PHYSICAL REVIEW LETTERS

VOLUME 37 5 JULY 1976 NUMBER 1

Weak Interactions and Eötvös Experiments*

Mark P. Haugan and Clifford M. Will†
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305
(Received 14 May 1976)

We use a current-current model for weak interactions to show that the weak-interaction contribution to the ground state energies of typical nuclei is about a part in 10^8 of their rest masses. By comparing this contribution with the results of the recent versions of the Eötvös experiments, we conclude that weak-interaction energies obey the principle of equivalence to better than a part in 100, thus refuting claims that those experiments did not test weak-interaction effects.

In recent years, substantial theoretical and experimental effort has been directed toward the principle of equivalence and the Eötvös experiment. Improved versions of Baron von Eötvös's original experiment have verified the equality of acceleration in a gravitational field of aluminum and gold to one part in 10^{11} (Princeton University experiment) and of aluminum and platinum to one part in 10^{12} (Moscow State University experiment),[1] and further refinements are planned.[2] The experiment has been interpreted as a test of the universality of free fall (UFF) for different forms of matter and energy,[3] and as a test of nonmetric theories of gravity.[4]

One of the motivations for making improvements in the experiment beyond the part-in-10^{12} level has been to test the effects of weak interactions on UFF.[2] If weak-interaction energies violate UFF, then the ratio of passive gravitational mass to inertial mass for a body should have the form

$$m_p/m_I = 1 + \eta E_W/m_I , \tag{1}$$

where E_W is the contribution to the mass of the body due to weak interactions in the nuclei and η is a dimensionless parameter that depends on the theory of gravity being used. The value $\eta = 0$ implies agreement with UFF. Eötvös experiments set an upper limit on the difference in m_p/m_I for different materials (say A and B) and so would set a limit on the quantity

$$\eta |(E_W/m_I)_A - (E_W/m_I)_B| .$$

Most published estimates give $E_W/m_I \lesssim 10^{-12}$ for typical nuclei[2,5] and so claim that the most recent Eötvös experiments are not sufficiently accurate to test UFF for weak interactions; rather, a decent test would require Eötvös experiments accurate to better than a part in 10^{14}. However, these estimates take into account only the parity-nonconserving parts of the weak interactions, which make no contribution to the energy of a nucleus in its ground state, to first order in the weak-interaction coupling constant, G_W. Nordtvedt[6] has suggested that the parity-conserving parts of the weak interactions will contribute at first order in G_W and so may yield a much larger estimate for E_W/m_I. In this paper, we show that in fact $E_W/m_I \sim 10^{-8}$.

We use a general form for the weak-interaction Hamiltonian valid for low energies that includes both charged and neutral current contributions. Using first-order perturbation theory we evaluate the expectation value of the Hamiltonian for a nuclear ground state consisting of Z protons and N neutrons. Our result is

$$E_W/m_I = 2.2\times 10^{-8}(NZ/A^2)f(N,Z)[1+g(N,Z)], \tag{2}$$

where $A=N+Z$. The function $f(N,Z)$ describes the effects of nuclear structure on E_W. For a free Fermi gas, $f(N,Z)=1$; for realistic nuclear models ranging from aluminum to gold $f(N,Z)$ probably varies by less than a 1%.[7] The function $g(N,Z)$ depends only on the theory of neutral-current weak interactions being used. For the Weinberg-Salam theory,[8] for example, $g(N,Z)$ has the form

$$g(N,Z) = 0.295[\tfrac{1}{2}(N-Z)^2/NZ + 4\sin^2\theta_W + (Z/N)\sin^2\theta_W(2\sin^2\theta_W - 1)], \tag{3}$$

where θ_W is the "Weinberg angle." Because the coefficient NZ/A^2 varies by only a few percent between aluminum ($Z=13, N=14$) and platinum ($Z=78, N=117$) or gold ($Z=79, N=118$), the difference in E_W/m_I between these elements will be rather sensitive to the functions f and g. However, barring an unlucky cancellation of effects, we feel that a reliable estimate of this difference may be made by taking $f(N,Z)=1$ and $g(N,Z)=0$ (no neutral-current effects) in Eq. (2). Then we obtain (for either platinum or gold),

$$(E_W/m_I)_{\mathrm{Al}} - (E_W/M_I)_{\mathrm{Pt(Au)}} \sim 2\times 10^{-10}. \tag{4}$$

Comparing this estimate with the results of the Princeton and Moscow versions of the Eötvös experiment we obtain the limits

$$|\eta| < 5\times 10^{-2}\ \text{[Princeton experiment]}, \quad |\eta| < 5\times 10^{-3}\ \text{[Moscow experiment]}. \tag{5}$$

Hence we conclude that *weak interactions obey the universality of free fall to within one part in 100.*

The remainder of this paper sketches details of the derivation of Eq. (2).

