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Stability of Field-Reversed Ion Rings in a Background Plasma*

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The low-frequency stability of a field-reversed ion ring/layer of large Larmor radius in a background plasma is treated by a generalized form of the "energy principle."

Recent advances in the production of multikiloampere ion beams[1] are applicable to the creation of field-reversed ion rings/layers by single pulse injection in the manner already demonstrated for E layers.[2] It has been pointed out[3] that the field-reversal factor $\zeta \equiv \delta B/B_0 = N r_i R_L/R^2$ can be increased by adiabatic magnetic compression, i.e., $\zeta \propto B_0^{1/2}$; here B_0 is the external field, δB is the diamagnetic change in the field on axis, N is the total number of ring ions, $r_i = e^2/mc^2$ is the ion classical radius, R is the mean ring radius, and $R_L = mV_0/eB_0$ is the ion Larmor radius in the external field B_0. Thus injection of a pulse of intense ion beam followed by magnetic compression may be a reasonable technique for the production of a field-reversed ion ring. The low-frequency stability of the combined ion-ring/background-plasma system has not yet been satisfactorily resolved although there have been studies of (i) specific modes,[4] with and without a background plasma, and (ii) stability of plasma confined by ion ring/E layer but with the ring/layer assumed to be rigid.[5] More recently Lovelace[6] has addressed himself to the stability of the combined system. The present calculation is more general and differs from his in that the Vlasov formalism is employed for the ion-ring dynamics as opposed to his approach which limits consideration to rigid displacements of the center of mass of the ring cross section.

We resolve the system stability by a technique employed initially by Newcomb[7] and applied subsequently by Davidson and Krall[8] to unneutralized E layers. The addition of a background plasma enormously complicates the problem. We give below the main results of the analysis, deferring the details to a future publication. Since our objective is to obtain an energy principle that furnishes sufficient conditions for stability we begin by recognizing the following constant of motion of system, viz.,

$$C = U - (\Omega L_z - \int d^3x\, d^3v\, f \ln f),$$

where the total system energy U is composed of plasma kinetic energy $\int d^3x\, \frac{1}{2} nmu^2$, pressure energy $\int d^3x\, p/(\gamma - 1)$, electromagnetic energy $\int d^3x \times (|\vec{E}|^2 + |\vec{B}|^2)/8\pi$ and the ring kinetic energy $\int d^3x \times d^3v\, \frac{1}{2} mv^2 f$, where $f(\vec{x}, \vec{v}, t)$ is the ring distribution; L_z is the z component of the total angular momentum,

$$L_z = \int d^3x\, \hat{z} \cdot \{nm\vec{r} \times \vec{u} + (4\pi c)^{-1} \vec{r} \times (\vec{E} \times \vec{B}) + m \int d^3v\, \vec{r} \times \vec{v} f\},$$

and Ω is an arbitrary constant. The quantity $\Omega L_z - \int d^3x\, d^3v\, f \ln f$ may be regarded as a generalized entropy of the ion ring in which case C is interpreted to be the system free energy when the plasma pressure is neglected. In what follows we limit our discussion to a pressureless cold plasma governed by the continuity equations, the plasma ion momentum balance,

$$nm\, d\vec{u}/dt = ne(\vec{E} + \vec{u} \times \vec{B}), \qquad (1)$$

the electron force balance,

$$\vec{E}+\vec{u}_e\times\vec{B}=0, \tag{2}$$

and flux conservation,

$$\partial\vec{B}/\partial t=\nabla\times\vec{u}_e\times\vec{B}. \tag{3}$$

In equilibrium $\vec{u}_e=\vec{u}=\vec{E}=0$ and

$$n_e=n+n_b \quad \text{(charge neutrality)}, \tag{4}$$

$$\vec{J}\times\vec{B}=T\nabla n_b-n_b m r\Omega^2\hat{r} \quad \text{(ring force balance)}. \tag{5}$$

