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Locally Supersymmetric Maxwell-Einstein Theory

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The spin-2–spin-$\frac{3}{2}$ gauge multiplet for supergravity is coupled to the spin-1–spin-$\frac{1}{2}$ vector multiplet in a locally supersymmetric way.

The gauge action for local supersymmetry (supergravity) was constructed by Freedman, van Nieuwenhuizen, and Ferrara,[1] and contains the massless spin-2 and spin-$\frac{3}{2}$ gravitational and Rarita-Schwinger fields. Deser and Zumino[2] gave an alternative simpler derivation in which they stressed the importance of torsion in supergravity, and Nath and Arnowitt[3] showed that the theory of Refs. 1 and 2 can be obtained from their superspace approach by taking a singular limit. In this Letter we discuss the next step in supergravity theory: coupling to matter. We couple the spin-1–spin-$\frac{1}{2}$ vector multiplet of global supersymmetry[4] in a locally supersymmetric way to the supergauge action. The key is to start in lowest order of the gravitational constant by coupling the spin-$\frac{3}{2}$ field to the Noether current of global supersymmetry, and to restore local gauge invariance in higher orders by successively adding terms to action and transformation laws in a systematic way. This is thus the same approach as was followed successfully in Ref. 1. We work in second-order formalism without auxiliary fields, but our final results indicate the presence of other auxiliary fields in addition to torsion.

Before commenting on the derivation, we give our final results. The complete action is the sum of the supergauge action $\mathcal{L}^G$ and the matter action $\mathcal{L}^M = \hat{\mathcal{L}}^M + \mathcal{L}_4{}^M$ to be defined below. The latter contains a massless Abelian spin-1 vector field, A_μ (the photon), and a Majorana spin-$\frac{1}{2}$ field, λ, coupled to the spin-$\frac{3}{2}$ Majorana spinor, ψ_μ, and to the gravitational vierbein field, $e_{a\mu}$ (the graviton), by means of two terms: a coupling of ψ_μ to the Noether current of global supersymmetry which is linear in the gravitational constant κ, and a direct four-fermion interaction $\mathcal{L}_4{}^M$ of gravitational strength κ^2:

$$\hat{\mathcal{L}}^M = -\tfrac{1}{4}eg^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \tfrac{1}{2}e\bar{\lambda}\gamma^\mu D_\mu\lambda + \tfrac{1}{4}e\kappa\bar{\psi}_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\lambda F_{\alpha\beta}, \tag{1}$$

$$\mathcal{L}_4{}^M = (e\kappa^2/8)[-(\bar{\psi}\cdot\psi)(\bar{\lambda}\lambda) + (\bar{\psi}_\alpha\gamma_5\psi^\alpha)(\bar{\lambda}\gamma_5\lambda) + \tfrac{1}{4}(\bar{\psi}\cdot\gamma\gamma\cdot\psi)(\bar{\lambda}\lambda) + \tfrac{1}{4}(\bar{\psi}\cdot\gamma\gamma_5\gamma\cdot\psi)(\bar{\lambda}\gamma_5\lambda)$$
$$- \tfrac{1}{2}(\bar{\psi}\cdot\gamma\gamma_5\psi_\alpha)(\bar{\lambda}\gamma_5\gamma^\alpha\lambda) + \tfrac{1}{4}(\bar{\psi}_\alpha\gamma_5\gamma_\rho\psi^\alpha)(\bar{\lambda}\gamma_5\gamma^\rho\lambda) + \tfrac{3}{2}(\bar{\lambda}\lambda)(\bar{\lambda}\lambda)]. \tag{2}$$

The symbol $\bar{\psi}\cdot\psi$ stands for $\bar{\psi}_\alpha\psi^\alpha$ and $\gamma\cdot\psi$ for $\gamma^\alpha\psi_\alpha$.

