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Use of Dipole Sum Rules to Estimate Upper and Lower Bounds for Radiative and Total Widths of $\chi(3414)$, $\chi(3508)$, and $\chi(3552)$*

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Upper and lower bounds on the widths for $\chi_J \to \gamma\psi(3095)$ can be estimated by assuming $E1$ transitions and approximate Russell-Saunders coupling for the $c\bar{c}$ system. Experimental widths for $\psi(3684) \to \gamma\chi_J$ make the lower bound more restrictive, giving radiative widths of 160 → 240, 230 → 400, and 280 → 480 keV for 3414-, 3508-, and 3552-MeV states, respectively. Cascade branching ratio data permit estimation of the total widths as > 1.6, 0.3–1.5, and 0.6–4 MeV, respectively.

In the spectroscopy of new particle states uncovered in e^+e^- annihilation it is now rather clearly established that the three states[1-3] generically labeled as χ have $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ for the 3414-, 3508-, and 3552-MeV states, respectively.[4] The spin and parity values and ordering of these states are just what is expected of the triplet p states in any $q\bar{q}$ bound-state model that parallels positronium.[5,6] The χ states are formed by the radiative decay $\psi(3684) \to \gamma\chi$. They are observed to decay into hadrons and also, for the $J=1$ and $J=2$ (and marginally for the $J=0$) via the two-photon cascade, $\psi(3684) \to \gamma_1\chi \to \gamma_2\psi(3095)$. Recently, branching ratios have been reported for the $\psi(3684) \to \gamma\chi_J$ transitions[7,8] and also products of branching ratios for the cascade transitions.[8-10] These are summarized in Fig. 1.

The view that these states are describable to a good approximation by a nonrelativistic potential model, with v^2/c^2 corrections, receives increasing support from the data.[6] I adopt this picture here. In the Russell-Saunders limit (J^2, J_z, L^2, and S^2 diagonal) the states have the designations shown in Fig. 1. The details of the binding potential need not concern us, but I make the assumption from the outset that tensor forces, relativistic effects, coupled channel effects, etc. are unimportant enough that they do not vitiate my use

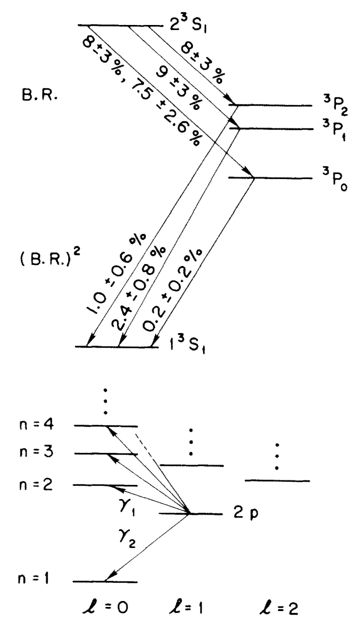

FIG. 1. (Top) Observed radiative transitions through the χ states. For the first transitions the numbers are branching ratios (Ref. 7; for the $J=0$ final state the second number is from Ref. 8). For the second step the numbers are the *products* of the branching ratios (Refs. 8 and 10). (Bottom) Schematic diagram showing the transitions involved in the second sum rule, used to set *lower limits* on the radiative widths.

of the second sum rule.

The branching ratios shown in Fig. 1 for $\psi(3684) \to \gamma\chi_J$ can be converted into radiative widths using $\Gamma_t \cong 228$ keV[11]: $\Gamma(\psi' \to \gamma\chi_J) = 17.5 \pm 6$, 20 ± 7, and 18 ± 7 keV, for the $J^{PC} = 0^{++}$, 1^{++}, and 2^{++} states, respectively.[12] Values in the range from 10 to 50 keV emerge from bound-state models, provided the quark charges are $e_Q = \pm\frac{2}{3}$.[13,6] Furthermore, with the experimentally favored J assignments, the experimental products $\Gamma_J/(2J+1)k^3 = (10 \pm 3) \times 10^{-4}$, $(13 \pm 5) \times 10^{-4}$, and $(16 \pm 6) \times 10^{-4}$ GeV^{-2} show constancy within errors. This indicates that the $E1$ rate formula,

$$\Gamma(\psi' \to \gamma\chi_J) = (4/27)\alpha e_Q^2(2J+1)k^3|\langle 2p|r|2s\rangle|^2, \qquad (1)$$

is approximately valid, with a common matrix element for all three transitions. Though other multipoles are possible in principle for the $J = 1, 2$ states, I assume complete $E1$ dominance for the transition rates of concern here.[14]