f_0 is a monotonically decreasing function of $\eta=H-\Omega P_\theta$ and for purposes of detailed calculation it is convenient to choose $\exp(-\eta/T)$; $H\equiv\frac{1}{2}mv^2$, $P_\theta=mrv_\theta-erA_\theta/c$; $\vec{J}=n_b er\Omega$; and n_b, n_e, and n are beam, plasma electron, and ion densities, respectively. We now cause a perturbation δf in $\vec{f}$ which gives rise to field perturbations $\delta\vec{E}$ and $\delta\vec{B}$. From these perturbations we compute the first-order perpendicular displacements $\vec{\xi}_{\perp e}\equiv\vec{\xi}_\perp$ and $\vec{\xi}_{\perp i}$ of the electron and ion fluids, respectively, from Eqs. (3), (2), and (1) with $\nabla\times\delta\vec{A}=\delta\vec{B}$, i.e.,

$$\delta\vec{A}=\vec{\xi}\times\vec{B}=\vec{\xi}_i\times\vec{B}-\frac{mc}{e}\frac{\partial}{\partial t}\vec{\xi}. \tag{6}$$

The first-order changes in density are

$$n_e^{(1)}=-\nabla\cdot n_e\vec{\xi}_e, \quad n^{(1)}=-\nabla\cdot n\vec{\xi}_i. \tag{7}$$

Second-order changes are given by

$$\vec{u}_e^{(2)}\times\vec{B}+\vec{u}_e^{(1)}\times\delta\vec{B}=0, \tag{8}$$

$$\vec{u}^{(2)}\times\vec{B}+\vec{u}_i^{(1)}\times\delta\vec{B}=\frac{mc}{e}\left(\frac{\partial}{\partial t}\vec{u}^{(2)}+\vec{u}^{(1)}\cdot\nabla\vec{u}^{(1)}\right), \tag{9}$$

$$\frac{\partial}{\partial t}\delta n_e^{(2)}=-\nabla\cdot(n_e\vec{u}_e^{(2)}+n_e^{(1)}\vec{u}_e^{(1)}), \tag{10}$$

etc. We now compute the variation of C to second order, $\delta C\equiv\delta C^{(2)}$, since the first-order variation vanishes to satisfy equilibrium. The system is enclosed by a conducting container at whose inner surface the tangential component of the electric field vanishes. After some vector manipulations in which the boundary conditions at the container are employed to eliminate several surface integrals, we obtain

$$\delta C=\frac{1}{2}\int d^3x\left\{\frac{1}{4\pi}(|\delta\vec{E}|^2+|\delta\vec{B}|^2)-\frac{r\Omega}{c}\hat{\theta}\cdot\delta\vec{E}\times\delta\vec{B}+nm|\vec{u}^{(1)}|^2+\Omega\left[er(n_e^{(2)}-n^{(2)})A_\theta-m(n^{(1)}u_\theta^{(1)}+nu_\theta^{(2)})\right]+T\int d^3v\,(\delta f)^2/f_0\right\}. \tag{11}$$

We assume that $\xi_\parallel=(\vec{\xi}\cdot\vec{B})/B$ adjusts itself to maintain charge neutrality under perturbed conditions also, i.e.,

$$\int d^3v\;\delta f=n_e^{(1)}-n_i^{(1)}, \tag{12}$$

and when $\xi_\parallel$ is disallowed then $\int d^3v\;\delta f=-\nabla\cdot n_b\vec{\xi}_\perp$. This assumption enables us to neglect the contribution of the electric fields which in any case are positive definite.[9] Without loss of generality we may write $\vec{\xi}=\vec{\xi}(r,z)\exp[i(l\theta-\omega t)]$, where ω is the perturbation frequency. From the perturbed Vlasov equation we obtain

$$\delta f=(f_0/T)\{-(\vec{\xi}\cdot\vec{J}\times\vec{B})/n_b+eg\}, \tag{13}$$

where

$$g=i(l\Omega-\omega)\int_{-\infty}^{t}dt'\vec{\xi}(t')\cdot\vec{v}(t')\times\vec{B}(t').$$

