Maximum Rate for the Proton-Proton Reaction Compatible with Conventional Solar Models*

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(Received 19 January 1976)

Recently reported experimental results have been interpreted to suggest that the rate of the proton-proton reaction may have been underestimated by a large factor. Although these results are in conflict with current theories of β decay and it is quite unlikely that the proton-proton rate could be seriously in error, we have investigated the implications of the suggestion for solar models. An enhancement of more than a factor of about 50 cannot be accommodated by conventional solar models.

Because of the serious nature of the solar neutrino problem,[1] the rates of the nuclear reactions involved in the *pp* chains and the CNO tri-cycle have been closely examined in recent years. For the most part these reactions can be measured in the laboratory, and the remaining uncertainties are small.[2] The proton-proton reaction itself, however, is believed to proceed via the weak interaction, and the resulting small cross section makes the direct measurement of its rate in the laboratory impossible with present techniques. But the theoretical calculation of this rate by first-order perturbation theory[3] is straightforward, employing as it does only the better established aspects of β-decay theory, and the uncertainty in the p-p rate is commonly regarded as small. Nonetheless, the recent results of Slobodrian, Pigeon, and Irshad[4] (SPI) bring into question that calculation, and motivate an investigation of the effects of an enhanced rate for the proton-proton reaction. There is some controversy concerning that experiment, however, and a number of laboratories are repeating the measurement to ascertain the validity of the reported results.

SPI have investigated the well-studied reaction $^3\mathrm{He}+{}^3\mathrm{He}$ (whose usual result is $2p+{}^4\mathrm{He}$ and which constitutes the termination of the *pp*-I chain), and report the observation of deuterons, which they attribute to $^3\mathrm{He}+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+{}^2\mathrm{H}+e^++\nu$, or to the production and subsequent decay of $^2\mathrm{He}$. This reaction is seen to be quite similar to the proton-proton reaction $^1\mathrm{H}+{}^1\mathrm{H}\rightarrow{}^2\mathrm{H}+e^++\nu$; in the former case the β decay which allows the formation of the deuteron can occur in the field of the tightly bound α particle, in the latter during the scattering of the two protons. To result in the production of a measurable flux of deuterons, the cross section of the reaction considered by SPI must be many orders of magnitude larger than the theoretical estimate—they suggest the effect of an intermediate vector boson and an enhancement factor of some 10^8. The similarity of that reaction to the proton-proton reaction would then cast doubt upon the rate calculated for p-p also. We will see that it does not seem possible to construct a solar model with a proton-proton rate very different from its standard value.

It should be emphasized that the weak-interaction theory used in the calculation of the proton-proton rate is that which has been so successfully applied to the interpretation of laboratory β decay, and that the circumstances of the proton-proton reaction differ only in minor regard from those occurring in several well-studied decays, as well as in SPI's $^3\mathrm{He}+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+{}^2\mathrm{H}+e^++\nu$. An analogous situation is the superallowed ($\log ft=3$) Gamow-Teller transition $^{10}\mathrm{C}(0^+,T=1)\rightarrow{}^{10}\mathrm{B}^*(1^+,T=0)$ in which $^8\mathrm{Be}$ may be regarded as a spectator to the conversion of a proton pair to a deuteron. (This transition does *not* have $\log ft=-5$ which is to be expected if the enhancement factor is the same as SPI claim.) Indeed, we may view the $^8\mathrm{Be}$ as a pair of α particles, and the similarity to the reaction of SPI is very close—the single α has been replaced by a pair, and bound-state wave functions replace the scattering wave functions. Perhaps an even better analog is the decay $^6\mathrm{He}(0^+,T=1)\rightarrow{}^6\mathrm{Li}(1^+,T=0)$. In this case a single α particle is spectator to the decay of a neutron pair to form a deuteron, and we appeal to charge invariance. Both of these decays are well understood, and there is little reason to expect $^3\mathrm{He}+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+{}^2\mathrm{H}+e^++\nu$ to behave differently. We must conclude that the observations of SPI are not likely to have been due to this reaction. Nonetheless, in what follows we investigate the consequences for solar models of an enhanced proton-proton rate, and conclude that the claim of SPI is implausible on astrophysical grounds as well.

In order to qualify as a solar model a $1M_\odot$ star must attain the solar luminosity and the solar radius at the solar age. One normally varies the initial composition until a model is achieved with $L = L_{\odot}$ at time $t_{\odot} \cong 4.7\times10^9$ yr, and the parameter α (ratio of mixing length to pressure scale height) until $R = R_{\odot}$ at $t_{\odot}$. Increasing α serves to make convection in the outer regions of the Sun more efficient, and decreases the radius.