The widths for the radiative transitions $\chi_J \to \gamma\psi(3095)$ can be calculated in bound-state models, but cannot be compared with the data on the square of the branching ratio, R_B^2, without knowledge of the total widths of the χ_J states. I show here that, with $E1$ dominance of the $\psi' \to \gamma\chi_J$ and $\chi_J \to \gamma\psi$ transitions and the approximate validity of Russell-Saunders coupling, upper *and lower* limits can be set on the widths for $\chi_J \to \gamma\psi$, limits that are stringent enough to provide estimates of the total widths of the χ_J states. Given the experimental and theoretical uncertainties, these latter quantities are rather rough, but they may well be the only semi-experimental estimates available for some time. The present experimental widths are just upper limits of perhaps 10 MeV, based on energy and measurement uncertainties.[2-4]

I use two dipole sum rules.[15] The first is the well-known Thomas-Reiche-Kuhn sum rule,

$$2\mu\sum_j \omega_{ji}|\langle j|\vec{r}|i\rangle|^2 = 3, \qquad (2)$$

where μ is the reduced mass of the two-particle system ($\hbar = c = 1$ here). With the ground state $\psi(3095)$ as the initial state, Eq. (2) permits an upper limit to be set on the $E1$ widths of the transitions $\chi_J \to \gamma\psi$ in the well-known way[16]:

$$\Gamma(\chi_J \to \gamma\psi) < 2\alpha e_Q^2 k^2/3\mu. \qquad (3)$$

With $e_Q = \frac{2}{3}$ and $2\mu = m_c = 1.65$ GeV, this gives the values shown in Table I. These upper bounds are of course dependent on our assumptions about quark charges and masses. The charge choice of $\frac{2}{3}$ is strongly indicated by the semiquantitative agreement of the radiative widths for $\psi' \to \gamma\chi_J$, already mentioned—a factor of 4 smaller calculated rates seems unreasonable. The effective quark mass is perhaps less certain, but the remarkable agreement of the calculations of De Rújula, Georgi, and Glashow[17] with the observed masses and mass splittings of the charmed baryons[18] indicates that my choice cannot be appreciably wrong.

TABLE I. Upper and lower bounds on radiative widths for $\chi_J \to \gamma_2\psi$. [Masses and photon energies in MeV, widths in keV. $e_Q = 0$ column is second term only from Eq. (5).]

M	k_1	k_2	Upper bound	Lower bounds $e_Q = \frac{2}{3}$	Lower bounds $e_Q = 0$
3414	260	304	240	160	80
3508	172	389	400	230	100
3552	130	428	480	280	120

A lower bound can be obtained by use of a dipole sum rule[19] that involves only the transitions $n, l \to n', l' = l-1$. I apply it to the $2p \to ns$ transitions shown in the bottom half of Fig. 1. For these the sum rule reads

$$2\mu\sum_n \omega_{ns,2p}|\langle ns|r|2p\rangle|^2 = -1. \qquad (4)$$

The beauty of this sum rule is twofold. The -1 on the right-hand side shows that the downward $2p \to 1s$ transition (γ_2 in Fig. 1) dominates the sum since it is the only term with a negative energy difference. This means that we can obtain a *lower bound* on the γ_2 rate. Furthermore, the $2p \to 2s$ contribution is known from $\psi' \to \gamma\chi_J$. This will raise the lower bound significantly. Expressing the lower bound for the width of $\chi_J \to \gamma\psi$ as much as possible in terms of experimental quantities, I write[20]

$$\Gamma(\chi_J \to \gamma_2\psi) \geq \frac{2\alpha k_2^2}{9\mu}e_Q^2 + \frac{3}{2J+1}\left(\frac{k_2}{k_1}\right)^2\Gamma(\psi' \to \gamma_1\chi_J). \qquad (5)$$

