COMMENTS

Avogadro Constant—Corrections to an Earlier Report

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Corrections are required to values of silicon crystal density and lattice parameter used in our determination of N_A. The corrections are of the order of 1 ppm but taken together result in a change of the previously reported value of 0.03 ppm. A published suggestion that the lattice-parameter measurement might require a much larger correction appears unsupported.

A new determination of the Avogadro constant, N_A, has been reported.[1] It was based on combining measurements of the lattice constant a_0, density ρ, and atomic weight A of well-characterized silicon crystals by means of the equation of Bragg, $N_A = nA/\rho a_0^3$. We have subsequently determined significant corrections to the values of lattice constant and density. The sources and effects of these corrections are reported here as well as is a reply to recent criticism of the a_0 measurement.

Detailed descriptions of the density[2] and atomic-weight[3] determinations have now been published elsewhere while essential features of the reckoning of a_0 were given in Ref. 1 and in a previous report.[4] Density values were obtained by hydrostatic comparisons of silicon crystal specimens with highly spherical steel balls whose volumes were determined by optical interferometry and whose masses were obtained with respect to kilogram replica No. 20. Atomic-weight values were obtained by an isotopic-abundance-weighted combination of the nuclidic masses involved. Abundances were determined by null comparisons against synthetic mixtures of separated isotopes. Lattice parameters were obtained by simultaneous optical and x-ray interferometry of a common baseline yielding the Si (220) repeat distance in terms of the wavelength of an $^{129}I_2$ stabilized $^3He^{22}Ne$ laser.

Regarding these three quantities (or the appropriate averages as given in Ref. 1), it has been found that corrections are needed for ρ and a_0 but not, as far as is now known, to A. The corrections are mainly for aspects of the measurements which were either inadequately or incorrectly evaluated. One other correction, as will be seen below, could be applied to either density or unit-cell volume. Although these corrections are each of the order of 1 ppm, the net effect is to change the previously reported result[1] by only 0.03 ppm.

Re-examination of the density procedures used in Ref. 2 showed that two corrections are required to the values assigned to the volumes of the steel spheres.[5] One correction is associated with the diameter measurement while the second is needed because of a deficiency in the cleaning procedure originally used. Diameters were determined from the interference patterns formed between the spheres and the (flat) end plates of a hollow Fabry-Perot etalon. The values thus depend directly on the length assigned to the empty etalon. An improved procedure for obtaining data from the photographic interferograms reduced the scatter of results by a factor of 2 and decreased the value for the etalon length from that reported by 4.8 nm (in a nominal length of 6.75 cm).

Also in connection with measurement of the steel spheres, cleaning procedures have been restudied. It was noted early in the study that interferometric results varied systematically with cleaning technique. In particular, diameters obtained after vapor degreasing were slightly larger than after chemical cleaning. Vapor degreasing was used in the already reported work because it led to a smaller spread of data. It is now believed that this choice was wrong and that 34.7 nm should be subtracted from previously reported diameters (nominally, 6.35 cm).

Corrections to the volumes of the steel spheres for these changes result in volume reductions of 1.88 ppm. Of this, 1.64 ppm is from the cleaning correction and 0.23 ppm from the change in etalon length. (The total figure contains a reduction of 0.012 ppm due to a numerical error in the previous publication.) This volume reduction in the standards propagates through the entire measurement of Si crystal densities and requires the values previously reported to be increased by 1.88 ppm. Overall error estimates for density are not significantly different from those reported previously. In terms of the weighted mean of A/ρ, the result remains $\sigma_T = 0.75$ ppm.

In the earlier report,[1] we neglected to take into account the nonzero compressibility of the Si crystal specimens.[6] This is a problem since the lattice-parameter measurements were carried out in vacuum while density measurements were made near atmospheric pressure. This effect may be taken to reduce the value of a_0^3 from the x-ray interferometric result. Its magnitude is readily estimated from the usual expression for bulk modulus, viz. $-V\,dp/dV = (c_{11} + 2c_{12})/3$.[7] Elastic constant values are $c_{11} = 1.657$ and $c_{12} = 0.639$ (in units of 10^{12} dyn/cm^2).[8] For a pressure change of 750 Torr one obtains a change in unit cell volume of 1.02 ppm.

As a final matter, we note a correction to the x-ray–optical-interferometric determination of a_0 and respond to a published criticism of this measurement (see last paragraph). Because x-ray and optical interferences do not occur on a common locus but are offset by an amount z (3.0 mm in the present case), path curvature produces an Abbe sine error. If the curvature is uniform, changing the direction by $\delta\theta$ in a path L, the fractional error is $z\,\delta\theta/L$. At the time of our report,[1] there was available an estimate of 0.20 ppm for this component of error which was probably not sufficiently clearly designated as such.

In subsequent work, reported recently,[9] it was possible to improve the trajectory characterization significantly. By use of a narrow, multiple-beam-diffraction feature (full width at half-maximum = 0.02 arcsec), a path curvature of $1.17 \pm \pm 0.06$ arc msec for a scan of 200 optical fringes was measured. This entails a correction of -0.27 ± 0.03 ppm to the previously reported value of a_0. The new result, 543.106 46 ppm, gives a revised value for a_0^3 of 0.160 197 194 nm^3 (0.45 ppm) (in vacuum).

The corrections given above for density and compressibility can now be combined with the remaining data to obtain a revised estimate for N_A. The density correction changes the invariant ratio A/ρ from its previous value of 12.059 043 to 12.059 020. The compressibility correction changes the value of a_0^3 given above to 0.160 197 031 nm^3. The combined effect is to give a revised value, $N_A = 6.022\,094\,1 \times 10^{23}$ mol^{-1} (0.88 ppm). This differs from that given in Ref. 1 by 0.03 ppm.