A sequence of models has been constructed with the proton-proton rate arbitrarily multiplied by a factor $f_{pp} \geqslant 1$ constant throughout the star. The evolution code used was that described by Newman[5] and Newman and Fowler,[6] except that the nuclear reaction rates employed (aside from the factor multiplying the p-p rate) were those of Fowler, Caughlan, and Zimmerman.[2] As f_{pp} was increased from unity the star tended to bloat, and the mixing-length parameter α was increased to counteract this tendency. In Fig. 1 is shown the behavior of the standard solar model of Newman and Fowler as f_{pp} is varied at $\alpha = 100$. At such a large value of α we are approaching the limit of perfectly efficient convection, and have left the region in which the mixing length can be given its usual physical interpretation. Little can be done to reduce the radius further. From the small radius and slightly enhanced luminosity to which it was driven by changing α from the more conventional value 1.8 to 100, the $X_0 = 0.737$ model increases in radius roughly proportional to the logarithm of f_{pp}. The solar radius is exceeded near $f_{pp} = 50$. The luminosity decreases at first, the usual response to an increased rate of energy generation.[7] But the regulating mechanism is soon overwhelmed, and the luminosity passes through a minimum at about $f_{pp} = 70$, and thereafter increases. The motion in the Hertzsprung-Russell diagram is then upward and to the left, on a curve with slope somewhat steeper than the main sequence. It is seen that the task of returning the star to solar conditions with the proton-proton rate enhanced by more than an order of magnitude or so is a formidable one. That it appears to be impossible for $f_{pp} \gtrsim 50$ is demonstrated in Fig. 2. Here we see the $X_0 = 0.737$, $\alpha = 100$ model "jumped" to various other initial compositions in the manner discussed by Clayton, Newman, and Talbot[7] and Newman[8] to generate an approximately isochronous sequence. Decreasing the initial hydrogen abundance increases the luminosity so that a solar model is attained at $f_{pp} = 50$ for $\alpha = 100$ and $X_0 \cong 0.69$. Decreasing X_0 further only serves to increase the luminosity above $L_{\odot}$, and does not serve to produce a solar model with $f_{pp} > 50$. Increasing X_0 reduces the luminosity below $L_{\odot}$ and does not produce a solar model. Decreasing α tends to shift the curves to the right, and allows construction of solar models with $f_{pp} < 50$. Increasing α above its already unphysical value of 100 does little to increase the efficiency of convection, and does not allow solutions with appreciably larger pro-

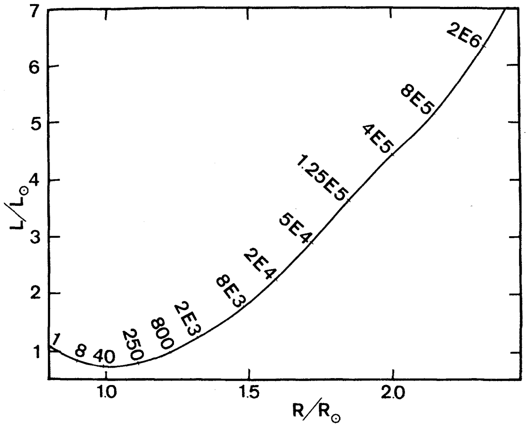

FIG. 1. Motion of the $X_0 = 0.737$ model with $\alpha = 100$ in the L-R plane as the proton-proton rate is increased. The curve is labeled with the value of the enhancement factor f_{pp} at representative points. The notation aEb stands for $a\times10^b$. The radius increases steadily with the logarithm of the proton-proton rate, exceeding the present solar value near $f_{pp} = 50$. The luminosity is driven down as the p-p rate is enhanced, reaches a minimum near $f_{pp} = 70$, and subsequently increases. For large enhancement factors the model is quite far from solar conditions.

