Radiative Decays of the Vector Mesons and the Meson Mixing Angles*

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A general SU(3) analysis of vector meson and pseudoscalar meson radiative decays is carried out. Two solutions are obtained corresponding either to an inverse-square mass formula or to a linear mass formula for vector mesons that lead to reasonable agreement with most of the radiative decays.

Recent measurements[1-3] of vector meson radiative decays have raised the question of the validity of the quark model and SU(3) predictions for these processes. The quark-model prediction for example, is 3 times the value of the recent measurement.[1] Since it is generally believed[4] that SU(3) predictions should be accurate to within 20 to 30%, it is important to determine whether in fact SU(3) symmetry is also badly violated.[5]

We will begin with a general SU(3) description of the radiative decays of the vector (V) and pseudoscalar (P) mesons,

$$V \rightarrow P + \gamma \tag{1}$$

and

$$P \rightarrow V + \gamma . \tag{2}$$

If we denote the SU(3) generating currents by $J_i{}^\lambda(x)$ ($i = 1, \ldots, 8$), then the electromagnetic current is given by $J_{\mathrm{em}}{}^\lambda = J_3{}^\lambda + (1/\sqrt{3})J_8{}^\lambda$ and the SU(3) structure of the matrix elements relevant to the decays (1) and (2) is[6]

$$\begin{aligned} \langle V_i | J_j | P_k \rangle &= g d_{ijk}, \\ \langle V_i | J_j | P_0 \rangle &= f d_{0ij}, \\ \langle V_0 | J_j | P_i \rangle &= f' d_{0ij}, \end{aligned} \tag{3}$$

where $i, j, k = 1, \ldots, 8$, $d_{0ij} = \sqrt{\tfrac{2}{3}}\,\delta_{ij}$, and V_i, V_0 (P_i, P_0) represent the vector (pseudoscalar) meson nonets. A complete description of the decays (1) and (2) requires, in addition to the three parameters introduced in Eq. (3), the vector and pseudoscalar mixing angles, θ_V and θ_P, respectively.

In a first attempt to find a solution, the parameter g was determined by a minimum-χ^2 fit to the $\rho^- \rightarrow \pi^- \gamma$ and $K^{*0} \rightarrow K^0 \gamma$ decay rates. The vector mixing angle and f' were then determined from the $\varphi \rightarrow \pi\gamma$ and $\omega \rightarrow \pi\gamma$ decay rates. This left θ_P and f to be determined from the $\varphi \rightarrow \eta\gamma$ decay rate. Thus, the pseudoscalar mixing angle could not be obtained uniquely, and was thereby assumed to be $-10°$, as predicted from the Gell-Mann–Okubo quadratic mass formula.

It was found that the parameters had the values $g = 0.476\ \mathrm{GeV}^{-1}$, $f = 0.769\ \mathrm{GeV}^{-1}$, and $f' = 0.889\ \mathrm{GeV}^{-1}$. The vector mixing angle was found to be 24°. This is very close to the value of 28°, which can be obtained from the inverse-square mass formula[7] for the vector mesons,

$$\begin{aligned} &\tfrac{1}{3}(4 m_{K^*}{}^{-2} - m_\rho{}^{-2}) \\ &\qquad = m_\varphi{}^{-2} \cos^2\theta_V + m_\omega{}^{-2} \sin^2\theta_V . \end{aligned} \tag{4}$$

This last mass formula is suggested by the spectral-function sum rules.[8] There are, of course, ambiguities in the signs of the amplitudes of the above decay processes. It was found that different choices of sign in the amplitudes led to parameters which violated the upper bounds on other decay rates. The results of this solution, and the experimental data used to determine the parameters, are summarized in Table I.

