For a $V-A$ interaction, $a \approx 1$ and $b \approx 0$, while a V or A interaction has $a=b=\frac{1}{2}$. The best fit is obtained by varying a and b until the expected value of $\sigma_N^{\bar{\nu}}/\sigma_N^{\nu}$ agrees with the corrected experimental value. For the best fit, $\sigma_N^{\bar{\nu}}/\sigma_N^{\nu}=0.48 \pm 0.20$, $a=0.85$, and $b=0.15$. These results are confirmed by the measured, essentially uniform dependences of R^{ν} and $R^{\bar{\nu}}$ on E_H which do not, however, sensitively discriminate among the possible forms of the neutral current.

In summary, measurements of neutral-current and charged-current inelastic scattering of ν and $\bar{\nu}$ rule out $V+A$, and are incompatible with a pure V or pure A form for the weak neutral current. The experimental results require a significant parity-nonconserving component in the weak neutral current, and are consistent with $V-A$.

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[1]G. Myatt, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973,* edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974), p. 389; F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973); A. Benvenuti *et al.*, Phys. Rev. Lett. 32, 800 (1974).

[2]B. Aubert *et al.*, Phys. Rev. Lett. 32, 1454, 1457 (1974).

[3]A. Benvenuti *et al.*, Nucl. Instrum. Methods 125, 447, 457 (1975).

[4]A. Benvenuti *et al.*, Phys. Rev. Lett. 37, 189 (1976).

[5]No account has been taken of a possible isoscalar component of the neutral current which is no $V-A$.

Remarks on Electromagnetic Splittings in Charmed Mesons*

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It has recently been predicted by De Rújula *et al.* that $m(D^+)-m(D^0)\sim 15$ MeV. The purpose of this Letter is to criticize this prediction and to examine in detail the mechanism responsible for meson mass splittings. It is concluded that electromagnetic splittings of hadrons cannot reliably be estimated using the present atomic models of hadrons. New terms are probably needed in the electromagnetic part of the potential.

The recent findings at SPEAR of a neutral particle at 1860 MeV has been interpreted as one of the predicted charmed mesons, $D^0(c\bar{p})$ or $\bar{D}^0(\bar{c}p)$. So far, the charged partners of these mesons, $D^{\pm}$ have not been detected and DGG2[1] have explained this suppression by hypothesizing that $D^{\pm}-D^0 \sim 15$ MeV. Although some suppression would exist for $D^{\pm}-D^0 \geqslant 5$ MeV, they argue the larger splitting on the basis of ideas to be criticized in this Letter. An independent criticism of their argument has been made by Lane and Weinberg[2] and where our results overlap, I refer to their work.

The $D^{\pm}-D^0$ mass difference is believed by DGG2[1] (and by Lane and Weinberg[2]) to originate from two sources: (a) "electromagnetic" one-photon exchange binding diagrams, and (b) n-p quark mass differences due to weak and electromagnetic quark mass renormalizations. In DGG2, the total splitting was presumed to be dominated by

$$\text{mass(meson 1)} - \text{mass(meson 2)} = \sum_{i=1}^{2}(m_1{}^i - m_2{}^i) + \alpha(Q_1{}^1 Q_1{}^2 - Q_2{}^1 Q_2{}^2)\langle 1/r\rangle_{\text{meson}}, \tag{1}$$

where $m_j{}^i$ and $Q_j{}^i$ are the mass and charge of the ith quark in the jth meson. In this picture, the π splittings are not affected by $m_n - m_p$, so $\langle 1/r\rangle_{\text{meson}}$ can be estimated directly to be 1260 MeV and then $m_n - m_p$ can be found from the K splittings. From this, DDG2 arrive at $m_{D^+} - m_{D^0} \sim 13$ MeV (the mass difference is estimated a bit higher due to the expectation that $\langle 1/r\rangle$ will increase with mass).

(i) Criticism of "electromagnetic" effects: One would expect that by using similar arguments for the baryons one would find comparable values of $\langle 1/r\rangle$. Indeed, from $\frac{1}{2}(\Sigma^+ - \Sigma^-) - \Sigma^0$ we estimate $\langle 1/r\rangle_{\text{baryon}} = 240$ MeV (cf. Lane and Weinberg[2]), which is *not* comparable to $\langle 1/r\rangle_{\text{meson}}$ (as computed by DGG2) and violates the prejudice that simple atomic models somewhat unify mesons and baryons. Indeed, in a one-gluon perturbed linear potential[3,4] (to be explained in detail later), a fit of the low-lying mesons leads to $\langle 1/r\rangle_{K^0} = 401$ MeV. (These masses are included in Table I.) This can be compared to Lane and Weinberg's and Kang and Schnitzer's[6] values for a linear potential, which are, respectively, $\langle 1/r\rangle_{K^0} = 520$ and 410 MeV. Those two estimates serve to quantify the suspicion (expressed above) that DDG2 have found too large a value for $\langle 1/r\rangle$. Lane and Weinberg believe that this is due to the inadvisability of using a potential model for π's. (However, we see from the table of meson masses that the π mass in this potential model is not too badly predicted.) There are two other possible explanations for their overestimate. These are as follows: (a) π^0 masses are difficult to estimate because, as observed in DDG1,[3] the effects of $q\bar{q}$ annihilation diagrams are large enough to significantly alter mass-splitting predictions [although, notice that pure one-photon annihilation diagrams lead to a term proportional to $(3+\vec{S}_1\cdot\vec{S}_2) = 0$ (for π)]. (b) They have ignored spin-orbit and spin-spin electromagnetic effects, as well as nonlinear effects arising from certain terms [left out in (1)] in the potential contributing to strong mass splittings. In the table, these are seen to be quite significant.

