E_7 as a Universal Gauge Group

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The exceptional group E_7 is proposed as a spontaneously broken gauge group underlying a unified field theory of the strong, electromagnetic, and weak interactions. The resulting model, which has only one gauge coupling constant and only one multiplet of elementary fermions, is shown to be compatible with the phenomenology of weak and electromagnetic interactions, including e^+e^- annihilation and high-energy neutrino scattering.

We propose a spontaneously broken gauge-theory model for the unification[1] of strong, electromagnetic, and weak interactions, which has the following attractive features. (1) The gauge group into which $\mathrm{SU}^{\mathrm{color}}(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$ is embedded is simple. As a consequence, there is only one coupling constant for all three interactions. (2) All the elementary fermions (leptons and quarks) of the theory belong to only one multiplet of the gauge group. (3) The relation which arises between the Weinberg angle and the strong-coupling constant is very well satisfied by experiment. (4) In addition to the well-established properties of the weak and electromagnetic interactions, the model can also reproduce the more recent experimental data on e^+e^- annihilation and high-energy neutrino scattering. (5) Parity is violated through spontaneous symmetry breakdown only. The weak current automatically has the right-handed pieces which have been invoked[2] to explain the $\Delta I=\frac{1}{2}$ rule.

The gauge group is the exceptional Lie group E_7. Since this group has only real and pseudoreal representations, the theory is free of Adler-Bell-Jackiw anomalies. $\mathrm{SU}(6)\otimes\mathrm{SU}(3)$ is a maximal subgroup of E_7. The SU(3) factor is an automorphism group of the octonion algebra, when E_7 is represented by matrices over octonions. For this reason, the SU(3) subgroup is believed to remain unbroken in the spontaneous breakdown of symmetry and is identified with the quark color group whose gauge vector bosons (the gluons) mediate the strong interactions[3]. We assume that the only observables of the theory are color-singlet states. All the elementary fermions of the theory belong to one multiplet, which under E_7 transforms as the 56-dimensional spinor representation and under the Lorentz group transforms as a left-handed, two-component spinor. The gauge vector bosons belong to the adjoint representation which is 133-dimensional. The $\mathrm{SU}(6)\otimes\mathrm{SU}^{\mathrm{color}}$ decompositions of these two representations are the following:

$$(\underline{56})=(\underline{20},\underline{1})+(\underline{6},\underline{3})+(\underline{6}^*,\underline{3}^*),\qquad(\underline{133})=(\underline{35},\underline{1})+(\underline{15},\underline{3})+(\underline{15},\underline{3}^*)+(\underline{1},\underline{8}).\tag{1}$$

The leptons belong to the $(\underline{20},\underline{1})$ submultiplet and will be represented by the totally antisymmetric tensor L^{abc} $(a,b,c=1,2,\ldots,6)$. The left-handed parts of the quarks belong to the $(\underline{6},\underline{3})$ submultiplet and will be denoted by Q_{Lai} $(i=1,2,3)$, while the left-handed parts of the antiquarks belong to the $(\underline{6}^*,\underline{3}^*)$ and will be denoted by $\hat{Q}_R{}^{ai}$ [we use the notation $\hat{\psi}_R=(\psi^c)_L$]. The gauge multiplet $(\underline{133})$ contains the $\underline{8}$ gluons, 35 color-singlet gauge fields that include the photon and the weak vector bosons, plus 90 leptoquark vector bosons which, in second order, mediate baryon-number-nonconserving decays and which consequently must be super heavy.

The electromagnetic current is given in terms of the SU(6) currents by

$$j_\mu=J_\mu{}^3+3^{-1/2}J_\mu{}^8+J_\mu{}'^3+3^{-1/2}J_\mu{}'^8,\tag{2}$$

where the unprimed generators act on the first three quarks and the primed ones on the last three. The charges of the quarks are thus $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$. The charges of the twenty two-component leptons are fixed as well: Four of them have charge $+1$, four have charge -1, and the twelve others are neutral. It is thus predicted that in addition to the electron and the muon there exist two other leptons with unit charge, one of which has presumably already shown itself at SPEAR.[4] The weak charged current is the sum of two charged SU(6) currents,

