[7]Earlier we used annealing temperatures exceeding 800°C. This did not lead to junctions showing the pronounced temperature dependence of the current. Cf. G. Baum, E. Kisker, A. H. Mahan, and K. Schröder, to be published.

[8]This is concluded from the field-emission pattern which exhibits a threefold symmetry if the tip is ⟨111⟩ oriented and a twofold symmetry (a rectangular pattern) if the tip is ⟨110⟩ oriented. A comparison in size of the W-EuS emission pattern with that of W, made by scaling the external magnetic field proportional to the square root of the extraction voltage (to insure equal electron optics), also agreed with this assignment. We do not yet know to what extent epitaxial growth is essential for the layer formation.

[9]L. W. Swanson, private communication.

[10]Typically the polarization vector $\vec{P}$ has an angle of about 80° with respect to the tip axis at low magnetic field. When a stronger magnetic field is applied, the direction of $\vec{P}$ turns longitudinally; typically an angle of 10° to the tip axis is observed in a field of 0.5 T. The dependence of current and polarization on external field strength is currently under investigation. At high field strengths a higher polarization as well as an increased emission current has been observed.

[11]P. Wachter, Phys. Kondens. Mater. 8, 80 (1968).

[12]R. H. Good and E. W. Müller, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1956), Vol. 21.

[13]W. Franz, in *Handbuch der Physik*, edited by S. Flügge (Springer, Berlin, 1956), Vol. 17, p. 205. Inside the EuS layer the effective electron mass was assumed to be equal to the free-electron mass (cf. Ref. 2).

[14]The barrier height $\varphi_i(T_c)$ should be equal to the difference between the work function (3.3 eV) and the electron affinity (2.4 eV) of EuS as pointed out by Müller *et al.* (Ref. 3). This is caused by a charged monolayer of $4f^6$ ions at the W-EuS boundary which enables a matching of the Fermi levels of the two materials. Our measured $\varphi_i(T_c) = 0.4$ eV is considerably smaller than that difference and may be caused by the influence of an additional dipole layer near the boundary.

[15]W. A. Thompson, F. Holtzberg, T. R. McGuire, and G. Petrich, in *Magnetism and Magnetic Materials —1971*, AIP Conference Proceedings No. 5, edited by C. D. Graham, Jr., and J. J. Rhyne (American Institute of Physics, New York, 1971), p. 827.

[16]J. Schoenes and P. Wachter, Phys. Rev. B 9, 3097 (1974).

[17]H. C. Siegmann, Phys. Rep. 17, 37 (1975).

[18]G. Busch, M. Campagna, and H. C. Siegmann, J. Appl. Phys. 41, 1044 (1970).

Black Holes in Thermal Equilibrium

G. W. Gibbons and M. J. Perry
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, England
(Received 1 December 1975)

It is argued that a black hole can remain in thermal equilibrium with a heat bath even in the presence of particle interactions. This is achieved by proving the identity of the Hartle-Hawking Feynman propagator and a certain thermal Green's function.

Hawking[1,2] has discussed the problem of particle emission from black holes using quantum field theory on classical background geometries. He has shown that if the particles do not interact among themselves, then the probability for the emission of a particle of energy E relative to infinity, in a state s, from a Schwarzschild black hole of mass M, $P_e(E,s)$, is related to the probability of absorption by the black hole from that state, $P_a(E,s)$, by

$$P_e(E,s) = \exp[-8\pi ME]P_a(E,s) \tag{1}$$

(in units such that $G = c = \hbar = k = 1$). This is, by the principle of detailed balance, a sufficient condition for the black hole to remain in thermal equilibrium with a heat bath of temperature

$$T = 1/8\pi M. \tag{2}$$

Because of the basic nature of this latter result, one would expect that it would remain valid in the presence of particle interactions even though Eq. (1) would no longer do so. This is especially important since the regime in which the Hawking process is of observational significance is that of possible miniature black holes formed in the early universe,[3,4] for which $T \sim 10^{12}$ K and the emitting region is $\sim 10^{-13}$ cm where strong interactions are obviously significant. In this Letter, we shall show that to all orders of perturbation theory for any renormalizable interaction, a nonrotating, neutral black hole can indeed be in thermal equilibrium with a heat bath at a temperature given by (2).