TABLE I. Meson masses: Masses are given in MeV. E is chosen so that $\lambda\bar{n}(J=0) = 494$.

	Calculated mass		Experiment[a]
Uncharmed			
π^0	205		135
$\pi^\pm - \pi^0$	3.2		4.6
ρ^0	764		770 ± 10
$\rho^\pm - \rho^0$	1.6		-2.6 ± 2.2
K^0	494		494
$K^0 - K^\pm$	4.07		3.99
$K^{*\pm}$	908		892
$K^{*0} - K^{*\pm}$	0.27		6.1 ± 1.5
φ	1035[b]		1019
Charmed	$m_c = 1650$	$m_c = 1800$	
D^0	1819	1972	1860
$D^+ - D^0$	1.37	1.02	Present work
$D^{*0} - D^{*+}$	0.57	0.74	Unknown
$D^{*0} - D^0$	171	158	~ 140[c]
$\psi(J=1)$	2826	3094	3105
$\psi(J=1) - \psi(J=0)$	79	71	Unknown

[a]Ref. 5.
[b]Assuming no mixing.
[c]Ref. 1.

(ii) Effect of n-p splitting: In atomic models, quark masses enter not only in Σm_i [present in (1)] but also in the kinetic energy denominators and in the perturbation to the SU(3)-independent potential. These effects tend to cancel the simple behavior of Σm_i. In particular, write in center-of-mass coordinates,

$$H = p^2/2\mu + V(r) + H_{\text{pert}} + \Sigma m_i, \tag{2}$$

where $\mu = m_1 m_2/(m_1+m_2)$. Then if $V(r) = a|r|^n$ ($n > -2, n\neq 0$; for binding, $a>0$ if and only if $n\geqslant 0$) WKB arguments applied to the ground state of $V(r) + p^2/2\mu$ imply that $E_0 \propto \mu^{-n/(2+n)}$

$$\Delta E_0 = \frac{dE_0}{dm}\,dm = \frac{-n}{2+n}\left(\frac{\mu}{m_p^{\,2}}\right)E_0\,dm, \tag{3}$$

where dm is the mass difference between $\bar{q}n$ and $\bar{q}p$, and for the potentials above (covering both the cases of infrared slavery and the Coulomb potential), ΔE_0 becomes increasingly negative as μ increases. For $E_0 \sim 500$ MeV (a typical value) with $m(n)\sim m(p) = 340$ MeV, $m(\lambda) = 540$ MeV, and $m(c) = 1650$ MeV

$$(\Delta M)_D - (\Delta M)_K \sim -0.4\left(\frac{n}{2+n}\right)(m_n - m_p). \tag{4}$$

H_{pert} contains terms expected to be fairly small, but dominated by some SU(3)-independent potential (alternatively, one can write V as a sum of, say, an infrared enslaver plus a Coulomb potential responsible for short-distance behavior) as in DGG1 (where this is $-\frac{4}{3}\alpha_S/|r|$). It is easily shown that

$$\langle r^m\rangle \propto \mu\left(\frac{-mn/2}{2+n}\right), \tag{5}$$

so if that perturbation is binding, it also contributes negatively to $(\Delta M)_D - (\Delta M)_K$. Notice that if $V(r) = |a|r^n - |b|r^{n-j}$ ($n>0, n>j>0, |a|>|b|$) then binding occurs but (5) suggests that such a potential might, for certain values of the parameters, lead to a positive value of $(\Delta M)_D - (\Delta M)_K$. That would be desirable, but at the moment there is no reason to believe in such a complicated potential.

In order to estimate the effect of these cancellations, consider a linear potential perturbed by $-\frac{4}{3}\alpha_S/|r|$ with $\alpha_S \sim 1.$[3] Assuming, as was discussed earlier, that $\langle 1/r\rangle \sim 400$ MeV, the K splitting implies that the effect due to the n-p difference, $\Delta^K M(n,p)$, is ~ 5 MeV. From (3) and (5), we calculate that $m_n - m_p$ is approximately equal to 10 MeV. Thus, by (4) [and (5)], $(\Delta M)_D - (\Delta M)_K \sim -3$ MeV and $\Delta^D M(n,p) \sim 2$ MeV. Adding this result to the electromagnetic effect implies that $D^\pm - D^0 \sim 4$–5 MeV, a value much lower than desired! Furthermore (see the table) the K^* and ρ splittings are also badly predicted.