$$j_\mu^{+} = J_{\mu 2}^{1} + J_{\mu 3}^{4} = Q_{L1i}^{\dagger}\sigma_\mu Q_{L2i} + Q_{L4i}^{\dagger}\sigma_\mu Q_{L3i} + Q_{R1i}^{\dagger}\bar{\sigma}_\mu Q_{R2i} + Q_{R4i}^{\dagger}\bar{\sigma}_\mu Q_{R3i} + \tfrac{1}{2}(L^{2ab})^{\dagger}\sigma_\mu L^{1ab} + \tfrac{1}{2}(L^{3ab})^{\dagger}\sigma_\mu L^{4ab}, \tag{3}$$

where $\sigma_\mu = (1, \vec{\sigma})$ and $\bar{\sigma}_\mu = (1, -\vec{\sigma})$. Among the leptons, there are four SU(2) doublets and two SU(2) triplets. The left-handed parts of the muon and its neutrino should be placed in an SU(2) doublet in order that the muon neutrino carry a neutral current. The $Q_{ai} = (Q_{Lai}, Q_{Rai})$ are not the physical quark fields which we denote by $q_{ai} = (q_{Lai}, q_{Rai})$ but are related to them by the $U_L(6) \otimes U_R(6)$ transformation

$$Q = \left(U_L \frac{1+\gamma_5}{2} + U_R \frac{1-\gamma_5}{2}\right) q, \tag{4}$$

which makes the quark mass matrix γ_5-free and diagonal. Similarly, a unitary transformation relates the L^{abc} to the physical lepton fields.[5] The mass matrix itself arises from vacuum expectation values of the Higgs scalar and pseudoscalar fields.

If we constrain U_L and U_R to be such that CP is conserved,[5] that the weak charged current is purely $V-A$ among the light quarks (q_1, q_2, and q_3), that its $\Delta S = 1$ piece is suppressed by a factor $\tan\theta$ (θ is the Cabibbo angle) relative to its $\Delta S = 0$ piece, and that the corresponding neutral current $j_\mu^{3} = \frac{1}{2}\int d^3x \times [j_0^{+}(x), j_\mu^{-}]$ has no $\Delta S \neq 0$ piece (in 0th order), then the hadronic part of j_μ^{+} must have the following general form:

$$\begin{aligned} j_0^{+\,\text{hadronic}} = {} & q_{L1}^{\dagger}[\cos\gamma(\cos\theta q_{L2} + \sin\theta q_{L3}) + \sin\gamma(\cos\omega q_{L5} + \sin\omega q_{L6})] \\ & + q_{L4}^{\dagger}[\cos\gamma'(\cos\theta' q_{L2} + \sin\theta' q_{L3}) + \sin\gamma'(\cos\omega' q_{L5} + \sin\omega' q_{L6})] \\ & + q_{R1}^{\dagger}(\cos\varphi q_{R5} + \sin\varphi q_{R6}) \\ & + q_{R4}^{\dagger}[\cos\beta'(\cos\rho' q_{R2} + \sin\rho' q_{R3}) + \sin\beta'(-\sin\varphi q_{R5} + \cos\varphi q_{R6})], \end{aligned} \tag{5}$$

where the various angles must satisfy the constraints

$$\rho' = 0 \text{ or } \rho' = \pi/2, \tag{6a}$$

$$\cos\gamma \cos\gamma' \cos(\theta - \theta') + \sin\gamma \sin\gamma' \cos(\omega - \omega') = 0, \tag{6b}$$

$$\cos^2\gamma \sin 2\theta + \cos^2\gamma' \sin 2\theta' = 0. \tag{6c}$$

Here (6b) follows from the unitary character of U_L. If $\cos\gamma$ is appreciably different from 1, a γ rotation must also be applied to the leptons in order to preserve lepton-hadron universality.

Let us write the $SU(3)^{\text{color}} \otimes SU(2) \otimes U(1)$ part of the gauge interaction Lagrangian:

$$L_{\text{int}} = g'' \sum_{\alpha=1}^{8} \bar{q}_i \gamma^\mu (\tfrac{1}{2}\lambda^\alpha)_i^{\ j} q_j G_\mu^{\alpha} + g \vec{j}_\mu \cdot \vec{A}^\mu + g' j_\mu^{0} B^\mu, \tag{7}$$