For simplicity we shall restrict our attention to a neutral scalar field φ, mass m. We enclose the black hole in a box with perfectly reflecting walls. For a small enough box (but not so small that it would be crushed by tidal forces), the total energy in the field will be negligible compared with the mass of the hole. We may treat φ as propagating in a fixed Schwarzschild background. There are now two approaches that one can take. Either one regards the problem as the creation of asymptotic particles from an initial no-particle state, or one considers a grand canonical ensemble of states of the field, from the point of view of an observer far from the hole. In both cases one constructs an appropriate propagator for the noninteracting fields, and uses it as the basis for a perturbation calculation of the interacting fields, for instance by setting out the Feynman-Dyson rules. In the first case, the appropriate propagator is the "Feynman Green's function" which has recently been calculated by Hartle and Hawking using the path-integral approach.[5] This describes fluctuations about the vacuum state. In the second case one would construct the "thermal Green's function."[6,7] This describes fluctuations about a system in thermal equilibrium at a temperature T. The point of our proof is to show that these two functions are one and the same thing.

The noninteracting Feynman propagator adopted by Hartle and Hawking is defined by

$$G_F(x, x') = i \lim_{\epsilon\to 0^+} \int_0^\infty \exp(-im^2W - \epsilon/W) \times F(W, x, x')\,dW, \tag{3}$$

and

$$F(W, x, x') = \int \delta[x(w)] \exp\left[\int_0^W \tfrac{1}{4} i g(\dot{x}, \dot{x})\,dw\right]. \tag{4}$$

The second integral is over all continuous paths with parameter $0 \le w \le W$ from x to x'. In order to give definite meaning to (4) it is necessary to analytically continue to complexified coordinates x and x'. $G_F(x, x')$ then becomes the boundary value of that complex solution of the inhomogeneous Klein-Gordon equation which satisfies reflecting boundary conditions at the surface of the box, and which is analytic in Kruskal coordinates except for those pairs of points which may be connected by a null geodesic, or a curve comprised of null geodesic segments which reflect off the surface of the box. The Kruskal advanced and retarded time coordinates, V and U, are related to the Schwarzschild coordinates r and t in the exterior region by

$$U = -\left(\frac{r-2M}{2M}\right)^{1/2} \exp\left(\frac{r-t}{4M}\right), \tag{5}$$

$$V = \left(\frac{r-2M}{2M}\right)^{1/2} \exp\left(\frac{r+t}{4M}\right). \tag{6}$$

This implies that $G_F(x, x')$ is periodic in the Schwarzschild time coordinate differences $t-t'$ with period $8\pi Mi$ and has singularities just above and just below the real $t-t'$ axis corresponding to the null geodesics mentioned above.

The noninteracting thermal Green's function $G_T(x, x')$ describes the state of thermal equilibrium with respect to observers outside and at rest with respect to the black hole. Such observers may describe their observations in terms of the static Schwarzschild coordinates t, r, θ, φ. The function is given by

$$G_T(x, x') = i\langle \tau\varphi(x)\varphi(x')\rangle. \tag{7}$$

τ is the Wick time-ordering operator and $\langle\ \rangle$ denotes the average over a grand canonical ensemble, i.e., for any operator A,

$$\langle A\rangle = \mathrm{Tr}(\rho A)/\mathrm{Tr}\rho. \tag{8}$$

ρ is the density matrix, which for our problem may be taken to be diagonal in a basis of noninteracting many-particle states described by the classical solutions of the homogeneous Klein-Gordon equation of the form

$$e^{-i\omega t}\chi(r, \theta, \varphi),$$

satisfying reflecting boundary conditions at the walls of the box. The diagonal elements will then be $\exp(-E/T)$ where E is the total energy, relative to infinity, of the state, and T the temperature of the enclosure.

