

cused on the V_K -like defect in the $\langle 110 \rangle$ direction. It is worth noting that at the surface the energy loss considerations are relaxed if the separation of halogen and interstitial occurs by ejection from the surface. Thus a $\langle 211 \rangle$ oriented molecule may successfully form and decay by ejection in the $\langle 211 \rangle$ direction so long as the collision sequence is sufficiently short (i.e., either direct ejection or possibly a one-step chain).

The differences between defect formation in the bulk of the solid and events at the surface have been mentioned before.¹⁵ If in sputtering measurements the material is derived from near surface events with very short replacement sequences then the yield from $\langle 110 \rangle$ and $\langle 211 \rangle$ processes will be comparable, as is observed.^{14,6} This situation is different from color-center formation in the interior of the solid where only $\langle 110 \rangle$ events will be important.

In this model the $\langle 211 \rangle$ ejection is an intrinsic process rather than a derivative of a $\langle 110 \rangle$ event. Such a simple explanation of the intense $\langle 211 \rangle$ spots avoids the sophistication of the model proposed by Smoluchowski.¹⁶

In conclusion we see that the photon sputtering experiments of Ref. 1 can be interpreted as evidence for very short replacement collision sequences of the halogen ions which result from the

capture and decay of excitons.

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Comment on the Stress-Induced Tricritical Point in MnO^\dagger

P. Bak and S. Krinsky

Brookhaven National Laboratory, Upton, New York 11973

and

D. Mukamel

Baker Laboratory and Materials Science Center, Cornell University, Ithaca, New York 14850

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Bloch *et al.* have observed that application of a [111] stress causes the first-order transition in MnO to become second order. We point out that the change of nature of the phase transition may be explained by noting that at zero stress the transition is described by an $n=8$ vector model which has no stable fixed point, while for large enough stress the transition is described by an $n=2$ model which has a stable fixed point. We suggest that a similar effect might be observed in UO_2 , Cr, and Eu by applying a symmetry-breaking field, such as a nonisotropic stress or a magnetic field.

Recently we have used the symmetry considerations of Landau and Lifshitz to derive the effective Hamiltonians for certain paramagnetic-to-antiferromagnetic transitions having order parameters with $n \geq 4$ components.^{1,2} We have suggested that the first-order nature of the transitions in MnO ($n=8$), UO_2 ($n=6$), Cr ($n=12$), and

Eu ($n=12$) can be explained by noting that the corresponding Hamiltonians possess no stable fixed points within the ϵ expansion (a similar discussion has been presented by Brazovskii and Dzyaloshinskii³). Since the stability and the nature of the fixed points are determined by symmetry considerations alone, the question naturally

arises as to what will happen if the symmetry is broken through the application of an external field, such as a nonisotropic stress or a magnetic field. Clearly, the dimensionality of the order parameter may be lowered, since the components of the *original* order parameter do not have to belong to the same irreducible representation of the *new* space group. For example, it is possible by carefully selecting the direction of the applied field to change the corresponding Landau-Ginzburg-Wilson Hamiltonian from an $n \geq 4$ Hamiltonian with *no* stable fixed point to an $n \leq 3$ Hamiltonian for which the isotropic fixed point is stable,⁴ and the transition may become *second-order*.

There has been a very interesting recent experiment using neutron scattering to study the antiferromagnetic order parameter in the type-II antiferromagnet MnO as a function of temperature and of uniaxial stress.⁵ It was found that a large [111] stress changes the transition from first order to second order. It was shown in a phenomenological way that this behavior *might* be explained within the mean-field approximation in terms of the coupling of spins to the compressible lattice, *provided* that this coupling exceeds a certain value. We have calculated this coupling using five *independent* sets of experimental data: (1), (2) Direct measurements of the spontaneous lattice distortion $\delta\alpha$ at 4 K.⁶ The coupling (jJ_1^0) is given by the formula⁶

$$\delta\alpha = 12.5N\sigma^2 jJ_1^0 / C_{44}V_m. \quad (1)$$

The notation is the same as that of Ref. 5. (3-5) Direct measurements of the stress-induced change in the exchange interaction, jJ_1^0 , using neutron scattering techniques. We are aware of *three* sets of data agreeing within experimental error.⁷⁻⁹ The five values of j agree within 15%. This supports the consistency of the theory. However, if this value is inserted in the formula⁵

$$T_{3c} = 966N(jJ_1^0)^2 / C_{44}V_m, \quad (2)$$

one finds that the *coupling is at least a factor of 2 too small to explain the observed first-order transition*. This calculation indicates that one has to go beyond mean-field theory to understand the discontinuous transition.