where $j_\mu^0 = j_\mu^3 - j_\mu$. The Weinberg angle θ_W is defined[6] by $\tan\theta_W = g'/g$. We emphasize however that since the weak currents in Eq. (7) are different from those in the original Weinberg-Salam model, the value for θ_W obtained by comparing Eq. (7) with the experimental data will be different as well. In the E_7 model, the unrenormalized value of the Weinberg angle is found to be $\sin^2\theta_W^{0} = \frac{3}{4}$. However, renormalization effects due to the presence of superheavy vector bosons are appreciable and must be taken into account. Using to this effect the method and assumptions of Georgi, Quinn, and Weinberg,[7] we find the following relations among the renormalized coupling constants:

$$\ln\left(\frac{M}{\mu}\right) = \frac{3(4\pi)^2}{22} \frac{1}{c^2 + 3c'^2}\left(\frac{1}{e^2} - \frac{c^2 + c'^2}{g''^2}\right), \tag{8a}$$

$$\sin^2\theta_W = \frac{c^2}{c^2 + 3c'^2}\left(1 + 2c'^2 \frac{e^2}{g''^2}\right), \tag{8b}$$

where, in the case of the E_7 model, $c^2 = 2$ and $c'^2 = \frac{2}{3}$, μ is the renormalization point, and M is the order of magnitude of the superheavy vector-boson mass. Using the value $g''^2/4\pi = 0.2$ at $\mu = 3$ GeV from a fit[8] of the ψ decay rate, we find $\sin^2\theta_W = 0.52$ and $M = 3\times 10^{23}$ GeV. This value is consistent with ex-

TABLE I. Increments in neutrino slope parameters.

Thresholds	ΔA_ν	ΔB_ν	$\Delta A_{\bar\nu}$	$\Delta B_{\bar\nu}$
q_1	$\cos^2\gamma\cos^2\theta$	0	0	0
q_2	0	0	0	$\cos^2\gamma\cos^2\theta$
q_3	0	0	0	$\cos^2\gamma\sin^2\theta$
q_4	$\cos^2\gamma'\cos^2\theta'$	$\cos^2\beta'\cos^2\rho'$	0	0
q_5	0	0	$\cos^2\varphi$	$\sin^2\gamma\cos^2\omega$
q_6	0	0	$\sin^2\varphi$	$\sin^2\gamma\sin^2\omega$

periment as shown below. The proton lifetime is then of the order of magnitude of 10^{63} years. It appears quite plausible—as had been suggested before[7]—that the same Higgs vacuum expectation values generate both the gravitational constant ($G^{-1/2}=1.2\times 10^{19}$ GeV) and the masses of the leptoquark vector bosons.

With its six quarks and two heavy charged leptons, the model is capable of reproducing the $R_{e^+e^-}$ ratio measured at SPEAR. For example, $R=4.67$ at energies where five quarks and one heavy lepton contribute. Assuming scaling and neglecting the contribution from quarks "in the sea," we find for high-energy neutrino scattering on $I_z=0$ nuclear targets

$$\frac{d^2\sigma}{dx\,dy}(\nu^{(\mu)}\to\mu^-)=\frac{G^2mE}{\pi}f(x)[A_\nu+B_\nu(1-y)^2]\sec^2\gamma, \tag{9a}$$

$$\frac{d^2\sigma}{dx\,dy}(\bar\nu^{(\mu)}\to\mu^+)=\frac{G^2mE}{\pi}f(x)[A_{\bar\nu}+B_{\bar\nu}(1-y)^2]\sec^2\gamma, \tag{9b}$$

where x and y are the usual scaling variables and where the slope parameters increase at each new threshold by the amounts given in Table I. We thus find that, independently of any particular choice of the weak interaction angles, q_5 and/or q_6 charm will be produced in antinuetrino beams at sufficiently high energies; the dimuon events seen by Benvenuti *et al.*[9] in the $\bar\nu^{(\mu)}$ beam might be an indication that the q_5 and/or q_6 threshold has already been passed.