Substituting for δf, $n^{(2)}$, $u_\theta^{(2)}$, etc., from Eqs. (13) and (7)–(10), respectively, and employing (12) and (5), we finally obtain after some algebra,

$$\delta C=\delta W_{\text{MHD}}+\delta W_c+\delta W_b, \tag{14}$$

where

$$\delta W_{\text{MHD}}=\tfrac{1}{2}\int d^3x\{|\delta\vec{B}|^2/4\pi+\vec{\xi}_\perp\cdot\vec{J}\times\vec{B}(\nabla\cdot\vec{\xi})-\vec{\xi}_\perp\cdot\vec{J}\times\delta\vec{B}\},$$

$$\delta W_c=\tfrac{1}{2}\int d^3x\;\vec{\xi}\cdot\vec{J}\times\vec{B}(mr\Omega^2/T)\vec{\xi}_\perp\cdot\hat{r},$$

$$\delta W_b=\tfrac{1}{2}(e^2/T)\int d^3x\,d^3v\,f_0|g|^2,$$

and $\delta\vec{B}=\nabla\times\vec{\xi}\times\vec{B}$. In arriving at (14) we have neglected diamagnetic terms arising from the background plasma of $O(\omega/\omega_{ci})$, where ω_{ci} is the ion-cyclotron frequency. Stability is assured if $\delta C>0$ for any arbitrary perturbation $\vec{\xi}$. Notice that $\delta W_{\rm MHD}$ is equivalent to the δW of the magnetohydrodynamic (MHD) energy principle in which the current $\vec{J}$ is carried by inertialess particles; δW_c is the destabilizing contribution from the centrifugal force on the ring; and δW_b is a positive definite term that emerges from the dynamics of the ring ions which corresponds in some sense to the $\gamma p(\nabla\cdot\vec{\xi})^2$ term of conventional MHD δW.

To illustrate the application of this extended energy principle (14) we treat the stability of an infinitely long field-reversed ion ring, i.e., ion layer with $\vec{B}=B\hat{z}$ and $f_0\propto\exp(-\eta/T)$. In this case one can readily calculate

$$\delta W_{\rm MHD}=(8\pi)^{-1}\int d^3x\,B^2[k^2\xi_\perp^2+(\nabla\cdot\vec{\xi}_\perp)^2],$$

for $\vec{\xi}=\vec{\xi}(r)\exp[i(l\theta+kz-\omega t)]$. Employing (5) we obtain

$$\delta W_c=-\tfrac{1}{2}\int d^3x\,n_b m\xi_r^2\Omega^2\{(mr^2\Omega^2/T)+2[1+(r/\xi_r)d\xi_r/dr]\}.$$

We evaluate the orbit integral g that occurs in δW_b in the limit of a thin layer $T/mR^2\Omega^2\equiv\epsilon\ll1$, and strong field reversal so that the radial betatron frequency ω_β satisfies $\Omega^2/\omega_\beta^2\approx\epsilon\ll1$. For marginally unstable ($\mathrm{Im}\omega\to0+$), nonlocalized modes, i.e., $r\,d\ln\xi_r/dr|_{r=R}\ll1$, we obtain in the limit $\epsilon\ll1$, and for $l\neq0$,

$$\delta W_b=\frac{1}{2}\int d^3x\,n_b m\xi_r^2\Omega^2\left\{\frac{mr^2\Omega^2}{T}+2r\frac{d}{dr}\ln\xi_r+l^2\left(1-\frac{\omega}{l\Omega}\right)^2+6-8\left(1-\frac{\omega}{l\Omega}\right)^{-1}+3\left(1+\frac{k^2r^2}{l^2}\right)\left(1-\frac{\omega}{l\Omega}\right)^{-2}\right\}.$$

Thus

$$\delta C=\frac{1}{2}\int d^3x\left\{\frac{B^2}{4\pi}[k^2\xi_r^2+(\nabla\cdot\vec{\xi}_\perp)^2]+n_b m\xi_r^2\Omega^2\left\{l^2\left(1-\frac{\omega}{l\Omega}\right)^2+4-\left(1-\frac{\omega}{l\Omega}\right)^{-1}\left[8-3\left(1+\frac{k^2r^2}{l^2}\right)\left(1-\frac{\omega}{l\Omega}\right)^{-2}\right]\right\}\right\}. \quad (15)$$

It is easy to establish that $\delta C>0$ for $l\geq2$. For $l=1$, $\omega/\Omega\to0$, it is also positive because of finite k. For arbitrary ω, i.e., $0<\omega/\Omega\ll1$ and $l=1$, the coefficient of $n_b m\xi_r^2\Omega^2$ in (15) is positive for $k^2R^2>0.15$. Thus the *sufficient* condition for stability against all modes $l\geq1$, viz., $\delta C>0$, is satisfied for $k^2R^2>0.15$.

