COMMENTS

Comment on the New Scaling Hypothesis of Dao *et al.* at Asymptotic Energies*

Robert J. Yaes
Department of Physics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada A1C 5S7
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I use the energy-momentum—conservation sum rules to investigate the consequences of the assumption that the "New Scaling Hypothesis" of Dao *et al.*, is valid at asymptotic energies for which $p_L \gg p_T, m$. Assuming that Koba-Nielsen-Olesen scaling, limited p_T, and the slow rise in multiplicity with energy also remain valid asymptotically, I obtain a scaling relation for the invariant inclusive distribution which is inconsistent with ordinary Feynman scaling, but consistent with the violation of Feynman scaling at $2p_L s^{-1/2} = 0$ recently found.

Dao *et al.*[1] have proposed a new scaling law for semi-inclusive cross sections and have shown that the data for p-p collisions between 13 and 300 GeV/c laboratory momentum of the incident proton are consistent with it for a wide range of multiplicities. However, it has been shown[2] that a fit to the same data can also be obtained assuming just Koba-Nielsen-Olesen (KNO) and Feynman scaling. The new scaling law for the noninvariant single-particle semi-inclusive cross section, integrated over transverse momentum, can be written

$$\frac{\langle p_L\rangle_n}{n\sigma_n}\frac{d\sigma_n}{dp_L} = \varphi\left(\frac{p_L}{\langle p_L\rangle_n}\right), \tag{1}$$

where σ_n is the n particle cross section, $\langle p_L\rangle_n$ is the average value of the magnitude of the c.m. longitudinal momentum for multiplicity n, and the right-hand side is assumed to be a function only of the scaling variable $p_L/\langle p_L\rangle_n$, independent of both s and n. It is clear that the normalization conditions,

$$\int (d\sigma_n/dp_L)\,dp_L = n\sigma_n, \tag{2}$$

$$\frac{\int |p_L|(d\sigma_n/dp_L)\,dp_L}{\int (d\sigma_n/dp_L)\,dp_L} = \langle p_L\rangle_n, \tag{3}$$

are satisfied provided the function φ satisfies

$$\int_{-\infty}^{\infty} dt\,\varphi(t) = \int_{-\infty}^{\infty} dt\,|t|\,\varphi(t) = 1. \tag{4}$$

Dao *et al.* propose a similar scaling law for p_T and show that it is also consistent with their data. However, since the available data[3] seem to indicate that $\langle p_T\rangle_n$ varies slowly, if at all, with both s and n, at high energies (especially if we exclude large-p_T events, which, in any event, contribute only a very small fraction of the total cross section, even for $\sqrt{s} = 63$ GeV), I shall concern myself here only with the scaling law in longitudinal momentum, Eq. (1).

In the paper of Dao *et al.*, n is the *charged* multiplicity, which is easiest to determine experimentally. However, it is reasonable to assume that if such a scaling law is valid, it will also hold when n is the *total* multiplicity (which is easiest to deal with theoretically), especially since there is evidence[4] that the average number of neutrals in an event is proportional to the number of charged prongs. In addition, it has been pointed out by Sivers[5] that both theoretical considerations and the trend of the data indicate that asymptotically, each particle species will obtain a fixed finite fraction of the total c.m. energy. If this is the case, the argument presented below, with slight modification, would also hold when n is the charged multiplicity. Hence, there is no loss of generality if we consider the scaling law in a model in which all produced particles are identical and spinless.

It is a popular belief (or, perhaps one should say, hope) that the regularities in the data[3] that seem to hold for $\sqrt{s}$ between 8 and 63 GeV—Feynman scaling, limited transverse momentum, the slow rise in multiplicity with energy, KNO scaling etc.—will remain valid asymptotically. I thus wish to investigate the consequences of the assumption that (1) is a true asymptotic scaling law that remains valid as $s \to \infty$. In a semi-inclusive reaction, in a model with all produced particles identical, the number of produced particles is fixed, while the energy available in the c.m. frame is $\sqrt{s}$. Hence, if we assume only that $\langle p_T\rangle_n$ rises significantly slower than $\sqrt{s}$, we must have,[6] for sufficiently large s, $p_L \gg p_T, m$ for each of the produced particles if energy is to be conserved. (The same argument would hold for an inclusive reaction if we assumed that the product $\langle n\rangle\langle p_T\rangle$ rises significantly more slowly than $\sqrt{s}$ at high energies. This assumption is supported by the data up to the highest observed cosmic-ray energies.) In that case, the energy is just