For further comparison with neutrino scattering data, we would like to consider three particular cases of Eqs. (5) and (6):

$$j_0{}^+=q_{L1}{}^\dagger q_{L2}(\theta)+q_{L4}{}^\dagger q_{L3}(\theta)+q_{R1}{}^\dagger q_{R5}(\varphi)+q_{R4}{}^\dagger[\cos\beta' q_{R2}+\sin\beta' q_{R6}(\varphi)], \tag{10a}$$

$$j_0{}^+=q_{L1}{}^\dagger q_{L2}(\theta)+q_{L4}{}^\dagger q_{L3}(\theta)+q_{R1}{}^\dagger q_{R5}(\varphi)+q_{R4}{}^\dagger[\cos\beta' q_{R3}+\sin\beta' q_{R6}(\varphi)], \tag{10b}$$

$$\begin{aligned} j_0{}^+ =& (q_{L1}{}^\dagger\cos\alpha/2+q_{L4}{}^\dagger\sin\alpha/2)(q_{L2}\cos\alpha/2+q_{L5}\sin\alpha/2)\\ &+(q_{L4}{}^\dagger\cos\alpha/2-q_{L1}{}^\dagger\sin\alpha/2)(q_{L6}\cos\alpha/2-q_{L3}\sin\alpha/2)\\ &+q_{R1}{}^\dagger q_{R5}(\varphi)+q_{R4}{}^\dagger[\cos\beta' q_{R3}+\sin\beta' q_{R6}(\varphi)], \end{aligned} \tag{10c}$$

with $\tan^2\alpha/2=\tan\theta_c$. These currents are on lines suggested earlier in Ref. 2 and by Glashow, Iliopoulos, and Maiani[10] for current (10a); Ref. 10, Wilczek *et al.*,[11] and Fritzsch and Minkowski[12] for current (10b); and Gürsey, Ramond, and Sikivie[13] for current (10c). For each of the three currents we obtain a $\Delta I=\frac{1}{2}$ rule provided $\cos\beta'$ is comparable to one. For choice (10b), q_4 charmed particles decay preferentially to strange-hadron final states, while for the other two choices, q_4 charmed particles decay about equally much to nonstrange as to strange hadron final states. Currents (10a) and (10c) possibly induce too large $\Delta S\neq 0$ neutral currents in higher orders.[11] Current (10b) does not have this drawback. Furthermore current (10a) is not consistent with current algebra.[14] The amount of q_4 charm excitation by a high-energy neutrino beam, as estimated from the rate of dimuon events, is predicted too low by current (10b) but is correctly predicted by currents (10a) and (10c).

The ratios $R_\nu=\sigma_{\rm tot}(\gamma^{(\mu)}\to\nu^{(\mu)})/\sigma_{\rm tot}(\nu^\mu\to\mu^-)$ and $R_{\bar\nu}=\sigma_{\rm tot}(\bar\nu^{(\mu)}\to\bar\nu^{(\mu)})/\sigma_{\rm tot}(\bar\nu^\mu\to\mu^+)$ have been calculated as functions of $\theta_{\rm W}$ for each of the three currents given in Eq. (10) and plotted in Fig. 1 together with the experimental data points.[15] Good agreement is found for the predicted value of $\sin^2\theta_{\rm W}$.

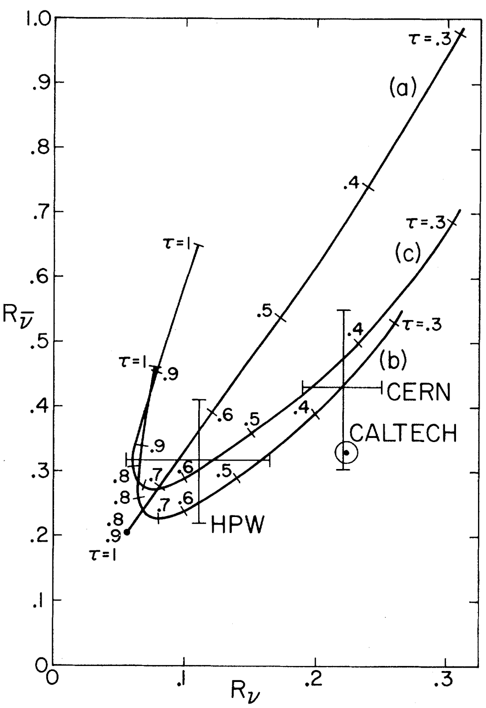

FIG. 1. Plot of R_ν and $R_{\bar{\nu}}$ as functions of $\tau = \sin^2\theta_W$ for each of the three currents given in Eq. (10). Data points are from Ref. 15.

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Note added.—Since this paper was submitted there has been new evidence about the possible existence of right-handed currents that also involve new quarks. This is discussed by M. Barnett (Harvard preprint) and A. de Rújula (invited talk at the 1976 Coral Gables Conference).

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