Comparison with Eq. (3) shows that the first term is one third of the upper bound. Note that the second term sets, within my assumptions, an "absolute" lower bound, independent of quark charges and masses. With the experimental widths for the upper transition I find the lower bounds and "absolute" lower bounds shown in

Table I. These values have experimental uncertainties of ~30%, at least for the "absolute" lower bound.

Table I shows that the radiative widths for $\chi_J \to \gamma\psi$ are rather closely delimited by the upper and lower bounds of Eqs. (3) and (5). In particular, the experimental branching ratios[7,8] for $\to \gamma\chi_J$ set relatively model-independent "absolute" lower limits of the order of 100 keV for all three transitions.

The branching ratios[8] for the cascade transitions $\psi' \to \gamma_1\chi_J \to \gamma_2\psi$ can be used, together with the bounds of Table I, to estimate the total widths of the χ_J states. With 0.08⁻, 0.09, and 0.08 for the branching ratios for $\psi' \to \gamma_1\chi_J$ [7,8,12] for $J=0, 1, 2$, the $\chi_J \to \gamma_2\psi$ branching ratios are estimated to be 0.025 ± 0.025 (or 0.065 ± 0.04 [10]), 0.27 ± 0.09, and 0.125 ± 0.075. The errors here are only the errors in the cascade R_B^2 values. There is an additional uncertainty of ~30% from the branching ratios for the first transition. A series of estimates for bounds on the total widths of the χ_J states are given in Table II. The "absolute" (A) lower bounds are computed by dividing the $e_Q=0$ bound from Table I by the *sum* of the central value of the radiative branching ratio and its estimated error. Similarly, an "absolute" upper bound uses the radiative upper bound from Table I and the *difference* of the central value and its associated error for the branching ratio. The plausible (P) upper and lower bounds come from the $e_Q = \frac{2}{3}$ columns in Table I, divided by the central values of the branching ratios.

The estimates in Table II for total widths are presently uncertain by ± 50% or more because of experimental uncertainties in the various branching ratios, apart from theoretical uncertainties. Nevertheless, they presumably provide at least order-of-magnitude estimates of the total widths of the χ_J states. The relative values within each column should be more reliable.

TABLE II. Estimated upper and lower bounds on the total widths of the χ_J states. [All masses and widths are in MeV. The estimates in parentheses for $J=0$ are based on the R_B^2 of Ref. 10. A means "absolute," P plausible.]

		Lower bounds		Upper bounds	
J	M	A	P	P	A
0	3414	1.6	6.4	9.6	∞
		(0.8)	(2.5)	(3.7)	(9.6)
1	3508	0.3	0.9	1.5	2.2
2	3552	0.6	2.2	3.8	9.6

Predictions[21,22,6] from an SU(4) ⊗ SU(3) color gluon gauge theory can be compared with the ranges in Table II. The annihilation rate for $\chi \to$ gluons and/or $q\bar{q}$ is supposed to represent the annihilation into ordinary hadrons. A typical rate is $\Gamma(\chi_0 \to gg) \simeq 96\alpha_s^2|R'(0)|^2/M^4$, with $\Gamma(\chi_2 \to gg) = \frac{4}{15}\Gamma(\chi_0 \to gg)$. For the $J^{PC} = 1^{++}$ and 1^{+-} states, the formula involves an additional factor of $\alpha_s \ln[4m^2/(4m^2 - M^2)]$ and is less reliable.[23] These rates are proportional to the square of the radial derivative of the p-state wave function at the origin, a quantity that varies as the fifth power of the scale parameter of the bound-state wave functions. Estimates range from $|R'(0)|^2 = 0.04$ GeV^5 [24] to 0.09 GeV^5.[21] A central value of 0.06 GeV^5 and $\alpha_s = 0.19$ gives $\Gamma(\chi_0 \to gg) \simeq 1.5$ MeV, $\Gamma(\chi_2 \to gg) \simeq 0.4$ MeV, and, less reliably, $\Gamma[\chi_1 \to g(\bar{q}q)] \simeq 0.13$ MeV. Including the radiative decays, I estimate the "theoretical" total widths to be ~1.5, ~0.2, ~0.45 MeV for $J=0, 1, 2$. These correspond roughly to the "absolute" lower bounds of Table II (unless the cascade branching ratio of Ref. 10 is used). No very compelling conclusion follows from this comparison. Because of sensitivity to $|R'(0)|^2$ it may be more reasonable to use the ranges in Table II to restrict the parameters in one's model of charmonium.