We begin with an independent-particle model for nuclear matter characterized by distinct proton and neutron Fermi energies, and describe the nucleon-nucleon weak interaction by the Hamiltonian,[9] valid for low momentum transfer,

$$\begin{aligned}&\hat{H}_W = 2^{-1/2}G_W[\hat{J}_\mu^{(c)\dagger}\hat{J}^{(c)\mu} + \hat{J}_\mu^{(n)\dagger}\hat{J}^{(n)\mu}], \quad J_\mu^{(c)} = \overline{\Psi}\gamma_\mu(1-\alpha\gamma_5)\tau_+\psi,\\ &J_\mu^{(n)} = A_s\overline{\Psi}\gamma_\mu(1-a_s\gamma_5)B\psi + A_v\overline{\Psi}\gamma_\mu(1-a_v\gamma_5)\tau_3\psi,\end{aligned} \tag{6}$$

where, in the standard isospin representation,

$$\psi = \begin{pmatrix}\psi_p\\ \psi_n\end{pmatrix}, \quad B = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \quad \tau_+ = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \quad \tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \tag{7}$$

and ψ_p and ψ_n are proton and neutron field operators. The dimensionless parameters A_s, A_v, a_s, and a_v depend on the theory of neutral-current weak interactions being used, and the parameter α has the value 1.21 ± 0.03 imposed by β-decay measurements.[10] Ignoring the creation of nucleon-antinucleon pairs, we write, in the language of second quantization,

$$\psi_p(\vec{x}) = V^{-1/2}\sum_{\vec{p},\vec{s}}\hat{b}(\vec{p},\vec{s})u(\vec{p},\vec{s})e^{i\vec{p}\cdot\vec{x}}, \quad \psi_n(\vec{x}) = V^{-1/2}\sum_{\vec{p},\vec{s}}\hat{c}(\vec{p},\vec{s})v(\vec{p},\vec{s})e^{i\vec{p}\cdot\vec{x}}, \tag{8}$$

where $\hat{b}$ and $\hat{c}$ are proton and neutron annihilation operators, u and v are Dirac spinors, and $\vec{p}$ and $\vec{s}$ denote three-momentum and spin, respectively. We choose as nuclear wave functions plane waves $e^{i\vec{p}\cdot\vec{x}}$ confined to a box of volume V.

In first-order perturbation theory the contribution of the weak Hamiltonian $\hat{H}_W$ to the total energy is

$$E_W = \tfrac{1}{2}\int d^3x\langle 0|\hat{H}_W(\vec{x})|0\rangle, \tag{9}$$

where $|0\rangle$ represents the unperturbed nuclear ground state. We substitute Eqs. (6), (7), and (8) into Eq. (9) and expand in powers of p/m, where m is the mass of a nucleon. Our result is

$$E_W = G_W 2^{-3/2}V^{-1}\{NZ[(3\alpha^2-1)+4A_nA_p] + \tfrac{1}{2}N^2A_n^2(1+3a_n^2) + \tfrac{1}{2}Z^2A_p^2(1+3a_p^2) + O(p/m)^2\}, \tag{10}$$

where

$$A_n = A_s - A_v, \quad a_n = (A_s a_s - A_v a_v)/(A_s - A_v), \quad A_p = A_s + A_v, \quad a_p = (A_s a_s + A_v a_v)/(A_s + A_v). \tag{11}$$

Those terms in the expansion of $\hat{H}_W$ that are linear in $\vec{p}$ are of odd parity and so make no contribution to E_W. The terms of $O(p/m)^2$ represent relativistic corrections to Eq. (10) that are of order $(p/m)^2 \sim (E_F/m) \sim 10^{-2}$. Using numerical values $V = 5.13\times 10^{-39}A$ cm^3 (corresponding to nuclear density $\rho = 1.95 \times 10^{38}$ cm^{-3}), $m_I = 0.931\times 10^3 A$ MeV, and $G_W = 0.896\times 10^{-43}$ MeV cm^3, we obtain

$$E_W/m_I = 2.2\times 10^{-8}(NZ/A)^2\{1 + 0.295[4A_nA_p + \tfrac{1}{2}(N/Z)A_n^2(1+3a_n^2) + \tfrac{1}{2}(Z/N)A_p^2(1+3a_p^2)]\}. \tag{12}$$

In Eq. (10), it is our use of a point-interaction Hamiltonian that is responsible for the simple dependence of E_W/m_I on the products of densities of the interacting particles: NZ/A^2, N^2/A^2, and Z^2/A^2.[11] Furthermore, the use of realistic nuclear wave functions (as opposed to plane waves) in the computation will not alter the result significantly[12]; the function $f(N,Z)$ that we have used in Eq. (2) should be unity to better than 1%.