$$E = (p_L{}^2 + p_T{}^2 + m^2)^{1/2} \cong |p_L| . \tag{5}$$

The energy-conservation sum rule for the semi-inclusive cross section[7] then states that the numerator on the left-hand side of (3) is just $\sigma_n \sqrt{s}$ so that $\langle p_L\rangle_n$ is determined by the energy sum rule to be[8]

$$\langle p_L\rangle_n = s^{1/2}/n , \tag{6}$$

which is just a statement of the fact that asymptotically essentially all of the c.m. energy goes into the longitudinal kinetic energy of the produced particles and that each particle has, on the average, $1/n$ of the total c.m. energy. Since by definition of the Feynman scaling variable, x,

$$p_L = \tfrac{1}{2}x\sqrt{s} , \tag{7}$$

we immediately obtain from (1) and (6) for the invariant semi-inclusive distribution

$$F_n(p_L, s) = \frac{1}{\sigma_n} E \frac{d\sigma_n}{dp_L} = \frac{n^2|x|}{2}\, \varphi\left(\frac{nx}{2}\right), \tag{8}$$

and, defining a new function

$$\tilde{\varphi}(y) = \tfrac{1}{2}|y|\, \varphi(\tfrac{1}{2}y) , \tag{9}$$

we obtain the simple scaling relation

$$F_n(p_L, s) = n\tilde{\varphi}(nx) . \tag{10}$$

The invariant inclusive distribution is given by

$$\begin{aligned} F(p_L, s) &= \sigma^{-1} E\, d\sigma/dp_L = \textstyle\sum_n \alpha_n F_n(p_L, s) , \\ \alpha_n &= \sigma_n/\sigma , \quad \textstyle\sum_n \alpha_n = 1 . \end{aligned} \tag{11}$$

If we assume KNO scaling

$$\alpha_n(s) = \langle n\rangle^{-1}\psi(n/\langle n\rangle) \tag{12}$$

and approximate the sum over n by an integral over the variable $z = n/\langle n\rangle$, we obtain

$$\begin{aligned} F(p_L, s) &= \langle n\rangle \textstyle\int z\, dz\, \psi(z)\tilde{\varphi}(\langle n\rangle xz) \\ &= \langle n\rangle\, \Phi(\langle n\rangle x) . \end{aligned} \tag{13}$$

The scaling relation (13) in the variable $\langle n\rangle x$ is obviously inconsistent with ordinary Feynman scaling in the variable x which would require a scaling relation of the form $F(p_L, s) = f(x)$, as (13) would require a shrinkage of the inclusive distribution, expressed as a function of x, with increasing $\langle n\rangle$. One possible way of avoiding this contradiction is to assume that the scaling relations (10) are valid but that the scaling limits are reached at different energies for different n. Thus, at any fixed, finite energy, it would not be possible to sum over all n to obtain (13). However, in this case the new scaling law would lose much of its usefulness as it could not be used to compare semi-inclusive cross sections over a wide range of n and s.

However, one can approach this result from another point of view.[9] At $x = 0$, (13) becomes

$$F(p_L, s) = \langle n\rangle\, \Phi(0) \sim \ln s \tag{14}$$

which is consistent with the 12% rise in the inclusive $\pi^\pm$ distributions at $x = 0$ when $\sqrt{s}$ goes from 22 to 63 GeV found by H. Bøggild *et al.*[10] Away from $x = 0$, most people believe that there is no clear indication of a violation of Feynman scaling in the data. However, because $\langle n\rangle$ is a slowly varying function of s, (13) is probably not ruled out by the present data for $p + p \to \pi^\pm + X$. The data presented by Dao *et al.* in support of their scaling hypothesis involved only negative pions (with a possible 10% K^- contamination). We would not expect (13) to be valid for $p + p \to p + X$ because of the prominent diffraction peak, but since the proton multiplicity varies much more slowly than the pion multiplicity for $\sqrt{s}$ between 8 and 63 GeV, a definitive statement on this point can not yet be made either.

We conclude that if the new scaling hypothesis remains valid at asymptotic energies, a contradiction with either Feynman scaling or KNO scaling would result. The fit to the data by Svensson and Sollin[2] would involve no such contradiction and thus might be considered preferable on that ground. More data, and a more careful analysis of present data, particularly for $\sqrt{s} > 25$ GeV, would be useful in resolving this question.

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[1]F. T. Dao, R. Hanft, J. Lach, E. Malamud, F. Nezrick, V. Davidson, A. Firestone, D. Lam, F. Nagy, C. Peck, A. Sheng, R. Poster, P. Schlein, W. Slater, and A. Dzierba, Phys. Rev. Lett. 33, 389 (1974); see also D. B. Lichtenberg, Phys. Rev. Lett. 33, 1520(C) (1974).