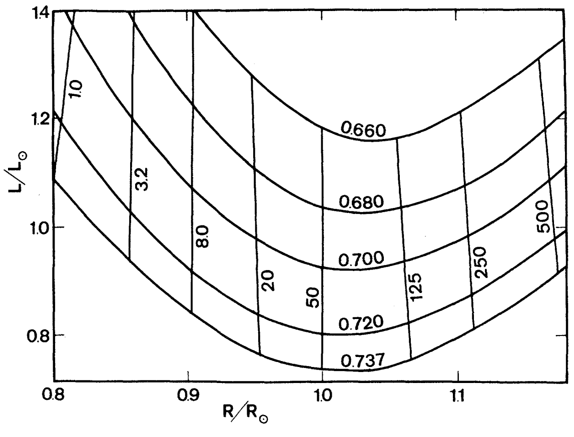

FIG. 2. Searching for a solar model at $\alpha = 100$. Vertical curves are labeled with the proton-proton rate enhancement factor f_{pp}, lateral curves with the initial hydrogen mass fraction X_0. There is a solution ($L = L_{\odot}$, $R = R_{\odot}$) near $f_{pp} = 50$, $X_0 = 0.69$. Solar models with larger enhancement factors would require mixing lengths even larger than the already unphysical value used here.

ton-proton rates.

It would thus seem that there is a maximum allowed value, in the context of conventional solar models, of about 50 times the usual rate of the proton-proton reaction. The limitation is imposed, however, by the constraints of the efficiency of energy transport. A similar problem was faced by Hoyle[9] in attaining the solar radius in his solar models with cosmological cores. He appealed to acoustical waves to carry a large fraction of the energy flux and allow $R = R_\odot$. Subsequently Newman and Fowler[6] have investigated the effects of acoustical waves and other nonradiative processes of energy transport as neutrino-quenching mechanisms in their own right. If such unconventional mechanisms are invoked to supplement convection, it is perhaps possible to achieve solar models with the p-p rate enhanced by a somewhat larger factor. An enhancement factor many orders of magnitude larger than unity is not likely to be accommodated by such measures, as Fig. 1 would suggest.

Indeed, a limit is imposed by elementary requirements of mechanical equilibrium. The response of the solar core to an enhancement of the proton-proton rate is to expand to lower central temperature and density. It is this tendency which produces the low neutrino-counting rates shown in Fig. 3. (Note that the *pep* reaction does not compete as effectively with p-p at the very low densities achieved at large enhancement factors f_{pp}. Counting rates below 0.3 solar neutrino unit can result.) However, powerful integral theorems[10] based on quite basic considerations constrain the central pressure: The basic requirement of supporting the solar mass at the solar radius fixes very nearly the pressure at the solar center. If hydrogen is too reactive a fuel to allow so small a luminosity as $L_\odot$ at the temperatures and densities required to produce the minimum pressure, then the radius must exceed $R_\odot$ or the luminosity must exceed $L_\odot$, or both. In either event we do not have a solar model.

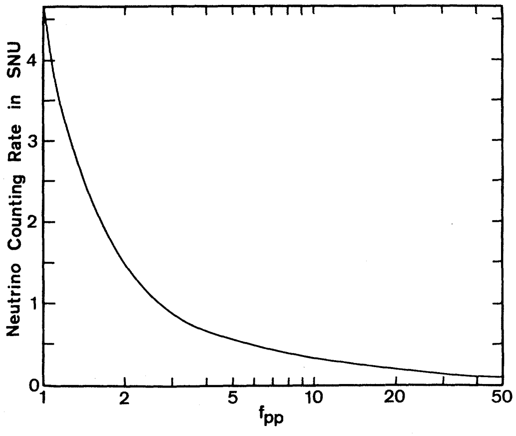

FIG. 3. Predicted neutrino-counting rate in solar neutrino units as a function of the proton-proton rate enhancement factor f_{pp}. Very low counting rates can be obtained for enhancement factors larger than about 3, and could resolve the solar neutrino problem if the proton-proton rate could be in error by such a large factor. These rates were calculated with ^{3}He taken to be always in equilibrium. This assumption begins to break down at the low temperatures encountered, and may significantly affect the results.

While a slight increase in the rate of the proton-proton reaction can produce solar models of low neutrino-counting rate, and so resolve the solar neutrino problem (the $f_{pp} = 50$ model results in less than 0.1 solar neutrino unit), a rate very different from that usually taken is difficult to reconcile with considerations of solar structure. Enhancement factors larger than about 50 seem to be incompatible with conventional solar models. By appealing to exotic processes of energy transport one might attain slightly larger values. Enhancement factors many orders of magnitude larger than unity are almost certainly incompatible with current ideas of stellar structure. Reversing the argument that cast doubt on the proton-proton rate, one must therefore suspect that the increase of many orders of magnitude in the cross section for the production of deuterium by $^3\mathrm{He} + {}^3\mathrm{He}$ suggested by SPI is unlikely.

We are grateful to Dr. Charles Barnes and Dr. Jack Overley for interesting discussions of relevant experiments.

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*Work supported in part by the National Science Foundation, Grants No. MPS71-02670 A05 and No. AST75-01398 A01.

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