to the anisotropy of ^{3}He-A, the B phase is believed to exhibit an isotropic superfluid density. For the region near the polycritical point, it is predicted that the value of $\rho_{s\perp}$ in ^{3}He-A is larger by a factor of 6/5 than the superfluid density for the B phase at the same temperature.

Although the experiments mentioned above have provided considerable information on the superfluid properties of liquid ^{3}He, they do not address directly the problem posed by the anisotropic nature of the A-phase liquid. As a result there is disagreement between experiments and between experiment and theory, particularly with respect to a comparison of the superfluid density of the anisotropic A phase and that of the isotropic B phase at the AB transition.

In the work reported here, we have used the torsional pendulum method of Andronikashvili[6] to obtain the superfluid density in both the A and B phases. For the A-phase measurements, we have empolyed a magnetic field to control the $\vec{l}$-vector orientation or "texture" of the fluid. By varying the direction of the magnetic field between runs we are able to measure different combinations of the components of the superfluid density tensor and to establish clearly the anisotropic nature of the superfluid density in the A phase.

The apparatus is shown in schematic form in Fig. 1. An epoxy torsional pendulum contains an open disk-shaped region 50 μm high and 0.84 cm in diameter. This region is filled from a larger reservoir through a hollow torsion rod. Cooling is achieved by indirect contact through a copper heat link to a Pomeranchuk compression cell. The fluid within the torsional rod provides the thermal contact between the liquid in the torsional cell and the reservoir. The thermal time constants are less than a minute for temperatures below the superfluid transition. The temperature of the liquid in the reservoir is determined through the use of a Pt^{195} NMR thermometer and also by observation of the frequency shift of the transverse NMR resonance in the A phase. The 190-G magnetic field used for these NMR measurements

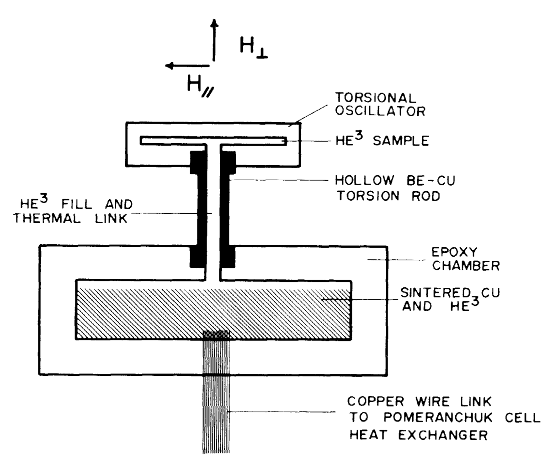

FIG. 1. Schematic of the apparatus. The drive and detector electrodes are not shown. The ^{3}He and ^{195}Pt NMR coils have also been omitted.

also serves to control the structure of the superfluid ^{3}He-A. A detailed account of the NMR measurements and the cooling method will be given elsewhere.[8]

Torsional oscillations of the pendulum bob are excited and detected electrostatically. In normal operation the torsional oscillator is placed in a feedback loop and allowed to oscillate continuously at its resonant frequency near 771 Hz. The period of the oscillation is monitored with an electronic counter. For temperatures near the superfluid transition, the viscous penetration depth is much larger than the 50 μm height of the ^{3}He region; therefore, the normal component is almost completely clamped to the pendulum bob. The moment of inertia of the clamped fluid adds to that of the pendulum bob in determining the oscillator frequency. Below the superfluid transition the moment of inertia of the normal fluid decreases as the superfluid fraction grows. Under the assumption of complete locking of the normal fluid to the pendulum bob,[9] an average superfluid density can be obtained from the temperature dependent period, $P(T)$, through the relation

$$\frac{\rho_s}{\rho}=\frac{P(T_c)^2-P(T)^2}{P(T_c)^2-P(0)^2},$$

where T_c is the transition temperature and $P(0)$ is the period of the empty cell. At high temperatures (greater than 50 mk) the viscous penetration depth is much less than the height of the ^{3}He space, and the clamped fluid makes only a small contribution to the moment of inertai. We obtain the empty-cell period from an extrapolation to the limit of zero penetration depth.

At the superfluid transition the oscillator has a Q of about 2×10^5. This high Q makes possible period measurements with a resolution of a few parts in 10^8. Although the ratio of the moment of inertia of the fluid to that of the torsional bob is only 2.8×10^{-4}, it is still possible to achieve superfluid density measurements with a precision of better than 0.1%. Realization of this precision in the ρ_s measurements requires continuous regulation of the pressure in order to avoid spurious frequency shifts.