On the other hand, we note that the observed behavior is in agreement with the ideas outlined above. MnO is an fcc crystal whose paramagnetic space group is $Fm\bar{3}m$. It exhibits a type-II antiferromagnetic structure with the sublattice magnetization \vec{m} perpendicular to the ordering wave vector \vec{K} . The Landau-Ginzburg-Wilson Hamiltonian for this system is¹

$$\mathcal{H} = -\frac{1}{2} \sum_{i=1}^4 [\mathbf{r}(\varphi_i^2 + \bar{\varphi}_i^2) + (\nabla\varphi_i)^2 + (\nabla\bar{\varphi}_i)^2] - \sum_{i=1}^6 u_i O_i(\varphi_i, \bar{\varphi}_i), \quad (3)$$

where $O_i(\varphi_i, \bar{\varphi}_i)$ are the six fourth-order invariants of the group $Fm\bar{3}m$. This Hamiltonian has no stable fixed point, which explains the first-order nature of the transition at zero stress. When a uniaxial [111] stress is applied, the paramagnetic group becomes $R\bar{3}m$, which is a subgroup of $Fm\bar{3}m$.

The $n=8$ representation which is associated with the zero-stress transition decomposes now into one $n=2$ representation and two $n=3$ representations of group $R\bar{3}m$. The $n=2$ representation corresponds to an order parameter with wave vector \vec{K} parallel to the stress direction [111], while the $n=3$ representations correspond to wave vectors \vec{K} along the $[\bar{1}\bar{1}1]$, $[11\bar{1}]$, and $[\bar{1}\bar{1}\bar{1}]$ directions. Experimentally it was found that application of a [111] stress favors the [111] wave vector.⁵ The transition is therefore described by the $n=2$ Hamiltonian

$$\mathcal{H}' = -\frac{1}{2} [\mathbf{r}'(\varphi_1^2 + \bar{\varphi}_1^2) + (\nabla\varphi_1)^2 + (\nabla\bar{\varphi}_1)^2] - u_1'(\varphi_1^2 + \bar{\varphi}_1^2)^2, \quad (4)$$

where $\mathbf{r}' = \mathbf{r} + \mathbf{a}(p)$ and $u_1' = u_1 + b(p)$. $\mathbf{a}(p)$ and $b(p)$ are functions of the stress p , such that $\mathbf{a}(0) = b(0) = 0$. This Hamiltonian has *one* stable fixed point. For sufficiently large stress the critical behavior can be calculated using the Hamiltonian (4) and neglecting the effect of the order parameters which belong to the two $n=3$ representations. If $u_1' = u_1 + b(p) > 0$, the Hamiltonian is within the domain of attraction of its stable fixed point and we expect the transition to be second order. When the applied stress is small, the two $n=3$ representations are almost degenerate with the $n=2$ representation and they cannot be neglected in calculating the critical behavior of the system. These order parameters will renormalize the coupling constants \mathbf{r}' and u_1' . If the renormalized interaction, $(u_1')_R$, is negative the Hamiltonian lies outside the domains of attraction and the transition is expected to be first order as in the case of zero stress. In the region where the transition is second order we expect the critical exponent β to be ≈ 0.33 which is the exponent for the X-Y model.¹⁰ We would also expect that a sufficiently strong applied magnetic field along the [111] direction would cause the transition to

become second order. The order parameter which describes the transition has either $n=2$ or $n=3$ components, depending on which representation is favored by the magnetic field. We urge that similar experiments be performed on other systems predicted to be first order because of a lack of a stable fixed point. For example, a [100] stress in the type-I, $\vec{m} \perp \vec{k}$ antiferromagnets (UO_2) reduces n from 6 to 2, and a [100] stress in the cubic sinusoidal magnet Cr or the helical magnet Eu changes n from 12 to 4. Each case has to be treated separately according to the type of magnetic ordering which actually develops in the external field. A list of systems is given in Ref. 2. For some of the systems the effect may be more difficult to observe than in MnO, since the coupling to the lattice, which actually splits the degeneracy of the order parameters, is probably less than for MnO, where a 0.1% strain may induce a 1% change in the exchange.

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