For the mode $l=0$ we may also take $\omega\to0$ thereby reducing the positive definite term δW_b to zero. We are thus left with

$$\delta C=\frac{1}{2}\int d^3x\,|\xi_r|^2\left\{\frac{B^2}{4\pi}\left[k^2+\left(\frac{\partial\ln(r\xi_r)}{\partial r}\right)^2\right]-nm\Omega^2\left[\frac{mr^2\Omega^2}{T}+2\left(1+\frac{r\,d\ln\xi_r}{dr}\right)\right]\right\}. \quad (16)$$

On minimizing δC we obtain the Euler-Lagrange equation for the vector potential $A=\xi_r B$:

$$\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\rho A-[k^2\delta^2-2\rho^2 n_b(\rho)/\bar{n}_b]\rho A=0, \quad (17)$$

where $\rho=r/\delta$, $\delta^4=(c^2/4\pi\bar{n}_b e^2)(2T/\Omega^2)$, and $n_b=\bar{n}_b\,\mathrm{sech}^2\tfrac{1}{2}(\rho^2-\rho_0^2)$ is the equilibrium density distribution for the ion layer. Equation (17) is identical to that derived by Marx[10] and thus we recover the conventional results on the tearing instability of a long layer.

These results indicate that a short layer, stable to the tearing instability, has improved stability properties against all other modes. However, in the above treatment the destabilizing effect of field curvature does not occur because of the long-layer approximation. In the opposite limit which accentuates this effect, Lovelace[6] has shown that the stability of a "bicycle tire" equilibrium is achieved against the kink mode for $R/a<(\pi/g)^{1/2}$, where R and a are the major and minor radii and g is a numerical factor of order unity, i.e., stable configurations are those which are "fat" and hence do not truly lie within the "bicycle tire" approximation $R/a\gg1$. Thus our calculations and those of Lovelace lead us to the belief that the most likely equilibrium that is stable to low-frequency perturbations is one in which $L\sim\Delta\sim R$, characteristic of experimentally observed relativistic electron rings in a gas background,[2] and also obtained numerically by particle simulation.[11]

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Strong Coupling between Liquid ^{3}He and Electron Spins at the Magnetic Phase Transition

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A steep dip in the nuclear-spin-lattice relaxation time, T_1, of liquid ^{3}He in contact with CuTA [copper tetrammine sulfate monohydrate, $Cu(NH_3)_4SO_4 \cdot H_2O$] has been observed near the ordering temperature, $T_N = 0.43$ K, of CuTA. Data show that the electron-spin fluctuation in CuTA plays an important role for the relaxation mechanism of liquid ^{3}He through the boundary interaction. An effective way of liquid-^{3}He cooling through this type of interaction is suggested.

Since the observation of the anomalous thermal boundary resistance between liquid ^{3}He and cerium magnesium nitrate (CMN) in the ultralow-temperature region by Abel *et al.*[1] in 1966, many theoretical[2-4] and experimental[5,6] investigations have been carried out to explain the origin of the phenomenon. Leggett and Vuorio[2] carried out an explicit calculation of this phenomenon in terms of the magnetic interaction between electron spins in CMN and ^{3}He nuclear spins. Guyer[3] rederived the boundary resistance for the same system using the relationship between the boundary resistance and the longitudinal relaxation time of interacting spin systems, and he pointed out that the strength of the coupling between electron spins and ^{3}He nuclear spins depends on the degree of synchronism between the motion of the two spins. Mills and Beal-Monod[4] re-examined theoretically the rate of energy transfer produced by the interaction between ^{3}He nuclear spins and electron spins. They suggest that the study of the longitudinal relaxation time, T_1, of the ^{3}He nuclei in contact with the magnetic salt, which has the magnetic phase transition in the low-temperature region, would be fruitful for the investigation of this boundary resistance problem. The present experimental work has been motivated by this suggestion.

I have chosen several magnetic salts as electron-spin systems to interact with the ^{3}He nuclear spin, and made ^{3}He NMR and T_1 measurements by the cw method. The results observed