The gravitational action is the same as in Refs. 1 and 2,

$$\mathcal{L}^G = -\tfrac{1}{2}\kappa^{-2}eR - \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma - (e\kappa^2/32)[(\bar{\psi}^b\gamma^a\psi^c)(\bar{\psi}_b\gamma_a\psi_c + 2\bar{\psi}_a\gamma_b\psi_c) - 4(\bar{\psi}_a\gamma\cdot\psi)^2]. \tag{3}$$

The total action $\mathcal{L}^G + \mathcal{L}^M$ is invariant under the following local supersymmetry transformations on the fields:

$$\delta\lambda = (F_{\mu\nu} + \kappa\bar{\psi}_{\mu}\gamma_{\nu}\lambda)(\sigma^{\mu\nu}\epsilon), \tag{4}$$

$$\delta A_{\mu} = -(\bar{\epsilon}\gamma_{\mu}\lambda), \tag{5}$$

$$\delta e^{a}{}_{\mu} = \kappa(\bar{\epsilon}\lambda^{a}\psi_{\mu}), \tag{6}$$

$$\delta\psi_{\mu} = 2\kappa^{-1}D_{\mu}\epsilon + (\tfrac{1}{4}\kappa\sigma^{ab}\epsilon)[2\bar{\psi}_{\mu}\gamma_{a}\psi_{b} + \bar{\psi}_{a}\gamma_{\mu}\psi_{b}] + \tfrac{1}{4}\kappa(\bar{\lambda}\gamma_{5}\gamma^{\rho}\lambda)[\gamma_{\rho}\gamma_{\mu} - \sigma_{\rho\mu}]\gamma_{5}\epsilon. \tag{7}$$

In these results, D_μ denotes the ordinary gravitationally covariant derivative (without torsion); local Lorentz and world indices are denoted by Latin and Greek letters; $\gamma_5\gamma_5 = \gamma_a\gamma^a = 1$ $(a = 1, 2, 3, 4)$, $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/4$; in flat space $g_{\mu\nu} = g^{\mu\nu} = \delta_\mu{}^\nu = (+, +, +, +)$ (the Pauli metric), and $e = \det(e^a{}_\mu)$.

In order to arrive at these results, we consider the flat-space globally supersymmetric vector multiplet and consider space-time–dependent anticommuting parameters $\epsilon(x)$. The requirement of invariance of the action at zero order in κ fixes uniquely the strength and the form of the third term in $\hat{\mathfrak{L}}^{\mathrm{M}}$. This comes about by demanding cancellation of the $F\lambda\partial\epsilon$ terms in $\delta\hat{\mathfrak{L}}^{\mathrm{M}}$.

As expected, ψ_μ couples to the Noether current[5] in agreement with the consistency requirement[2] that to lowest order in κ the source of the ψ_μ field be conserved.

Invariance to all orders in κ is expected to be arrived at by adding higher-order terms to the action and transformation laws according to the construction of Ref. 1. We also expect electromagnetic gauge invariance to be unbroken, which restricts the number of possible extra terms considerably. This was the reason for considering the vector multiplet instead of, say, the scalar multiplet.

At the order κ level, there are $F^2\psi\epsilon$ and $\partial\lambda\,\lambda\psi\epsilon$ terms in $\delta\hat{\mathfrak{L}}^{\mathrm{M}}$. The first kind of terms does indeed vanish as a consequence of the fact that $F_{\mu\alpha}\epsilon^{\alpha\nu\rho\sigma}F_{\rho\sigma}$ is proportional to $\delta_\mu{}^\nu$. This is a minor miracle, analogous to the cancellation of the order ψ terms in pure supergravity. The $\partial\lambda\,\lambda\psi\epsilon$ terms do not vanish by themselves, but they can be rewritten in a unique and systematic way as either terms obtained by varying $\delta\psi_\mu = 2\kappa^{-1}D_\mu\epsilon$ in $\psi^2\lambda^2$ terms, or terms proportional to the field equations of the λ and ψ fields. Cancellations of the $\partial\lambda\,\lambda\psi\epsilon$ terms can now be obtained by adding to the matter action all $\psi^2\lambda^2$ terms as given in (2) and by modifying the transformations for λ and ψ according to (4) and (7).

To order κ^2 the $\lambda^3F\epsilon$ terms do not vanish, but the same strategy now fixes uniquely the λ^4 term (which happens to be unique for Majorana spinors). At this point, the complete theory has been determined and in order that the theory be fully locally supersymmetric, all other variations have to cancel by themselves. To the same order κ^2, there are only $\psi^2\lambda F\epsilon$ terms which indeed vanish. Finally to order κ^3 the $\lambda^4\psi\epsilon$ and $\lambda^2\psi^3\epsilon$ terms also vanish and we have therefore succeeded in coupling a matter system to supergravity.