There are two possible implications of these results. The first is that although either DGG2 or Lane and Weinberg may turn out to be right, that will be fortuitous and not based on a complete analysis of the meson electromagnetic splittings. Furthermore, because of the above potential-dependent ambiguities, the value of $D^+ - D^0$ will not determine whether or not an atomic model of meson masses is valid. The second and most important implication of the linear potential calculation above is that if the one-gluon perturbed linear potential model is to be taken seriously (as I believe it should[4]), then either the DGG2 phase space charged meson suppression mechanism is in error and $D^+ - D^0 < 5$ MeV, or some terms have been left out of the "electromagnetic" part of the interactions. The latter view is more appealing to me and is supported as follows: A possible source of error in our electromagnetic estimates might be the fact that we have ignored all diagrams involving combinations of one photon (or W meson) and gluons. Such diagrams would be expected to differ according to whether we are looking at light baryons, light mesons, heavy mesons, etc. Indeed, $m_n - m_p$ may be an effective measure of these effects and I found for baryons that $m_n - m_p \sim 3$–4 MeV, a value much smaller than the 10 MeV calculated for the light mesons. Such calculations are model dependent and probably not worth doing until bound-state calculations for strongly interacting theories are better understood. In the computations done below with the harmonic potential, $m_n - m_p$ was allowed to vary over a wide range and it was observed that $D^+ - D^0 < 10$ MeV in all cases with $|m_n - m_p| < 30$ MeV. This leads to the speculation that even with the considerations above $D^+ - D^0$ is expected to be less than 10 MeV.

Validity of the linear potential DGG1 model: Following the method of Celmaster,[4] the meson mass splittings were computed in a one-gluon perturbed harmonic potential. To within 25% of the splittings these are expected to be the same as for a linear potential.[4] I worked in a center-of-mass frame and $H = H_0 + H_{\text{pert}}$ where[2]

$$H_0 = m_1 + m_2 + \frac{p^2}{2\mu} + \frac{\mu\omega^2 r^2}{2} + E\ ,$$

and

$$\begin{aligned} H_{\text{pert}} = {} & \sum_{i=1}^{2}\left[(p_i^2 + m_i^2)^{1/2} - m_i - \frac{p_i^2}{2m_i}\right] + (\alpha Q_1 Q_2 - \tfrac{4}{3}\alpha_S)\left[\frac{1}{r} - \frac{1}{2m_1 m_2}\left(-\frac{p^2}{r} - \frac{r(\vec{r}\cdot\vec{p})p}{r^3}\right)\right] \\ & - \frac{\pi}{2}\delta^3(r)\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16\vec{S}_1\cdot\vec{S}_2}{3m_1 m_2}\right) - \frac{1}{2r^3}\left\{\left[\frac{1}{m_1^2}\vec{S}_1 + \frac{1}{m_2^2}\vec{S}_2 + \frac{2}{m_1 m_2}(\vec{S}_1 + \vec{S}_2)\right]\cdot\vec{r}\times\vec{p}\right. \\ & \left. - 2\vec{S}_1\cdot\vec{S}_2 + \frac{6(\vec{S}_1\cdot\vec{r})(\vec{S}_2\cdot\vec{r})}{r^2}\right\} + O\left(\frac{\alpha_S}{\pi}, \alpha\right), \end{aligned}$$

where $\vec{S}_i$, Q_i, and m_i are the quark spins, charges, and masses, respectively. The terms are explained by Celmaster[4] and the splittings are computed by a first-order perturbation of the H_0 ground state. ω is chosen to be 600 MeV, which is consistent with the value used for the baryons (details for this are in Celmaster[4]) and α_S was chosen to be 1, in keeping[4] with the fact that the sum of kinetic energies of the meson constituents is slightly smaller than that for the proton constituents (where α_S is found to be 0.8). Masses used were $m_p = 340$, $m_n = 347$, $m_\lambda = 540$, and $m_c = \{^{1650}_{1800}$. [Two m_c masses were chosen because we expect the heavy meson masses to be fairly sensitive to the precise (and unknown) form of the potential (although the charm splittings can be expected to be less sensitive to the potential)]. $I = 0$ meson masses were not predicted because (DDG1) I expect annihilation diagrams to make an important contribution.

From the table, we see that, except for the K^* splittings, the observed mean meson masses agree reasonably well with the predictions. (By the above discussions, the electromagnetic splitting predictions should be modified in some, as yet, unknown way.) Note, however, that the perturbative splittings are of the same order as the H_0 energy levels, so that perturbation theory should serve at most as a guide.

It is satisfying to observe that the charmed system satisfies $D^* - D \sim 160$, which is close to the value needed by DGG2 in order to lead to phase space suppression of $D^{\pm}$.

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[1]A. De Rújula, Howard Georgi, and S. L. Glashow, Phys. Rev. Lett. 37, 398 (1976), referred to as DGG2.
[2]Kenneth Lane and Steven Weinberg, Phys. Rev. Lett. 37, 717 (1976).
[3]A. De Rújula, Howard Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975), referred to as DGG1.
[4]William Celmaster, to be published.
[5]V. Chaloupka, Phys. Lett. 50B, 1 (1974).
[6]J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 461 (1975).