The detailed nucleon-nucleon weak interaction is probably more complex than the model we have used, yet simple arguments from a current-current Hamiltonian do agree with the magnitude of observed parity-nonconserving decays in nuclei, in contrast with calculations involving exchanges of vector mesons.[13] We also note the work of Adams[14] which suggests that correlations due to the strong interactions do not alter this picture by more than a factor of 2 or so for final decay-rate or energy-shift results. Hence we feel that Eq. (12) is a realistic estimate for the magnitude and functional dependence of E_W/m_I.

In obtaining Eq. (4), we ignored neutral-current effects. To estimate such effects, we use the Weinberg-Salam model extended to hadrons[15]; using the neutral-current parameters

$$A_v = \tfrac{1}{2}(1 - 2\sin^2\theta_W), \quad A_s = -\sin^2\theta_W, \quad a_v = (1-2\sin^2\theta_W)^{-1}, \quad a_s = 0, \tag{13}$$

appropriate to the model, we find for aluminum and platinum

$$(E_W/m_I)_{\rm Al} - (E_W/m_I)_{\rm Pt} = 2.2\times 10^{-10}(0.40 - 0.98\sin^2\theta_W + 4.24\sin^4\theta_W). \tag{14}$$

For gold, the result differs from Eq. (14) by a few percent.

Allowed values for $\sin^2\theta_W$ fall in a broad range. Weinberg[15] estimated $\sin^2\theta_W = 0.45 \pm 0.15$ in 1974. More recent results indicate a somewhat smaller value. Morfin[16] estimates $\sin^2\theta_W < 0.4$ from results of $\bar{\nu}_\mu e \to \bar{\nu}_e e$ using only the Weinberg-Salam model in interpretation; other, theory-dependent results indicate a lower limit $\sin^2\theta_W > 0.3$.

No choice of θ_W leads to a zero result for the E_W/m_I difference but a minimum of 7.5×10^{-11} occurs when $\sin^2\theta_W \approx 0.12$. For $\sin^2\theta_W = 0.3$ we find 1.1×10^{-10} and for $\sin^2\theta_W = 0.4$ we have 1.5×10^{-10}. These are somewhat reduced from our pure charged-current estimate [Eq. (4)] but do not significantly alter our final conclusion that weak-interaction energies obey UFF to a part in 100.

We are grateful to J. D. Walecka and R. V. Wagoner for helpful comments.

---

*Work supported in part by the National Science Foundation Grant No. MPS 74-15524 A01, and by the National Aeronautics and Space Administration.

†Alfred P. Sloan Foundation Research Fellow.

[1] P. G. Roll, R. Krotkov, and R. H. Dicke, Ann. Phys. (N.Y.) 26, 442 (1964); V. B. Braginsky and V. I. Panov, Zh. Eksp. Teor. Fiz. 61, 873 (1972) [Sov. Phys. JETP 34, 463 (1971)].

[2] P. W. Worden, Jr., and C. W. F. Everitt, in *Experimental Gravitation, International School of Physics "Enrico Fermi," Course LVI,* edited by B. Bertotti (Academic, New York, 1974); P. W. Worden, Jr., Ph. D. thesis, Stanford University, 1976 (unpublished); P. K. Chapman and A. J. Hanson, in *Proceedings of the Conference on Experimental Tests of Gravitation Theories,* edited by R. W. Davies, NASA-Jet Propulsion Laboratory Technical Memorandum No. 33-499 (National Technical Information Service, Springfield, Va., 1971).

[3] L. I. Schiff, Proc. Nat. Acad. Sci. U. S. 45, 69 (1959); R. H. Dicke, in *Relativity, Groups and Topology,* edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964).

[4] A. P. Lightman and D. L. Lee, Phys. Rev. D 8, 364 (1973); C. M. Will and M. P. Haugan, unpublished.

[5] H. Y. Chiu and W. F. Hoffman, in *Gravitation and Relativity,* edited by H. Y. Chiu and W. F. Hoffman (Benjamin, New York, 1964), p. xv; Dicke, Ref. 3, p. 172.