It is of interest to consider the predicted radiative decays which are based on mixing angles that arise from linear mass formulas for both the vector and the pseudoscalar mesons. The linear mass formulas

$$\tfrac{1}{3}(4 m_{K^*} - m_\rho) = m_\varphi \cos^2\theta_V + m_\omega \sin^2\theta_V \tag{5}$$

and

$$\tfrac{1}{3}(4 m_K - m_\pi) = m_\eta \cos^2\theta_P + m_{\eta'} \sin^2\theta_P \tag{6}$$

imply $\theta_V = 37°$ and $\theta_P = -24°$. With these values and the widths $\Gamma(\varphi \rightarrow \pi\gamma)$, $\Gamma(\omega \rightarrow \pi\gamma)$, and $\Gamma(\varphi \rightarrow \eta\gamma)$ as input, we obtain the results shown in Table II. The coupling constants turn out to be $g = 0.746\ \mathrm{GeV}^{-1}$, $f = 0.876\ \mathrm{GeV}^{-1}$, and $f' = 0.790$

TABLE I. Radiative decay widths predicted by SU(3) symmetry. These widths were calculated assuming $\theta_P = -10°$. The remaining parameters are predicted to have the values $g = 0.476\ \mathrm{GeV}^{-1}$, $f = 0.769\ \mathrm{GeV}^{-1}$, $f' = 0.889\ \mathrm{GeV}^{-1}$, and $\theta_V = 24°$.

Decay	Theoretical width (keV)	Experimental width (keV)
$\rho \rightarrow \pi\gamma$	35	35 ± 10[a]
$\rho \rightarrow \eta\gamma$	26	<160[b]
$K^{*+} \rightarrow K^{+}\gamma$	20	<80[c]
$K^{*0} \rightarrow K^{0}\gamma$	78	75 ± 35[d]
$\omega \rightarrow \pi\gamma$	870	870 ± 61[c]
$\omega \rightarrow \eta\gamma$	24	<50[c]
$\varphi \rightarrow \pi\gamma$	6.5	6.50 ± 1.94[c]
$\varphi \rightarrow \eta\gamma$	81	81 ± 32[c]
$\varphi \rightarrow X^{0}\gamma$	0.84	...
$X^{0} \rightarrow \rho\gamma$	130	<270[c]
$X^{0} \rightarrow \omega\gamma$	2.6	<80[c]

[a] See Ref. 1.
[b] See Ref. 9.
[c] See Ref. 10.
[d] See Ref. 2.

GeV^{-1}. If we consider the conditions imposed by the Okubo-Zweig-Iizuka[11] rule on radiative transitions, then we find that the matrix elements

$$\langle V_8 + \sqrt{2} V_0 | (J_3 + (1/\sqrt{3}) J_8) | P_8 - (1/\sqrt{2}) P_0 \rangle$$

and

$$\langle V_8 - (1/\sqrt{2}) V_0 | (J_3 + (1/\sqrt{3}) J_8 | P_8 + \sqrt{2} P_0 \rangle$$

must vanish, which implies

$$g = f = f'. \qquad (7)$$

The continuity of quark lines is a qualitative rule for understanding the suppression of $\varphi \rightarrow 3\pi$ decay. The approximate equality of the parameters g, f, and f' found in the linear-mass-formula case suggests that a minimum-χ^2 fit to the decays $\omega \rightarrow \pi\gamma$, $\varphi \rightarrow \pi\gamma$, and $\varphi \rightarrow \eta\gamma$ with Eq. (7) imposed should yield a reasonable result. This fit is summarized in Table II.

There is evidence[12] in favor of θ_P lying in the range $-10°$ to $-24°$. It is significant that this mixing angle also yields consistent results for the decays $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, and $X^0 \rightarrow \gamma\gamma$. An SU(3) analysis of these two-photon decays shows that they should depend on three parameters, singlet and octet coupling constants similar to g and f in Eq. (3) and θ_P. If these coupling constants are determined from the $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ decay rates, the $X^0 \rightarrow \gamma\gamma$ decay rate can be predicted. In particular, using $\Gamma(\pi^0 \rightarrow \gamma\gamma) = 8.02 \pm 0.42$ eV[13] and $\Gamma(\eta \rightarrow \gamma\gamma) = 0.324 \pm 0.046$ keV,[14] we predict for $\theta_P = -10°$ the value $\Gamma(X^0 \rightarrow \gamma\gamma) = 3.93$ keV and for $\theta_P = -24°$ the value $\Gamma(X^0 \rightarrow \gamma\gamma) = 0.38$ keV. These compare favorably with the experimental upper bound $\Gamma(X^0 \rightarrow \gamma\gamma) < 19$ keV.[15] It is important to emphasize that these predictions as well as those presented in Tables I and II are based only on SU(3) and do not make use of vector-meson dominance or the quark model.