The upper and lower bounds in Table I are exact statements in the limit of $E1$ transitions only and Russell-Saunders coupling with small splittings and no configuration mixing. The reality is that the triplet-singlet splitting of the s states is apparently large, the p states are relatively widely split, and their successive spacings do not satisfy the Landé interval rule. To understand the p-state splittings it is necessary to include a tensor force contribution as well as the spin-orbit coupling.[25,6] There are, in addition, potential complications from relativistic effects and coupled channels above the charm threshold. For a relatively low-lying transition such as $\chi_J \to \gamma\psi$, the mixing of d states into the s and f states into the p states by the tensor force may not be a serious problem. Certainly we can say that the bounds in Table I are not strict bounds. I can only hope that they provide reasonable limits on the expected radiative widths from which the rough ranges of Table II for total widths follow.

Note added.—(a) The bounds in Eqs. (3) and (5) are strictly valid only for negligible spin-orbit interaction. They can be generalized to first order in the strength of the spin-orbit coupling. Let $\bar{k}_i = (k_{i0} + 3k_{i1} + 5k_{i2})/9$ be the weighted average photon energy for the first or second transition of Fig. 1. The extension of Eq. (3) is obtained by replacing $k^2 \to k_{2J}^3/\bar{k}_2$. The upper bounds in Table I then become 180, 390, and 510 keV, respectively. The generalization of Eq. (5) has k_2^2 in both terms replaced according to $k_2^2 \to k_{2J}^3/\bar{k}_2$, *and* $[3/(2J+1)]\,\Gamma(\psi' \to \gamma_1\chi_J)/k_1^2$ replaced by the unweighted arithmetic average of $\Gamma(\psi' \to \gamma_1\chi_J)/k_{1J}^2$ for $J=0, 1, 2$. The revised lower bounds of Table I (for $e_Q=\frac{2}{3}$) are then 110, 230, and 300 keV, respectively. The revised upper and lower bounds differ significantly from those in Table I only for the $J=0$ state where $240 \to 180$ and $160 \to 110$ keV. (b) The two sum rules can be used alternatively to set an interesting upper bound on the transition rates for $\psi' \to \gamma_1\chi_J$, namely,

$$\Gamma(\psi' \to \gamma_1\chi_J) \leq (4\alpha e_Q^2/27\mu)(2J+1)k_{1J}^3/\bar{k}_1.$$

This yields upper limits of 65, 56, and 40 keV for $J=0, 1, 2$. These are only a factor of 2 to 3.7 larger than observation and a factor of 6 less than the bound based on the Thomas-Reiche-Kuhn sum rule alone.[15]

This last remark on the new upper bound is entirely due to M. Chanowitz who also urged on me the correct treatment of the spin-orbit interaction at least to first order. I thank him for helpful correspondence and thank K. Gottfried for drawing Ref. 15 to my attention.

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[1] W. Braunschweig *et al.*, Phys. Lett. 57B, 407 (1975).

[2] G. J. Feldman *et al.*, Phys. Rev. Lett. 35, 821 (1975).

[3] W. Tanenbaum, Phys. Rev. Lett. 35, 1323 (1975).