In order to measure the anisotropic components of the superfluid density tensor, it is necessary to control the structure or texture of the fluid. A technique for achieving this is suggested by the work of Ambegaokar, de Gennes, and Rainer,[10] and Ambegaokar.[11] In the absence of a magnetic field or large superflows, a uniform texture with the $\vec{l}$ vector oriented perpendicular to the walls of the slab is expected. A magnetic field parallel to the plane of the slab stabilizes this structure. For the case of a sufficiently large magnetic field perpendicular to the slab, the A-phase superfluid undergoes an orientational transition from the uniform structure to a nonuniform texture.

Recently, Fetter[12] has considered this problem in greater detail. Assuming that $\rho_{s\perp}=2\rho_{s\parallel}$, he has computed the average superfluid density that would be measured for various values of the perpendicular magnetic field when the superflow is in the direction of the bending of the $\vec{l}$ vector in the nonuniform texture. For a perpendicular magnetic field of 290 G and a slab width of 50 μm, Fetter estimated the minimum value of the average ρ_s measured in an experiment such as ours to be $0.59\rho_{s\perp}$.

In Fig. 2, we show the superfluid density measured for two different orientations of the magnetic field. The data shown were taken at a pressure of 27 bars. The larger values of superfluid density are obtained with the magnetic field parallel to the slab of liquid. In this configuration the magnetic field enhances the stability of the uniform texture with the $\vec{l}$ vector perpendicular to the plane of the disk. Since the velocity arising from the torsional motion of the Andronikashvili cell lies in the plane of the disk, the component of the superfluid density tensor measured for this magnetic field configuration is $\rho_{s\perp}$. The lower values of superfluid density were obtained for the magnetic field normal to the plane of the disk; this configuration is expected to produce the nonuniform texture. Although a detailed interpretation of the measurements for the nonuniform texture would require model calculations, the basic anisotropic behavior of the A-phase superfluid is clear from the data shown in Fig. 2.

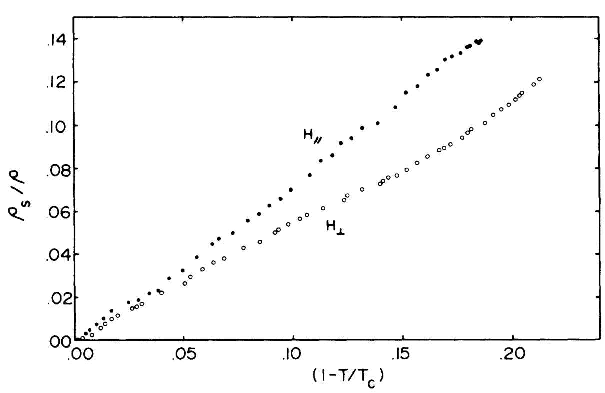

FIG. 2. Superfluid density data in the A phase at 27 bars is shown for two different orientations of the magnetic field.

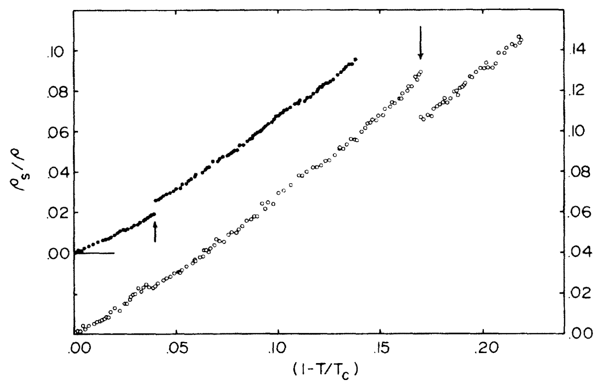

FIG. 3. Superfluid density data obtained while cooling through the AB phase change (indicated by arrows) is shown for two different A-phase textures. Open circles indicate uniform-texture data at 23 bars with $H_{\parallel}$. Data at 19 bars, with $H_{\perp}$ and nonuniform texture is shown as solid circles.

A direct comparison can be made between the superfluid densities of the A and B phases at the AB transition. Although the 27-bar data extend to temperatures well below T_{AB}, supercooling of the A phase prevented observation of the B phase at this pressure. At the lower pressures of 19 and 23 bars, the transition from the A to B phase does occur as indicated by the NMR and by a discontinuous jump in the viscosity and the superfluid density.[13] Superfluid density data for these two pressures for two different magnetic field orientations are shown in Fig. 3. For the 23-bar data the magnetic field has been set parallel to the plane of the torsional cell producing the uniform texture. In this case there is a decrease in the superfluid density in passing from the A to B phase. In contrast, an increase is seen in the 19-bar data where the magnetic field is set to produce the nonuniform texture.