In a recent review,[10] Hart has offered particular criticism of our lattice-parameter measurement. The principal suggestion made in this criticism is that we neglected the Abbe sine error and that besides that, we could not characterize the trajectory to the precision needed to sustain our earlier claim of accuracy. In a more detailed response submitted for publication,[11] we attempt to clarify the role of various x-ray measurements in trajectory characterization and examine alternative optical-design strategies. Without recapitulating these details, however, it is clear from the path-curvature measurements noted above that we were never troubled by effects from this source amounting to 20 ppm as posited in Ref. 10. Furthermore and specifically, this error as noted above has been evaluated and the required correction of 0.27 ± 0.03 ppm applied to the measured value of a_0.

[1] R. D. Deslattes, A. Henins, H. A. Bowman, R. M. Schoonover, C. L. Carroll, I. L. Barnes, L. A. Machlan, L. J. Moore, and W. R. Shields, Phys. Rev. Lett. 33, 463 (1974).

[2] H. A. Bowman, R. M. Schoonover, and C. L. Carroll, J. Res. Nat. Bur. Stand., Sect. A 78, 13 (1974).

[3] I. L. Barnes, L. J. Moore, L. A. Machlan, and W. R. Shields, to be published.

[4] R. D. Deslattes and A. Henins, Phys. Rev. Lett. 31, 972 (1973).

[5] H. A. Bowman, R. M. Schoonover, and C. L. Carroll, Nat. Bur. Std. (US) Report No. NBSIR 75-768, August 1975 (unpublished), available from the authors.

[6] A remark by M. Hart, Proc. Roy. Soc. London, Ser. A 346, 1 (1975) brought this effect to our attention.

[7] J. F. Nye, *Physical Properties of Crystals* (Oxford Univ. Press, Oxford, England, 1957), p. 146-147.

[8] H. J. McSkimin, J. Appl. Phys. 24, 988 (1953).

[9] R. D. Deslattes, in Proceedings of the Fifth International Conference on Atomic Masses and Fundamental Constants, Paris, France, June 1975 (to be published). This report contains a typographical error (1.7 arc

msec appears in place of 1.17) and a numerical mistake (the correction for path curvature shoyld be 0.27 ppm as given in the present communication rather than 0.18 ppm).

[10]Hart, Ref. 6.

[11]To be published.

Relation between Monopole Mass and Primary Monopole Flux*

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We give quantitative results concerning the simple idea that the probability of observing a monopole at Earth with $\beta \simeq 0.5$ decreases as the monopole mass decreases, for masses $\lesssim 10^4$ Gev/c^2: The effective cross section of Earth is roughly the geometric cross section times the ratio of the monopole mass to 8000 GeV/c^2.

Recently Price *et al.*,[1] reported a cosmic-ray event which they interpret as a magnetic monopole[2,3] of charge $137e$, and mass $\gtrsim 600M_p$ (M_p is the proton mass). It was observed near Sioux City, Iowa (latitude ~42.5°N) to be going generally downward with a velocity $\simeq 0.5c$. The purpose of the present Comment is to make quantitative a rather obvious point: The lighter the monopole, the less likely it is to be observed anywhere on its trajectory with such a low velocity.[4] The reason is that the potential energy at the surface of Earth of a monopole of magnetic charge $g = 137e$ in Earth's dipole field is roughly 8000 GeV, and a monopole of mass much less than this must nearly always be traveling at close to the speed of light. Only monopoles of mass $\gtrsim 10^4$ GeV/c^2 have much chance of retaining a small velocity in Earth's magnetic field.

For the purposes of this work, we assume that the alleged monopole came from outer space as a primary cosmic ray. Wilson[5] and Hungerford[4] have shown that, for kinematic reasons, the monopole could not have been produced in the atmosphere by a conventional cosmic-ray primary. We do not consider the possibility that the monopole migrated to the surface of Earth from the interior, and was then accelerated along a field line. One might expect a cosmic monopole to have been accelerated to very high energies by galactic magnetic fields, in which case the observed event is *a priori* an unlikely candidate for a monopole. However, it might have come from the Sun at low energies in which case its energy change in the solar magnetic field might well be comparable to its potential energy in Earth's field.

We have studied the orbits of a monopole in Earth's field by a combination of computer-generated orbits and approximate analysis. Let us begin with the analysis. Earth's magnetic field is approximated by a centered dipole of magnetic moment B_0a^3, where a is the radius of Earth (6371 km) and $B_0 = 0.31$ G. In units which are natural for the problem (length measured in units of a, velocity in units of c, and mass or energy in units of the monopole mass M), the equations of motion are

$$d\vec{p}/dt = -K\nabla[(\cos\theta)/r^2], \tag{1}$$

where $\vec{p} = \gamma\vec{\beta}$ is the momentum $[\gamma = (1-\beta^2)^{-1/2}]$, and

$$K = gB_0a/Mc^2 = 8641M_p/M. \tag{2}$$

The corresponding Hamiltonian is

$$H = \gamma + K(\cos\theta)/r^2, \tag{3}$$

and it is conserved, as is the azimuthal momentum $P_\varphi = \gamma r^2 \sin^2\theta\,\dot{\varphi}$.

The orbital motion can be reduced to quadratures for nonrelativistic motion. Such an analysis will be relevant in the near vicinity of the place where the cosmic-ray event was observed, where the motion is nearly nonrelativistic.

The nonrelativistic equation for θ reads

$$\frac{d(r^2\dot{\theta})}{dt} = \frac{K\sin\theta}{r^2} + \frac{P_\varphi{}^2\cos\theta}{r^2\sin^3\theta}, \tag{4}$$