[4] For the evidence that leads to these assignments, see G. J. Feldman, in Proceedings of the SLAC Summer Institute on Particle Physics: Weak Interactions at High Energies and the Production of New Particles, Stanford, California, 2–13 August 1976 (to be published).

[5] T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975), and Phys. Rev. D 12, 1404 (1975); T. Appelquist *et al.*, Phys. Rev. Lett. 34, 365 (1975); E. Eichten *et al.*, Phys. Rev. Lett. 34, 369 (1975)

[6] For an elementary review, see J. D. Jackson, Lawrence Berkeley Laboratory Report No. LBL-5500, August 1976 (unpublished), and in Proceedings of the SLAC Summer Institute on Particle Physics: Weak Interactions at High Energies and the Production of New Particles, Stanford, California, 2–13 August 1976 (to be published).

[7] D. H. Badtke *et al.*, in Proceedings of the Eighteenth International Conference on High-Energy Physics, Tbilisi, U. S. S. R. 15–21 July 1976 (unpublished).

[8] J. S. Whitaker *et al.*, to be published.

[9] In Ref. 8 four $\gamma\gamma$ events are associated with a state at 3455 MeV (or 3340 MeV). This might be the 0^{-+} partner of the $\psi(3684)$. I do not consider this state here.

[10] For the $J=0$ initial state, the R_B^2 shown in Fig. 1 is based on one event from SPEAR. If the two events reported by B. H. Wiik [in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California, 1975*, edited by W. T. Kirk (Stanford Linear Accelerator Center, Stanford, Calif., 1975), p. 69] are added, this ratio becomes $0.5 \pm 0.3\%$.

[11] V. Lüth *et al.*, Phys. Rev. Lett. 35, 1124 (1975).

[12] The quoted errors do *not* include the $\pm 25\%$ uncertainty in the total width of the $\psi(3684)$.

[13] E. Eichten *et al.*, Phys. Rev. Lett. 36, 500 (1976).

[14] The *charge* coupling invoices the matrix element of $\vec{\epsilon}\cdot\vec{r}\cos(\vec{k}\cdot\vec{r}/2)$. Replacement of the cosine by unity introduces errors in the rates of 15–20% at most, much less for the softest photons (Ref. 6).

[15] The application of the Thomas-Reiche-Kuhn sum rule to charmonium transitions has been considered also by T. N. Pham and T. N. Truong, Phys. Lett. 64B, 51 (1976).

[16] The $E1$ rate for $\chi_J \to \gamma\psi$ is obtained from Eq. (1) by the substitutions $(2J+1) \to 3$, $\langle 2p|r|2s\rangle \to \langle 2p|r|1s\rangle$.

[17] A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[18] E. G. Cazzoli *et al.*, Phys. Rev. Lett. 34, 1125 (1975); W. Y. Lee, in Proceedings of the SLAC Summer Institute of Particle Physics: Weak Interactions at High Energies and the Production of New Particles, Stanford, California, 11–13 August 1976 (to be published).

[19] H. A. Bethe and E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic, New York, 1957), p. 256, Eq. (61.4).

[20] There is some arbitrariness in using Eq. (4) because the p-state splittings are not negligible. Empirically, the quantity $\omega_{2s,2p}|\langle 2s|r|2p\rangle|^2$ deduced from $\Gamma(\psi' \to \gamma\chi_J)/(2J+1)k^2$ is remarkably constant for the three J values. Thus recipes different from (5) lead to results differing only modestly from those of Table I.

[21] R. Barbieri, R. Gatto, and R. Kögerler, Phys. Lett. 60B, 183 (1976).

[22] R. Barbieri, R. Gatto, and E. Remiddi, Phys. Lett. 61B, 465 (1976).

[23] For the 1^{+-} state $({}^1P_1)$, the rate into three gluons is found in Ref. 22 to be $\frac{5}{6}$ times of annihilation for χ_1.

[24] K. Gottfried and K. M. Lane, private communication.

[25] J. Pumplin, W. Repko, and A. Sato, Phys. Rev. Lett. 35, 1538 (1975); H. J. Schnitzer, Phys. Rev. Lett. 35, 1540 (1975).