[2]B. E. Y. Svensson and L. Sollin, Phys. Rev. Lett. 34, 1199 (1975).

[3]Most of the relevant data can be found in A. M. Rossi *et al.*, Nucl. Phys. B84, 269 (1975); J. Whitmore, Phys. Rep. 10C, 273 (1974); L. Foá, Phys. Rep. 22C, 1 (1975); and the extensive references to be found within.

[4]See, for example, F. T. Dao, in *Particles and Fields —1974*, AIP Conference Proceedings No. 23, edited by C. E. Carlson (American Institute of Physics, New York, 1975).

[5]D. Sivers, Phys. Rev. D 8, 4004 (1973).

[6]It is interesting to note that this is not yet satisfied by the 300-GeV data of Dao *et al.* For the high multiplicity events, $n = 20-26$, the longitudinal momentum must be shared by too many particles for $\langle p_L \rangle_n$ to be large. In the case of low multiplicity events, $n \leq 8$, a large contribution comes from the diffractive component where most of the energy is carried off by the leading positive particles. $\langle p_L \rangle_n$ for the negative tracks measured by Dao *et al.* is thus still small.

[7]A. Ballestrero and E. Predazzi, Nuovo Cimento 13A, 470 (1973).

[8]We have previously noted that the average values of certain quantities in inclusive reactions are uniquely determined by the energy-momentum conservation sum rules [R. J. Yaes, Phys. Rev. D 7, 2161 (1973), and Lett. Nuovo Cimento 4, 611 (1972)].

[9]See W. Ernst and I. Schmitt, Universität Bielefeld Report No. Bi-75/26 (to be published).

[10]H. Bøggild *et al.* (British-Scandinavian Collaboration), in Proceedings of the International Conference on High Energy Physics, Palermo, Italy, June 1975 (to be published); L. Foá, *ibid.*

Comments on "New Method to Measure Structural Disorder: Application to GeO_2 Glass"

J. Karle and J. H. Konnert
Laboratory for the Structure of Matter, Naval Research Laboratory, Washington, D. C. 20375
(Received 2 October 1975)

We discuss recent extended x-ray-absorption fine-structure work of Sayers, Stern, and Lytle. We note that their use of distance variation to rule out crystallinity is not valid and that the variation they find is also available from diffraction data. Finally, contrary to statements of Sayers, Stern, and Lytle, we have not described glasses in terms of microcrystals. Our radial distribution analysis reveals ordering in some glasses which resembles in part but cannot be equated to that in crystals.

In a paper with the above title,[1] Sayers, Stern, and Lytle (SSL) have presented a measurement, based on their interesting new extended x-ray-absorption fine-structure (EXAFS) technique, which is interpreted as eliminating the possibility of crystallinity in a material. This result is then used to infer that certain conclusions that we have allegedly drawn concerning the structure of GeO_2 glass are not correct. In fact, their description of our conclusions is quite erroneous. For example, we have neither concluded nor presented evidence for the conclusion that the glasses we have examined are composed of microscopically small crystals, microcrystals. In actuality, the measurements of SSL are in agreement with our conclusions, although they do not afford strong additional supporting evidence. We were aware of those values from our diffraction experiments and they concern only a small portion of the radial distribution function representing the distribution of distances.

It should be noted that the EXAFS experimental information of SSL on the α-quartz and the glassy forms of GeO_2 was limited to distance distributions for the bonded Ge-O and the smallest Ge-Ge distances. It was found that the distributions were essentially the same for the Ge-O distances. For the Ge-Ge distances, however, the distribution in the glass was much broader than in the α-quartz form. The excess root-mean-square deviation was reported to be 0.077 ± 0.014 Å. This result was interpreted by SSL (Ref. 1, p. 587) to imply that for crystal formation to be consistent with their experiment, "each microcrystalline region is about 3.8 Å in radius, only 1.8 Å of which is undistorted." In fact, it is easy to show that the excess distribution of the Ge-Ge distance can be consistent with any degree of ordering ranging from a continuous random network of tetrahedra to macroscopic crystalline regions. The distribution of the Ge-Ge distance found by SSL is in fine agreement with the measurements that we made and the conclusions that we drew. We found that ordered regions in GeO_2 glass extend to 15–20 Å but do not resemble the bonding topology of α-quartz. They do resemble the bonding topology, however, of a tridymite form which could indeed have the Ge-Ge distribution found for the glass by SSL. Since the parameters of the unit cell (space group $F1$)