Applying the usual formal arguments to the analytic continuation of $G_T(x, x')$, we see that

$$\begin{aligned} G_T(\underline{x}, t; \underline{x}', t') &= i\mathrm{Tr}[e^{-\beta H}\tau\varphi(\underline{x}, t)\varphi(\underline{x}', t)]/\mathrm{Tr}\,e^{-\beta H} \\ &= i\mathrm{Tr}[e^{-\beta H}\tau\varphi(\underline{x}, t)e^{\beta H}e^{-\beta H}\varphi(\underline{x}', t')]/\mathrm{Tr}\,e^{-\beta H} \\ &= i\mathrm{Tr}[e^{-\beta H}\tau\varphi(\underline{x}, t+i\beta)\varphi(\underline{x}', t')]/\mathrm{Tr}\,e^{-\beta H} \\ &= G_T(\underline{x}, t+i\beta; \underline{x}', t'), \end{aligned} \tag{9}$$

since

$$\varphi(\underline{x}, t) = e^{-\beta H}\varphi(\underline{x}, t-i\beta)e^{\beta H}, \tag{10}$$

where we have used the invariance of the trace under cycle permutation. Hence $G_T(x, x')$ is periodic in $t-t'$ with period $i\beta = i/T$, analytic in the strip $\epsilon < \mathrm{Im}(t-t') < 1/T - \epsilon$, and satisfies the inhomogeneous Klein-Gordon equation. In order to give precise mathematical meaning to (5) we may regard these conditions as defining $G_T(x, x')$. It is clear that for the choice $T = 1/8\pi M$, $G_F(x, x')$ and $G_T(x, x')$ are identical. This establishes completely the thermal character of the emitted radiation in the noninteracting case.

Now when interactions are present one wants to find the various "n-point functions" in which all the physical information of the theory is contained. These may be obtained in a perturbation calculation in which one uses the noninteracting propagator as the lowest-order approximation. From the thermodynamic point of view, one seeks expectation values over the grand canonical ensemble using the Hamiltonian (relative to an observer at infinity) of the exact theory. From the Feynman-propagator point of view one seeks the amplitudes for various processes in which particles travel from initial points $x_1, x_2, \ldots$, and arrive at final points $x_1', x_2', \ldots$. Since one starts from the same zeroth approximation the end results will be the same. Indeed, the general periodicity properties will still be true of the exact propagators.

More details and the straightforward generalization to fields of higher spin, and to the case of rotating and charged black holes, will be the subject of a future publication. However, we mention here that for fermions, the Green's functions will be antiperiodic in $t-t'$, and that for rotating and charged black holes the Green's functions will be quasiperiodic in $t-t'$. There also seems to be no obstacle to extending these results to other space-times containing event horizons.[8]

The arguments presented here also provide a partial justification for the heuristic hydrodynamical model adopted by Carter, Lin, and the present authors to discuss the emission process in the high-energy limit.[9]

We should like to thank S. W. Hawking for helpful discussions.

[1] S. W. Hawking, Nature (London) 248, 30 (1974).
[2] S. W. Hawking, Commun. Math. Phys. 43, 199 (1975).
[3] S. W. Hawking and B. J. Carr, Mon. Not. Roy. Astron. Soc. 168, 399 (1974).
[4] B. J. Carr, Astrophys. J. 201, 1 (1975).
[5] J. B. Hartle and S. W. Hawking, Phys. Rev. D (to be published).
[6] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).
[7] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, Reading, Mass., 1962).
[8] G. W. Gibbons and S. W. Hawking, to be published.
[9] B. Carter, G. W. Gibbons, D. N. C. Lin, and M. J. Perry, to be published.