[6] K. Nordtvedt, Jr., private communication.

[7]J. D. Walecka, private communication.
[8]S. Weinberg, Phys. Rev. D 5, 1412 (1972).
[9]We use the standard techniques and notation of, for example, J. D. Walecka and A. L. Fetter, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971); J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[10]See, for example, the discussion by Bjorken and Drell, Ref. 9.
[11]In fact, a point-interaction model permits more general statements about the (N,Z) dependence of E_W. For nuclear ground states, the third component of isospin, T_3, will be a good quantum number. In the Weinberg-Salam model, for instance, this feature is reflected in the $(N-Z)^2 \sim T_3^2$ dependence in Eq. (3). We thank J. D. Walecka for pointing out to us.
[12]We expect the isospin-dependent parts of nuclear wave functions to contribute only at the level (electromagnetic interaction energy)/(rest mass) $\lesssim 10^{-3}$.
[13]F. C. Michel, Phys. Rev. 133, B329 (1964).
[14]J. B. Adams, Phys. Rev. 156, 1611 (1967).
[15]S. Weinberg, Rev. Mod. Phys. 46, 255 (1974).
[16]J. G. Morfin, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California, 1975*, edited by W. T. Kirk (Stanford Linear Accelerator Center, Stanford, Calif., 1975).

Measurement of Nucleon Structure Function in Muon Scattering at 147 GeV/c*

H. L. Anderson, V. K. Bharadwaj, N. E. Booth, R. M. Fine, W. R. Francis, B. A. Gordon, R. H. Heisterberg, R. G. Hicks, T. B. W. Kirk, G. I. Kirkbride, W. A. Loomis, H. S. Matis, L. W. Mo, L. C. Myrianthopoulos, F. M. Pipkin, S. H. Pordes, T. W. Quirk, W. D. Shambroom, A. Skuja, L. J. Verhey, W. S. C. Williams, Richard Wilson, and S. C. Wright

Enrico Fermi Institute and Department of Physics, The University of Chicago, Chicago, Illinois 60637, and High Energy Physics Laboratory and Department of Physics, Harvard University, Cambridge, Massachusetts 02138, and Department of Physics, The University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, and Department of Nuclear Physics, The University of Oxford, Oxford, OX1 3RH, England

(Received 7 April 1976)

Results on the nucleon structure function, νW_2, are presented for $0.2 \leqslant q^2 \leqslant 50$ (GeV/c)^2 and $5 \leqslant \nu \leqslant 130$ GeV. They were obtained by scattering 147-GeV positiive muons inelastically from a liquid deuterium target.

In this Letter, we report the results on the nucleon structure function, νW_2, measured at Fermilab by scattering 2.1×10^{10} positive muons of energy 147 GeV from a liquid deuterium target. Preliminary results for νW_2 and some results for the distributions of muoproduced hadrons from hydrogen have already been reported.[1]

In the first Born approximation, the differential cross section for the scattering of muons of energy E to a final energy E' through an angle θ is related to the two inelastic structure functions W_1 and W_2 by[2]

$$\frac{d^2\sigma}{dq^2 d\nu} = \left(\frac{\pi}{PP'}\right)\frac{2\alpha^2}{q^4}\left(\frac{P'}{P}\right)\left[\left(2EE' - \frac{q^2}{2}\right)W_2(q^2,\nu) + (q^2 - 2m_\mu^2)W_1(q^2,\nu)\right],$$

where $\nu = E - E'$ and $q^2 = 2(EE' - PP'\cos\theta - m_\mu^2)$. The ratio of the inelastic structure functions can be expressed as $W_1/W_2 = (1+\nu^2/q^2)/(1+R)$, where $R \equiv \sigma_L/\sigma_T$ is the ratio of the photoabsorption cross sections for the longitudinal and transverse photons.[3] The values of the nucleon structure function $\nu W_2(\omega, q^2)$ are obtained by assuming $R = 0.18$,[4] where $\omega = 2M\nu/q^2$ is the Bjorken scaling variable.[5] We propose to measure the value of R in subsequent experiments.

Figure 1 is a schematic drawing of the apparatus. Positive muons of 147 GeV/c strike a 122-cm-long, 17.8-cm-diam, liquid deuterium target. The apparatus is triggered when the counter logic condition $\mathrm{B}\cdot\overline{\mathrm{N}}\cdot\mathrm{G}\cdot\mathrm{M}$ is satisfied. B signals the incident muon with no accompanying halo muon (vetoed by hodoscope V). N is the downstream