There is an elegant alternative to determine which four-point contact terms need to be added to a gauge theory, once all three-point couplings are known. One simply looks at all four-point tree graphs of the S matrix which are constructed from the three-point vertices (simple pole graphs) and verifies whether the S matrix is gauge invariant (in our case under the substitution $\psi_\mu \to k_\mu$). If not, the sum of all pole graphs (with $\psi_\mu \to k_\mu$) must be a local function proportional to k_μ (if there exists at all a gauge-invariant S matrix), from which a direct four-point coupling can be found, which restores the gauge invariance of the S matrix. In this way, we obtained the $\psi^2\lambda^2$ terms in the action, while we found that no $F^2\psi^2$ terms can be present. This same method, which has been used before in the context of the Yang-Mills theory,[6] also reproduces the purely gauge four-fermion coupling of Eq. (3).[7]

We now comment on our results. In addition to four local gauge invariances (coordinate, local Lorentz, electromagnetic, and supersymmetric invariances) the theory is invariant under global chiral transformations, as can be verified by inspection. Our method can in principle be applied to couple supergravity to other matter multiplets.[8] In particular, one might expect similar results for the Yang-Mills multiplet,[9] although the scalar multiplet[4] may have more complicated couplings.

Inspection of the transformation law of ψ as well as of the structure of the $\psi^2\lambda^2$ terms reveals that torsion[2] cannot be the only auxiliary field needed to linearize our Lagrangian. Our method avoids the need to guess the complete set of auxiliary fields. Certainly more auxiliary fields are needed to close the algebra.[10] It seems that introducing the notion of a supersymmetric covariant derivative[11] simplifies the formulation of the theory. It would be interesting to see whether our theory can be obtained from the Nath-Arnowitt superspace approach[12] or from the Zumino non-Riemannian geometry.[12]

Details of this work will be published elsewhere.

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[9]S. Ferrara and B. Zumino, Nucl. Phys. B79, 413 (1974); A. Salam and J. Strathdee, Phys. Lett. 51B, 353 (1974).

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[11]As in quantum electrodynamics and Yang-Mills theory, one can define a supersymmetric connection by replacing the gauge parameter ϵ by the gauge field ψ_μ in the variation of the fields. One finds the covariant derivatives $\hat{D}_\mu A_\nu = \partial_\mu A_\nu + \frac{1}{2}\kappa(\bar{\psi}_\mu \gamma_\nu \lambda)$ and $\hat{D}_\mu \lambda = D_\mu \lambda - \frac{1}{2}\kappa F_{\alpha\beta} \times \sigma^{\alpha\beta}\psi_\mu$, in terms of which one finds $\delta\lambda = \hat{F}_{\alpha\beta}\sigma^{\alpha\beta}\epsilon$, while half of the Noether coupling emerges as the minimal coupling contained in $-\frac{1}{2}\bar{\lambda}\gamma^\mu \hat{D}_\mu \lambda$. We thank Dr. P. Breitenlohner for discussions about supersymmetric covariant derivatives in general.

[12]P. Nath, in *Proceedings of the Conference on Gauge Theories and Modern Field Theory, Boston, 1975* (MIT Press, Cambridge, Mass., 1976), p. 281; B. Zumino, *ibid.* p. 255.

Experimental Confirmation of the Parity of the Antiproton

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The reaction $\bar{p}p \to 2\pi$ with polarized targets exhibits asymmetry. From this we extract the intrinsic parity of the antiproton.

With the availability of intense antiproton beams and polarized targets we are now in a position to investigate fundamental properties of antiprotons. In particular, we would like to raise the question of the *experimental* determination of the parity of the antiproton. On general theoretical grounds one would expect this parity to be such that a $\bar{p}p$ state with orbital momentum L has a parity $(-1)^{L+1}$. If the proton is described by a local relativistic field which transforms in a local manner under both Poincaré and space inversion transformations, then the $\bar{p}$ parity is opposite that to the proton.[1,2] This is most easily seen in terms of the asymptotic proton field which contains annihilation operators for the proton and creation operators for the antiproton. This requirement of quantum field theory transcends the minimum requirement of relativistic invariance, since it is not possible to transform particles into antiparticles by a real Lorentz transformation.

At the present time the true nature of the proton is not known. Many imaginative models of an extended nature are under intense investigation at the present time. It will be important for such models[3] to know whether the intrinsic parity (η)