The results in Table II agree well with the experimental radiative decay data with the exception of the $\rho \rightarrow \pi\gamma$ and $K^{*0} \rightarrow K^0\gamma$ decays. The predicted values for these decays are fairly close to those predicted[16] by the mixing angles of the quad-

TABLE II. Radiative decay widths predicted with $\theta_V = 37°$ and $\theta_P = -24°$, as predicted by the linear mass formula. The coupling constants are determined to be $g = 0.746\ \mathrm{GeV}^{-1}$, $f = 0.876\ \mathrm{GeV}^{-1}$, and $f' = 0.790\ \mathrm{GeV}^{-1}$ in the unconstrained fit. The constrained fit has $g = f = f' = 0.783\ \mathrm{GeV}^{-1}$.

Decay	Theoretical width (keV) Unconstrained	Theoretical width (keV) Constrained	Experimental width (keV)
$\rho \rightarrow \pi\gamma$	85	93	35 ± 10[a]
$\rho \rightarrow \eta\gamma$	84	82	<160[b]
$K^{*+} \rightarrow K^{+}\gamma$	48	53	<80[c]
$K^{*0} \rightarrow K^{0}\gamma$	190	210	75 ± 35[d]
$\omega \rightarrow \pi\gamma$	870	890	870 ± 61[c]
$\omega \rightarrow \eta\gamma$	12	10	<50[c]
$\varphi \rightarrow \pi\gamma$	6.5	1.9	6.50 ± 1.94[c]
$\varphi \rightarrow \eta\gamma$	80	95	81 ± 32[c]
$\varphi \rightarrow X^{0}\gamma$	1.3	1.1	...
$X^{0} \rightarrow \rho\gamma$	89	62	<270[c]
$X^{0} \rightarrow \omega\gamma$	8.9	6.9	<80[c]

[a] See Ref. 1.
[b] See Ref. 9.
[c] See Ref. 10.
[d] See Ref. 2.

ratic mass formula. However, the experimental errors quoted for these rates may be too conservative. For the $\rho^- \to \pi^- \gamma$ decay, in particular, the experimental analysis[17] cannot at present distinguish between the value 35 ± 10 keV, quoted in the literature, and a value of 80 ± 10 keV. The latter value would be in excellent agreement with that predicted here on SU(3) arguments, and also by simple vector-meson dominance[18] without SU(3). More experimental information on these decays is clearly required.

We can conclude from our results that there are solutions based on conventional SU(3) that are consistent with most of the new radiative decay data. In particular, the fact that one solution chooses linear mass formulas for the mesons and leads naturally to the Okubo-Zweig-Iizuka[11] quark-line rules is very interesting and deserved further study.

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$$[(2\pi)^3 4k_0 q_0]^{1/2} \langle V_i(k,\epsilon) | J_j^\lambda(0) | P_k(q) \rangle = g d_{ijk} \epsilon^{\lambda\mu\nu\rho} \epsilon_\mu k_\nu q_\rho,$$

etc., where k and ϵ are the four-momentum and polarization of the vector meson and q the momentum of the pseudoscalar meson. SU(3)-symmetry breaking is introduced by using physical phase space in calculating the widths, which are given by, e.g.,

$$\Gamma(\rho^- \to \pi^- \gamma) = \left(\frac{1}{96\pi}\right)\left(\frac{g}{3}\right)^2 \left(\frac{m_\rho^2 - m_\pi^2}{m_\rho}\right)^3$$

and

$$\Gamma(\chi^0 \to \rho^0 \gamma) = \left(\frac{1}{32\pi}\right)\left[g \frac{1}{\sqrt{3}} \sin\theta_P + f\left(\frac{2}{3}\right)^{1/2} \cos\theta_P\right]^2 \times \left(\frac{m_{\chi^0}^2 - m_\rho^2}{m_{\chi^0}}\right)^3$$

for decays of types (1) and (2), respectively.

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