We have made a number of observations of the jump in the superfluid density between A and B while both warming and cooling through the phase change. We summarize these observations in Fig. 4 by plotting the ratio of the A phase superfluid density to that of the B phase against the reduced temperature at which the phase change occurred. There is one anomalous observation where the phase change is clearly seen in the viscosity but is absent in the superfluid density data. With the exception of this point, all the ratios obtained for transitions between the uniform A phase texture and the isotropic B phase lie near a value of 1.24. This is to be compared with the value of 6/5, of the ratio, $\rho_{sA\perp}/\rho_{sB}$, expected from theory for the region near the polycritical

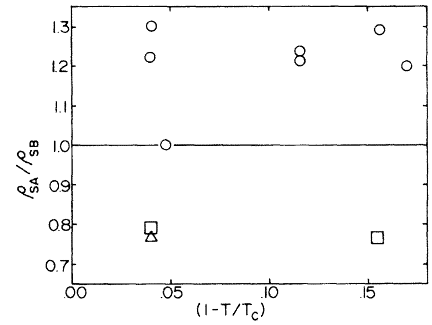

FIG. 4. Ratios of the A-phase superfluid density to the B-phase value are plotted against the reduced temperature at which the phase change was observed. Open circles indicate data at 23 bars with $H_{\parallel}$; open squares, 23-bar data with $H_{\perp}$; and the open triangle 19 bars with $H_{\perp}$.

point. A recent paper by Kuroda and Nagi[14] indicates that the ratio, $\rho_{sA\perp}/\rho_{sB}$, increases from a value of 1.03 at zero pressure to 1.28 at the melting curve, with a value of about 1.21 at 23 bars. The data at 23 bars, then, are in reasonable agreement with theoretical expectations.

The data of Fig. 4 may be used to obtain the ratio between the superfluid density values for the uniform and nonuniform A-phase textures. In our experiment, several weeks may pass between runs with different orientations of the magnetic field, so we use the superfluid density of the isotropic B phase as a reference to avoid systematic errors which could arise from uncertainty in the temperature or shifts in the sensitivity of the apparatus. At 23 bars, the ratio of the means of the values obtained for the two different directions of the magnetic field is 0.62, a value in close agreement with the estimate of Fetter.[12] One may infer from this agreement that the bending pattern of the nonuniform A-phase texture is dominantly in the azimuthal direction.

Comparison of our results with previous measurements[2-4] in less-well-defined textures must be qualitative since the anisotropy of the A-phase superfluid density is such a large effect. The results of the experiment by Kojima, Paulson, and Wheatley,[3] in a 50-μm parallel-plate geometry, where the boundary condition would be expected to favor a uniform texture, are also in reasonable quantitative agreement with our results. The major difference lies in the smaller difference in ρ_s between the A and B phases. When making intercomparisons between various sets of measurements, one must also keep in mind the fact that the temperature scales of different laboratories are still not in unverisal agreement.

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[13]Discontinuous changes in viscosity and superfluid density at the *AB* phase change were first observed by Alvesalo *et al.* (Ref. 2). Further work is in progress on the analysis and interpretation of the viscosity data from the present experiment. The data do show the quantity $(\eta_A - \eta_B)/\eta_B$ to have a value of about 0.13 at T_{AB}.

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New Applications of X-Ray Standing-Wave Fields to Solid State Physics

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An x-ray standing-wave–field apparatus has been constructed that demonstrates an impurity atomic position resolution of ~0.02 Å. The method is applied to the measurement of ion-implantation–induced lattice relaxation. It is pointed out that the method is also suited to determine surface impurity locations perpendicular to crystal surfaces.

It is by now well known that x-ray standing-wave fields are excited in nearly perfect crystals under the conditions of strong diffraction. The systematic study of this dynamical diffraction effect in the case of the Laue geometry has led to several interesting applications of x-ray diffraction to solid state and atomic physics via the Bormann effect,[1] pendellösung fringe measurements,[2] and x-ray interferometry.[3] The Bragg geometry, despite its simpler nature, appears in recent times to have played a somewhat minor role in terms of dynamical diffraction applications. Nevertheless, using the recent development of solid-state x-ray detectors of high-energy resolution, Golovchenko, Batterman, and Brown[4] were able to demonstrate the possibility of using the peculiar properties of the Bragg geometry along with x-ray fluorescence techniques to locate the atomic position of impurity atoms in a crystal.

This work, following the studies of Batterman,[5,6] utilized the dynamical-theory prediction[7] that for